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Research paper

FEM Formulation to Solve Hyperelastic Problem up to Large Strain

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Abstract

In this paper finite element method has been practiced for solving the hyperselastic problem upto large strain. Truss problem has been solved by using the finite element formulation. The aim of the paper is to discussed the algorithm used to solve the complex problem. For that FLagSHyp code (Finite Element Large Strain Hyperelastic Problem written by Javier bonet and Richard D .wood) algorithm has been discussed to understand the behaviour of hyperelastic material subjected to large strain. The result obtained is compared by the result obtained from abaqus software. Three models i.e. Odgen model, Incompressible neo-hookean model and Mooney-Rivlin model have been applied for the truss problem.

Keywords:*Finite Element, Large Deformation, Hyperelastic Program.*

1. Introduction

In linear and non-linear F.E analysis having large strains Incremental formulation of equation of motion are mainly in practice

Previously non-linear analyses were based on expansion of linear analyses and have been developed for particular applications. Initially the procedures were developed on direct basis for obtaining the results to the particular problems considered, but lately it has been found that nonlinear formulations based on principles of continuum mechanics can be efficiently applied.

The procedures are derived from the principle of virtual work. It has been found that, in theory, all the formulation give identical results. The only difference in the results arises because of the different definition of material behaviour

Now a day, analysis of non-linear behavior is done through software which provide better results. For that it become important to understand the fundamental continuum mechanics, non-linear finite element formulation and solution procedure. A number of books are available to provide background on this subject, for example the book of JN Reddy [1], Klaus Hackl and Mehndigoodarazi [2] explain the basic concepts of linear and non-linear continuum mechanics (stress and tensor, elasticity, Hooke's law, displacement functions. The book of Javier Bonet and Richard D. Wood [3], Morton E. Gurtin [4], J. Tensley Oden [5], explain the basic fundamentals related to different types of non-linearitieseg.material non-linearity, geometric non-linearity and contactnon-linearity; kinematics concepts viz deformation tensors, strains; kinetics concepts viz Cauchy stress, first and second piolakirchoff stress and different governing equations involved in it.

MeralBayraktar[6] studied the hyperelastic constitutive models for isotropic nonlinear material based on strain energy potentialeg. neo-hookean model, odgen model and mooney –rivlin model. The ability of model was compared in predicting uniaxial deformation states based on the data obtained from dumb-bell test specimen under the uniaxial loading condition. The result showed that the mooney-rivlin model, neo-hookean model and odgen model are more suitable for small, moderate and large deformation if the material is incompressible. E. Boudaia et al. [7] analysed the large deformation problem of ogdenhyperselastic model based on the finite element method follow updated lagrangian formulation.

Dr Mushin N. Hamza et al. [8] determined and compared the hyperelastic constitutive model for incompressible elastomers, vizOdgen model and mooney-rivlin model; the material parameter is obtained from using least square method and non-linear least square method respectively. The results obtained by the method have been compared with the behaviour of material obtained from treloar experiments. The results showed that the mooneyrivlin is most suitable when deformation is 100% while odgen is more appropriate when deformation exceed 100%.

Klaus-Jurgen Bathe et al. [9] derived and compared the two-finite element incremental formulations vizUpdatedlagrangian formulation and total lagrangian formulation for nonlinear static and dynamic analyses.

It has been observed that there are basically two different approaches used in FEM for solving large strain problem, first is the total lagrangian formulation and the second is updated lagrangian formulation.

The formulation include large strain and material non linearities. The formulations are listed below.

Total Lagrangian Formulation:

$$\int_{0_v}^{t} {}_0^t C_{ijrs}\, {}_0^t \varepsilon_{rs}\, \delta\, {}_0^t \varepsilon_{ij}\, {}_t^0 dv + \int_{0_v}^{t} {}_0^t S_{ij}\, \delta\, {}_0^t \eta_{ij} = {}^{t+\Delta t}_{0}R - \int_{0_v}^{t} {}_0^t S_{ij}\, \delta\, {}_0^t e_{ij}\, {}_t^0 dv \qquad (1)$$

Updated Lagrangian Formulation:

$$\int_{t_v} {}^{t+\Delta t}_{t}C_{ijrs}\, {}^{t+\Delta t}_{t}\mathcal{E}_{rs}\, \delta\, {}^{t+\Delta t}_{t}\mathcal{E}_{ij}\, {}^{t}_{0}dv + \int_{t}^{t+\Delta t} {}^{t}_{0}\zeta_{ij}\, \delta\, {}^{t+\Delta t}_{t}\eta_{ij}\, {}^{0}_{t}dv$$
$$= {}^{t+\Delta t}_{0}R - \int_{t_v}^{t+\Delta t} {}^{t}_{0}\zeta_{ij}\, \delta\, {}^{t+\Delta t}_{t}e_{ij}\, {}^{0}_{t}dv \quad (2)$$

Where $\mathbf{0}, \boldsymbol{t}\ \boldsymbol{and}\ \boldsymbol{t}+\Delta \boldsymbol{t}$ are time step/load level showing the reference configuration, new reference configuration and current configuration respectively; ${}^{t}_{0}\boldsymbol{C}_{\boldsymbol{ijrs}}, {}^{t+\Delta t}_{t}\boldsymbol{C}_{\boldsymbol{ijrs}}$ are the components of constitutive tensor at time $\boldsymbol{t}$ with reference in the configuration at time $\mathbf{0}$ and $\boldsymbol{t}$ respectively; ${}^{t}_{0}\mathcal{E}_{\boldsymbol{rs}}, {}^{t+\Delta t}_{t}\mathcal{E}_{\boldsymbol{rs}}$ components of strain increment tensor referred to configuration at $\mathbf{0}$ and $\boldsymbol{t}$ time respectively; ${}^{t}_{0}\boldsymbol{e}_{\boldsymbol{ij}}, {}^{t+\Delta t}_{t}\boldsymbol{e}_{\boldsymbol{ij}}$ are linear part of strain increment at time $\mathbf{0}$ and $\boldsymbol{t}$ respectively; ${}^{t}_{0}\boldsymbol{\eta}_{\boldsymbol{ij}}, {}^{t+\Delta t}_{t}\boldsymbol{\eta}_{\boldsymbol{ij}}$ are non linear part of strain imcrement at time $\mathbf{0}$ and $\boldsymbol{t}$ respectively; ${}^{t}_{0}\boldsymbol{S}_{\boldsymbol{ij}}$ are the components of second piola kirchoff stress tensor at time $\boldsymbol{t}$ with reference to configuration at time $\mathbf{0}$; ${}^{t}_{0}\boldsymbol{\zeta}_{\boldsymbol{ij}}$ are the components of Cauchy stress tencor at time $\boldsymbol{t}$ and ${}^{t+\Delta t}_{0}\boldsymbol{R}$ is the external virtual work expression corresponding to configuration at time $\boldsymbol{t}+\Delta\boldsymbol{t}$. Plate with hole and cantilever beam problems were considered. Result show that both give same results but UL formulation requires less time to calculate the element matrices

$$C = F^T F = U^2 \quad (5)$$

David S. Malkus, et al. [10] studied the application of penalty method for incompressible non-linear problem. They examine the tangent stiffness matrix K should be positive definite throughout the whole sequence of iteration for maintaining the stability of solution process for that the load increment and refinement strategies explained by oden [5].

2. Finite Element Procedure

In this procedure the problem is solved in incremental steps. It uses the iterative methods to give the results. The steps involved in the procedure

2.1. Deformation

Deformation defined as the change of the shape or size of the body from an initial configuration to a deformed configuration

2.1.1 Deformation Gradient tensor:

The deformation gradient tensor is defined as the ratio of line element emanating from position X in the reference configuration to the line element in the current configuration and expressed in mathematical form as.

$$F = dx/dX \quad (3)$$

In indicial notation the deformation gradient tensor is expressed as

$$F = \sum_{i,I=1}^{3} F_{iI}\, e_i \otimes E_I; \qquad F_{iI} = \frac{\partial x_i}{\partial X_I}; \quad i, I = 1,2,3 \quad (4)$$

Where e_i and E_I are the unit base vectors of initial geometry and deformed geometry respectively.

2.1.2 Right Cauchy-Green deformation tensor:

Where U is the right stretch tensor

It can also be expressed in terms of eigen values and eigen vectors as

$$C = \sum_{i=1}^{3} \lambda_i^2 N_i \otimes N_i \quad (6)$$

Where $\boldsymbol{\lambda_i^2}$ are the eigen values (principal value) of C and N_i are the eigen vectors (principal direction) of C

2.1.3 Left Cauchy-Green Deformation Tensor or Finger Tensor:

It has been defined as

$$B = FF^T \quad (7)$$

It can also be expressed in terms of eigen value and eigen vector as

$$b = \sum_{a=1}^{3} \lambda_a^2 n_a \otimes n_a \quad (8)$$

Where $\boldsymbol{\lambda_a^2}$ are the eigen values of b and n_a are the eigen vectors of b

2.2 Strain

Strain ($\mathcal{E}$) is a measure of deformation and has been defined as

$$\mathcal{E} = \frac{\vartheta(x - X)}{\vartheta x} = F' - I \quad (9)$$

Where I is the identity tensor

2.2.1 Engineering Strain:

It has been defined as

$$H = \frac{\Delta L}{L} = \frac{l - L}{L} \quad (10)$$

where L is the initial length and l is the final length.

2.2.2 Logarithmic Strain:

It is obtained by integrating the incremental strain $\int_{L}^{l} \frac{\delta l}{l}$ when the stretching is taking place from its initial length to its final length

$$L = \int_{L}^{l} \frac{\delta l}{l} = \ln\left(\frac{l}{L}\right) \quad (11)$$

2.2.3 Green Lagrangian Strain:

The green lagrangianstrain(E) deals with the large deformation and expressed as

$$E = \frac{1}{2}\left(\frac{l^2 - L^2}{L^2}\right) = \frac{1}{2}(C - I) \quad (12)$$

Where C is the left Cauchy green deformation tensor

2.2.4 Eulerianalmansi Strain:

The eulerianalmansistrain(e) is a symmetric positive definite spatial tensor and expressed as

$$e = \frac{1}{2}\left(\frac{l^2 - L^2}{l^2}\right) = \frac{1}{2}(I - b^{-1}) \quad (13)$$

Where b is the right Cauchy green deformation tensor.

2.2.5 Stretch ratio:

The stretch ratio or extension ratio(λ) is the ratio of final length and the initial length of the material line.

$$\lambda = \frac{l}{L} \quad (14)$$

2.3 Stress

Stress is defined as the intensity of internal resisting force developed at a point against the deformation caused due to load. It is generally expressed as

$$\sigma = \lim_{\Delta a \to 0} \frac{\Delta P}{\Delta a} \quad (15)$$

Where $\Delta \boldsymbol{a}$is the elemental area to normal $\boldsymbol{n}$ in the neighborhood of point$p$, $\Delta \boldsymbol{P}$ is the internal resisting force against the load on the elemental area.

2.4 Virtual Work Principle

The principle of virtual work is used to define the equilibrium of the body

$$\int_{S_t} \delta u^T T ds + \int_v \delta u^T f dv = \int \delta \mathcal{E}^T \sigma dv \quad (16)$$

For linear and nonlinear problem, we cannot obtain the solution directly as above equation give the solution in discrete time step so we use the lagrangian formulation [11] to solve the problem which integrate the unknown variables (displacement and strain) with reference to initial position or first position. If there is nonlinear problem, then it became necessary to linearize it after the lagrangian formulation

2.5 Formulation of Continuum Mechanics Incremental Equation

Assume that the kinematic and static variables solutions for all time steps have been solved, and the solution for next time t+Δt would be next. It has been noted that the solution for the next needed equilibrium position is complicated and would be applied continuously until the entire path of the solution has been solved.
The virtual work principle is applied to express the equilibrium of the body in the configuration at the time $+\Delta \boldsymbol{t}$.

$$\int_{t+\Delta t_v}^{0} {}^{t+\Delta t}_{0}\sigma_{ij}\ \delta\ {}_{t+\Delta t}^{0}e_{ij}\ {}^{t+\Delta t}_{0}dv = {}^{t+\Delta t}_{0}R \quad (17)$$

Where ${}^{t+\Delta t}_{0}\boldsymbol{R}$is the external virtual work expression and shown as,

$${}^{t+\Delta t}_{0}R = \int_{t+\Delta t_v}^{0} {}^{t+\Delta t}_{t+\Delta t}f_k^0\ \delta u_k^{0}\ {}^{t+\Delta t}_{0}dv + \int_{t+\Delta t_a}^{0} {}^{t+\Delta t}_{t+\Delta t}t_k^0\ \delta u_k\ {}^{t+\Delta t}_{0}dv \qquad k = 1,2, = 3 \quad (18)$$

Where${}^{t+\Delta t}_{0}\boldsymbol{f}_k^0$, ${}^{t+\Delta t}_{t+\Delta t}\boldsymbol{t}_k^0$are the body force vector and surface traction vector in the configuration at the time$\boldsymbol{t} + \Delta \boldsymbol{t}$respectively,$\boldsymbol{\delta u}_k^0$ is the changes in the current displacement components ${}^{t+\Delta t}_{0}\boldsymbol{u}_k$and $\boldsymbol{\delta}_{t+\Delta t}^{0}\boldsymbol{e}_{ij}$ are the corresponding variation in strains. as

$$\delta_{t+\Delta t}^{0}e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad (19)$$

The difficulty arises that the body configuration at time $\boldsymbol{t} + \boldsymbol{\Delta t}$ is unknown so the above equation cannot be solved directly. A solution can be attained by considering all the variables to a previously known equilibrium configuration for that two approaches have been used i.e. Total lagrangian formulation and Updatedlagrangian formulation

2.6 Equilibrium Iteration

The equilibrium equation of virtual workit need to iterate in each load step until the equations are satisfied to a required tolerance to maintain the solution stability. The equation used in the Total lagrangean formulation and updated lagrangean formulation are

$$({}_{0}^{t}K_L + {}_{0}^{t}K_{NL})\Delta u^i = {}^{t+\Delta t}_{0}R - {}^{t+\Delta t}_{0}F^{i-1} \quad (20)$$

$$({}_{t}^{t}K_L + {}_{t}^{t}K_{NL})\Delta u^i = {}^{t+\Delta t}_{0}R - {}^{t+\Delta t}_{t}F^{i-1} \quad (21)$$

Where ${}^{t+\Delta t}_{0}\boldsymbol{u}_0^{\boldsymbol{i}} = {}^{t+\Delta t}_{0}\boldsymbol{u}_0^{\boldsymbol{i-1}} + \Delta \boldsymbol{u}^{\boldsymbol{i}}$ and i is the number of iterations

3. Truss Problem

A four node truss of dimension 140mm ×1.414mm ×0.707mm has been chosen. The initial orientation angle is 45 degree. There is a hinge support on node 1 and roller support on node 2 which constrained the motion in x direction and allow to move in y direction as shown in fig.1 (all dimension in mm). The top node displacement is prescribed downward in increment of 1mm.

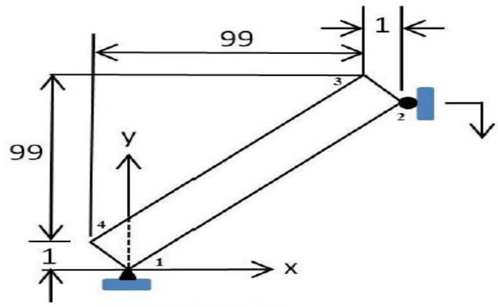

Fig. 1: Truss with prescribed displacement in downward direction

The problem is solved using 198 increments (1 mm increment in every increment step). A truss is divided into 700 elements as shown in fig.2. The convergence criterion is set to 1.e-10 in the code with a maximum 25 iterations in every step.

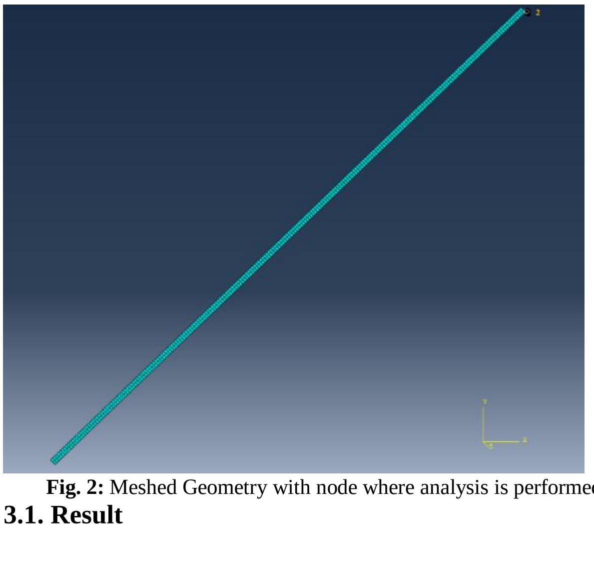

Fig. 2: Meshed Geometry with node where analysis is performed

3.1. Result

The vertical Reaction vs displacement graph of node 2 obtained from FLagSHyP code is shown in fig.3

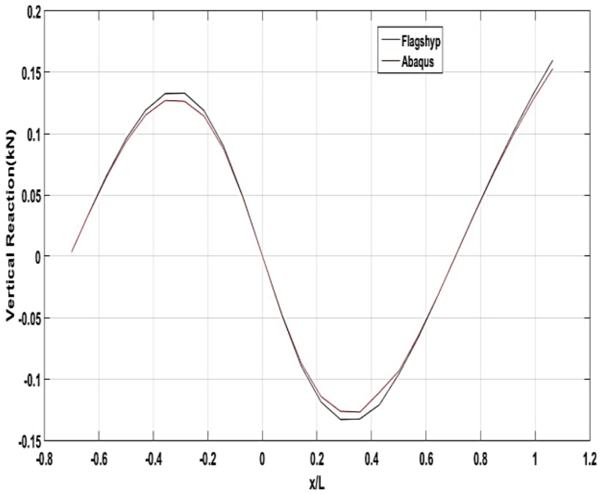

Fig. 3: Vertical Reaction v/s Displacement of node 2

In fig.3, x is the vertical position of node 2 in every increment step and L is the length of the truss. It can be observed that the truss exhibit non-linear snap through behaviour (also see fig.4). In this non-linear instability region the equilibrium path goes from one stable point (at which force is 0.1322KN) to another stable point(at which force is 0.1329 KN).

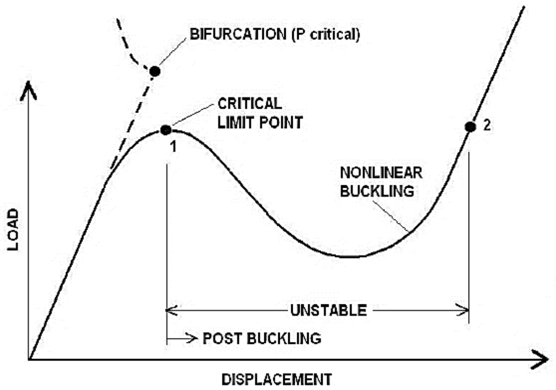

Fig. 4: Non-Linear Snap through behaviour of Truss [12]

4. Conclusion

The following conclusions are illustrated in this work.

1. Only material non-linearity is considered as the program is structured using Updated lagrangian formulation.
2. The results depict that the material is highly nonlinear elastic.
3. The non-linear snap through behaviour is obtained in truss problem

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