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# Optimization of Permanent Magnet Machines using Analytical Sub-Domain Model and Differential Evolution Algorithm

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## Abstract

This paper presents an optimization of permanent magnet synchronous machines (PMSMs) using Analytical Sub-domain Model together with Differential Evolution Algorithm (ASDEA). A three-phase, 6-slot/4-pole, surface-mounted PMSM is selected in the design with initial motor parameters which are determined from the sizing equations. Five motor parameters are to be optimized i.e. magnet thickness, airgap length, slot-opening width, magnet arc, and stator inner radius. Four objective functions are chosen i.e. to have lowest total harmonic distortions in the induced back-emf, lowest cogging torque, highest output torque and highest efficiency. Results show a good agreement between the analytical method and finite element analysis (FEA). The optimization of 6-slot/4-pole PMSM is further analyzed by comparing with other optimization algorithms i.e. Analytical Sub-domain with Genetic Algorithm (ASGA), and Analytical Sub-domain with Particle Swarm Optimization (ASPSO). It is observed that ASPSO has the fastest computing time compared to ASGA and ASDEA. Whereas ASDEA is approximately 50% faster than ASGA. The design work for PMSMs can potentially become faster without compromising the accuracy. While repetitive changes in motor parameters in finite element modeling could be avoided after applying this Analytical Sub-domain with Differential Evolution Algorithm.

***Keywords***: *analytical sub-domain; differential evolution; permanent magnet synchronous machines; back-emf; cogging torque.*

## 1. Introduction

The conventional approach to design an electrical machine using finite element method (FEM) normally requires intensive computational time to get high accuracy in the results [1-3]. Each time the electrical machine parameters are changed, the motor model should be rebuilt and computed again which creates unproductive or redundancy work. Consequently, it leads to longer electrical machine design process to be completed. In today industry, competition between product developers to have faster electrical machine design process is crucial. Therefore, an optimization tool is aptly appropriate to mitigate this problem [4-6]. Analytical sub-domain model (ASM) is capable to predict quite accurately the performance of permanent magnet synchronous machine (PMSM) i.e. cogging torque, induced back-emf, output torque, output power and unbalanced magnetic force with faster computational time compared to FEM [7, 8]. On the other hand, the optimization algorithm is able to reduce the redundancy work because it could iteratively and intelligently compute to yield the optimal electrical machine parameters under given objective functions. The combination of analytical sub-domain together with optimization algorithm should be able to shorten the computational time and eliminate the redundancy work issues. Genetic Algorithm (GA) can be used in the optimization due to its accuracy but it has longer computational time. On the other hand, Particle Swarm Optimization (PSO), has faster computational time, but exhibits lowest accuracy. Differential Evolution (DE) is another optimization algorithm that has better accuracy without sacrificing the computational time. Due to that, in this paper, DE is applied together with Analytical Sub-domain, which can be called as ASDEA.

In this paper, the machine parameters and dimensions of a three-phase, 6-slot/4-pole PMSM are first determined using the sizing equations as described in [9]. This 6-slot/4-pole PMSM has stator windings with 60 turns/phase, 18AWG wire gauge, single-tooth wound coils, silicon steels for the core, surface-mounted permanent magnet of *NdFeB* grade with radial magnetization, 5A peak sinusoidal phase current, 35mm stator outer radius, and 50mm axial length. Then, its torque output is validated with finite element software. i.e. Opera2D. Next, five motor parameters i.e. magnet arc, $\alpha$, slot-opening width, $b_o$, magnet thickness, $h_m$, airgap length, $l_g$, and stator inner radius, $R_{si}$ are selected to be optimized using ASDEA. Furthermore, the motor optimization is also conducted with Analytical Sub-domain with Genetic Algorithm (ASGA) and Analytical Sub-domain with Particle Swarm Optimization (ASPSO). Finally, the results obtained from these three optimization tools are compared for motor performance, allowing one to choose the best optimization algorithm that can be coupled with the Analytical Sub-domain.

## 2. Analytical Modeling and Sizing Equations

Characteristics and performance of PMSMs can be analytically evaluated if the magnetic field distributions particularly in the airgap region can be accurately estimated. In the past decade, many researchers have used quite successfully the analytical subdomain model (ASM) for this purpose. ASM applies separation of variables technique using the Laplace's equation in the airgap and slot opening regions; while using the quasi-Poisson's equation in the magnet and winding slot regions [10]. There are four regions in this modelling i.e. magnet, airgap, slot opening, and winding slot [11]. Using ASM, the magnetic field distributions, induced back-emf, electromagnetic torque, and winding inductances can be determined [7, 8]. However, the magnetic field solutions in ASM are derived and developed based on the following simplifying assumptions:(a) end-effects are neglected, meaning that the magnetic variables are independent of axial length *z*; (b) infinite permeability in the rotor and stator cores, indicating that saturation effect in the cores is ignored; (c) no electrical conductivity in the materials, where eddy-currents induced in the copper wires, rotor magnets and steel cores are null; (d) stator teeth are spoke shaped (radial slot boundaries); and (e) linear BH curve for the rotor magnets. The induced back-emf for each phase in PMSMs can be calculated by [12]

$$E_{rms} = \omega_e K_w N_t \phi \tag{1}$$

where $\omega_e$ is the electrical angular frequency, $K_w$ is the winding factor, $N_t$ is the winding turns per phase, and $\phi$ is the total flux entering in one pole-pitch which is determined using (2).

$$\phi = \frac{\pi R_{si} B_{avg} l_{axl}}{p_{pair}} \tag{2}$$

where $B_{avg}$ is the average flux density in the airgap, $R_{si}$ is the stator inner radius, , $p_{pair}$ is the pole-pair number, and $l_{axl}$ is the axial active length of the motor. Cogging torque in PMSMs is generated due to the interaction of rotor magnets with stator slots. Normally, cogging torque can be analytically estimated using (3) [8], where Maxwell stress tensor is applied in the middle of the airgap region.

$$T_{cogging} = \frac{R_{mid}^2 l_{axl}}{\mu_0} \int_0^{2\pi} B_r \cdot B_\theta d\theta \tag{3}$$

where $R_{mid}$ is the mid radius of the airgap, $\mu_0$ is the permeability of vacuum, $\theta$ is the rotor angular position in mechanical degree, while $B_r$ and $B_\theta$ are the radial and tangential components for the airgap flux density distributions respectively. Additionally, the efficiency of PMSMs is based on ratio of output power to the input power as shown in (4).

$$Efficiency(\%) = \frac{\sum_{k=1}^{n=3} E_{rms} I_{rms}}{\sum_{k=1}^{n=3} {I_{rms}}^2 R_\phi + \sum_{k=1}^{n=3} E_{rms} I_{rms}} \tag{4}$$

where $E_{rms}$ is the rms induced back-emf per phase, $I_{rms}$ is the rms phase current and $R_\phi$ is the phase winding resistance. The power loss is mainly contributed by the copper windings. While core losses, rotor losses and other losses are first assumed to be negligible for simplicity. The phase resistance is estimated as:

$$R_\phi = \frac{8 N_t \rho_{copper}}{\pi {D_c}^2} (l_{axl} + \pi R_{end}) \tag{5}$$

where $R_{end}$ is the average radius of the end-windings, $\rho_{copper}$ is the copper resistivity at 20°C, and $D_c$ is the diameter of copper wire. The number of winding turns per phase can be calculated using (6).

$$N_t = 2\pi \left[ \left( R_{so} - \frac{\pi R_{si} B_g}{B_{sat} N_s} \right)^2 - \left( R_{si} + \frac{(2\pi R_{si} - w_{tb} N_s) B_g}{2 B_{satx} N_s} \right)^2 \right] - \left[ R_{so} - R_{si} - \frac{\pi R_{si} B_g}{B_{sat} N_s} - \frac{(2\pi R_{si} - w_{tb} N_s) B_g}{2 B_{satx}} \right] \left( \frac{2\pi R_{si} B_g}{B_{sat}} \right) \tag{6}$$

$$w_{tb} = \frac{2\pi R_{si}}{N_s} \cdot \frac{B_g}{B_{sat}} \tag{7}$$

$$B_g = \frac{h_m}{h_m + \mu_r l_g} B_r \tag{8}$$

where $R_{so}$ is the stator outer radius, $R_{si}$ is the stator inner radius, $B_g$ is the average airgap flux density, $B_{sat}$ is the saturation flux density in the stator core, i.e. 1.6T, $N_s$ is the stator slots number, $w_{tb}$ is the tooth body width, $B_{satx}$ is the saturation flux density in the tooth tip region, i.e. 1.0T, $B_r$ is the magnet remanence, $h_m$ is the magnet thickness, $l_g$ is the airgap length, and $\mu_r$ is the relative permeability of the magnets.

## 3. Differential Evolution Algorithm

Differential evolution (DE) is one of the artificial intelligent algorithms that is originated from genetic algorithm family. It has four design processes, namely population, mutation, crossover, and selection [13]. It is a population based on optimization method. It begins with a randomly initiated population of *Np* D-dimensional real-valued parameter vectors. Each vector, also known as genome/chromosome, forms a candidate solution to the multi-dimensional optimization problem. The $i^{th}$ vector of the population at the current generation with each parameter has minimum and maximum bounds which can be represented in (9).

$$\vec{P}_{i,j} = \left[ p_{1,j}, p_{2,j}, p_{3,j}, \dots, p_{D,j} \right], \ \vec{p} \in R^D \tag{9}$$

After initialization, DE creates a donor vector $Q_{i,j}$ corresponding to each population member $P_{i,j}$ in the current generation through mutation. A mutation strategy as shown in (10) is applied, where $F_m$ is a constant parameter called mutation scale factor.

$$\vec{Q}_{i,j} = \vec{P}_{ro,j} + F_m \cdot \left( \vec{P}_{r1,j} - \vec{P}_{r2,j} \right) \tag{10}$$

To enhance the potential diversity of the population, a crossover operation is run after generating the donor vector through mutation. The common binomial crossover is applied on each *D* variables whenever a randomly generated number between 0 and 1 is less than or equal to the crossover constant $C_r$ value as shown in (11).

$$\vec{R}_{i,j} = r_{k,i,j} = \begin{cases} q_{k,i,j} \ if \ r_k \le C_r \ or \ k = k_{rand} \\ p_{k,i,j} \ if \ r_k > C_r \end{cases} \quad k = 1..n \tag{11}$$

The selection step is to determine whether the target or the trial vector survives to the next generation. The selection operation is shown in (12).

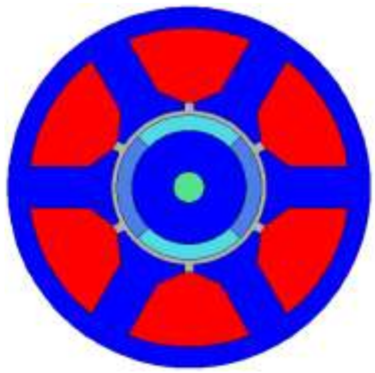
$$\vec{p}_{i,j+1} = \begin{cases} \vec{R}_{i,j} \; if \; f(\vec{R}_{i,j}) \leq f(\vec{P}_{i,j}) \\ \vec{P}_{i,j} \; if \; f(\vec{R}_{i,j}) > f(\vec{P}_{i,j}) \end{cases} \tag{12}$$

## 4. Results and Discussion

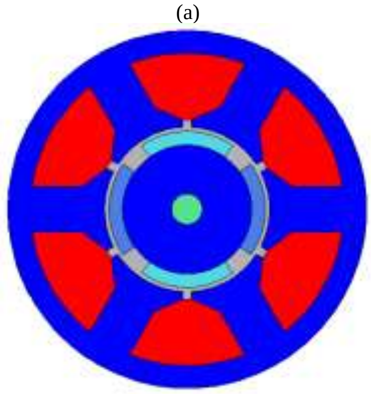
After having completed the optimization process and its calculation, it is observed that ASGA has the longest computational time compared to ASPSO and ASDEA respectively. While ASPSO has the fastest computational time, whereas ASDEA is about 50% faster than ASGA. It is really time consuming to design PMSM using FEA since one needs to build a new model for each change in the parameters. Assuming that each parameter is varied over ten points, then for this 6-slot/4-pole PMSM, it will require a computational time of approximately 250 hours to complete, but still the results obtained may not be fully satisfactory. ASGA and ASDEA are based on mutation of genes and this leads to longer computational time compared to ASPSO whose modelling is based on schooling of birds. Even though ASGA and ASDEA need more time than ASPSO, they both managed to provide results more accurate than ASPSO. Results from ASDEA for the 6-slot/4-pole PMSM show that the magnet arc is optimally reduced to 0.8 from full-pitch, yielding 20% reduction of magnet volume, hence a cost-saving. The stator inner radius, $R_{si}$ is increased to 16.09mm, resulting in slot winding area slightly reduced to 191.79mm$^2$. This will affect other electrical machine dimensions i.e. stator body width $w_{tb}$, stator yoke thickness $w_{sy}$, and tooth teeth height $w_{tt}$. The average torque and average power are increased after optimization which is good. The output torque ripple has approximately increased by 6.8%. The motor efficiency has increased by 2%. The copper loss is about 5.1W with input power and output power for the respective optimization techniques as shown in Table 1. FEA models for 6-slot/4-pole PMSM before and after optimization are shown in Figure 1. Figure 2 shows the magnetic flux density contour before and after optimization. After the slot winding area is reduced, the magnetic flux densities in the stator body width and stator yoke are reduced from 1.58T to about 1.41T, which can potentially reduce the core losses in the stator iron further.

**Table 1:** 6-slot/4-pole PMSM parameters before and after optimization

| Parameters | Initial | ASGA | ASPSO | ASDEA |
|---|---|---|---|---|
| time, hours | - | 8.3 | 1.6 | 4.3 |
| α | 1.00 | 0.76 | 0.82 | 0.79 |
| $b_o$, mm | 2.00 | 1.95 | 1.88 | 1.88 |
| $h_m$, mm | 3.00 | 2.33 | 2.81 | 2.67 |
| $l_g$, mm | 1.00 | 0.82 | 0.76 | 0.82 |
| $R_{si}$, mm | 15.00 | 16.71 | 14.74 | 16.09 |
| $w_{tb}$, mm | 8.10 | 8.90 | 8.40 | 8.90 |
| $w_{sy}$, mm | 4.10 | 4.50 | 4.20 | 4.50 |
| $w_{tt}$, mm | 3.10 | 3.50 | 3.10 | 3.40 |
| $A_{slot}$, mm$^2$ | 244.4 | 181.81 | 221.67 | 191.79 |
| $E_{rms}$ $THDv$, % | 20.3 | 9.6 | 7.9 | 9.9 |
| $T_{cogging}$, Nm | 0.0697 | 0.0731 | 0.0803 | 0.0766 |
| $P_{avg}$, W | 79.7 | 85.7 | 80.1 | 85.1 |
| $T_{avg}$, Nm | 0.5071 | 0.5454 | 0.5099 | 0.5417 |
| $T_{ripple}$, Nm | 0.1460 | 0.1468 | 0.1628 | 0.1566 |
| $P_{out}$, W | 79.7 | 85.7 | 80.1 | 85.1 |
| $P_{in}$, W | 86.3 | 90.9 | 85.3 | 90.4 |
| *Efficiency*, % | 92.4 | 94.3 | 93.9 | 94.1 |

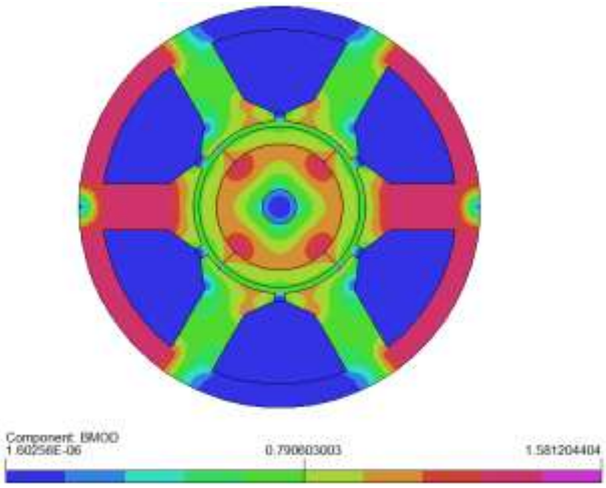


(a)

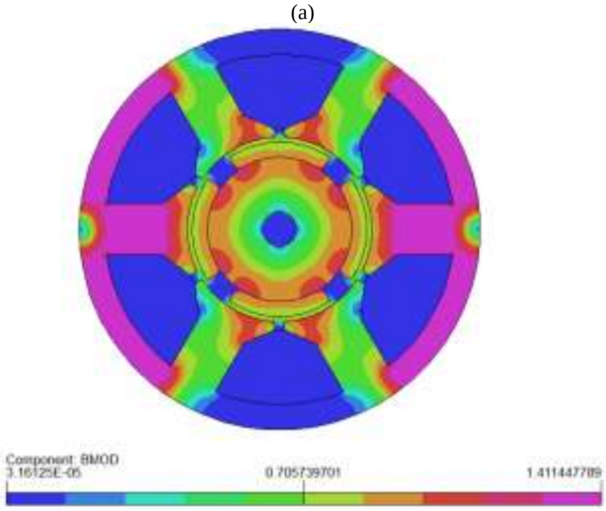


(b)

**Fig. 1:** FEA model of 6-slot/4-pole PMSM (a) before and (b) after optimization



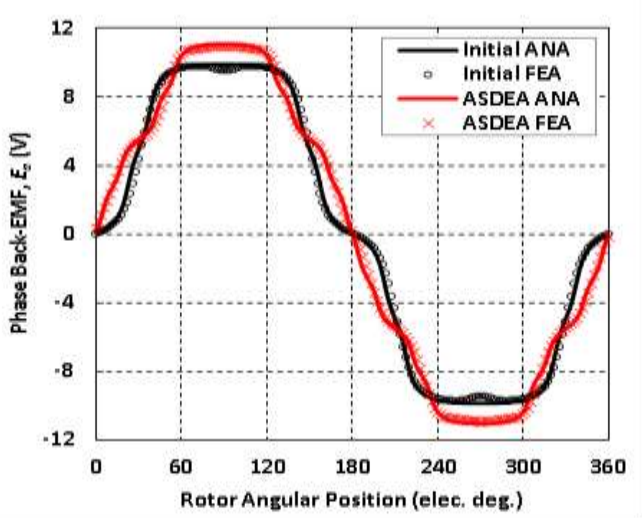

(a)



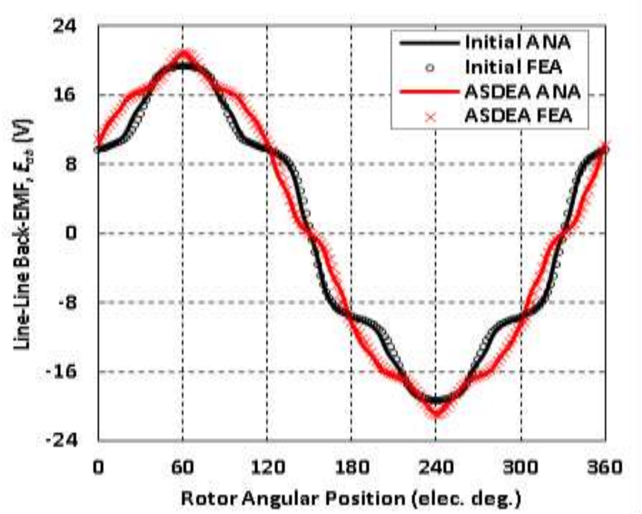

(b)

**Fig. 2:** Magnetic flux density contour of 6-slot/4-pole PMSM (a) before and (b) after optimization

Initially, the phase back-emf is slightly trapezoidal in shape, then it becomes more sinusoidal after the motor has been optimized as shown in Figure 3(a), and for the line-line back-emf as shown in Figure 3(b). The results from multi-objective optimization based on ASDEA are compared with that obtained from 2D FEA where good agreement has been achieved. The change in magnet pole arc can affect the shape of phase back-emf waveform. The sinusoidal shape of back-emf exhibits low harmonics distortion. In these figures, the legend "Initial ANA" means the analytical results of the motor using initial parameters after the sizing equations. Initial FEA indicates the results from 2D FEA motor model also using the initial parameters. Whereas, ASDEA ANA means the results from ASDEA optimization technique, while ASDEA FEA is the results from 2D FEA motor model built using the optimal parameters from ASDEA.




(a)




(b)

**Fig. 3:** (a) Phase back-emf and (b) Line-line back-emf of PMSM before and after optimization

The highest output torque with minimal torque ripple is shown in Figure 4 before and after optimization for the 6-slot/4-pole PMSM. It also indicates the results from FEA motor model after optimization. The output torque ripple as shown in Figure 4 has been reduced from 63.0mNm into 45.3mNm. The average output torque has increased from 0.507Nm to 0.542Nm, an increase of 6.9% after applying ASDEA optimization technique. It also shows similar improvement for the output power as shown in Figure 5, reduced ripple which is intended for a good motor design.

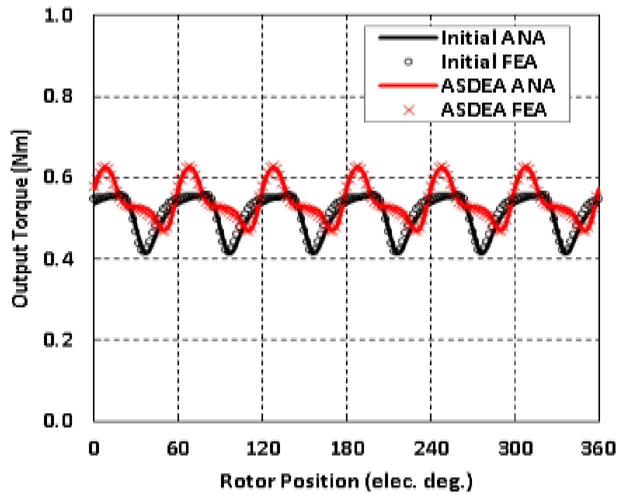



**Fig. 4:** Output torque of PMSM before and after optimized.

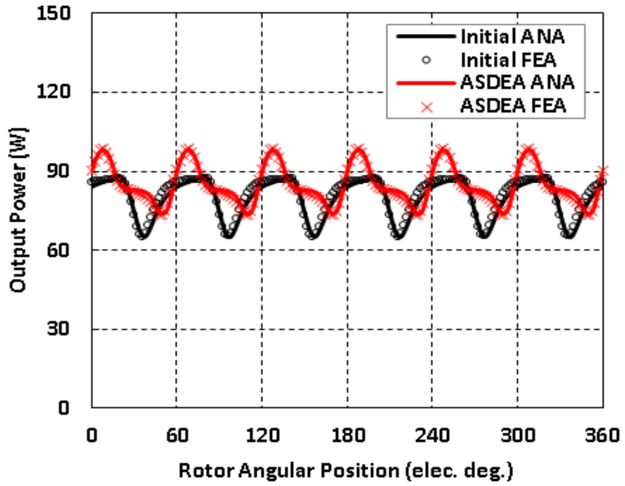



**Fig. 5:** Output Power of PMSM before and after optimized

The cogging torque can be further reduced by skewing the stator as shown in Figure 6. The cogging torque is reduced from 76.5mNm to 36.1mNm when the stator is skewed by 15° mech. If the stator is skewed by 30° mech., then the cogging torque is reduced to 4.9mNm. The shape of phase and line-line back-EMFs becomes more sinusoidal as represented in Figures 7 and 8 respectively. The torque ripples have also been reduced from 152.3mNm to 30.5mNm when stator is skewed by 15° mech., and reduced to to 12.3mNm when stator is skewed by 30° mech. as illustrated in Figure 9.

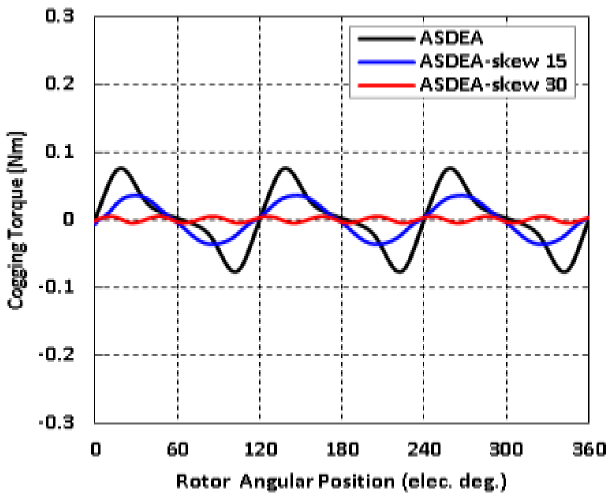



**Fig. 6:** Cogging torque of PMSM before and after stator skewing

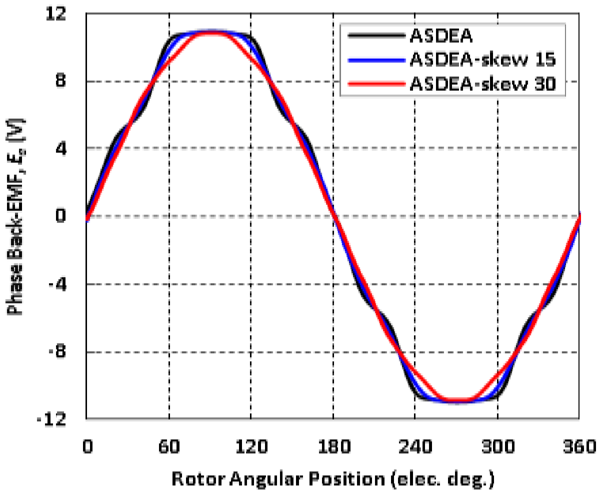



**Fig. 7:** Phase back-EMF of PMSM before and after stator skewing

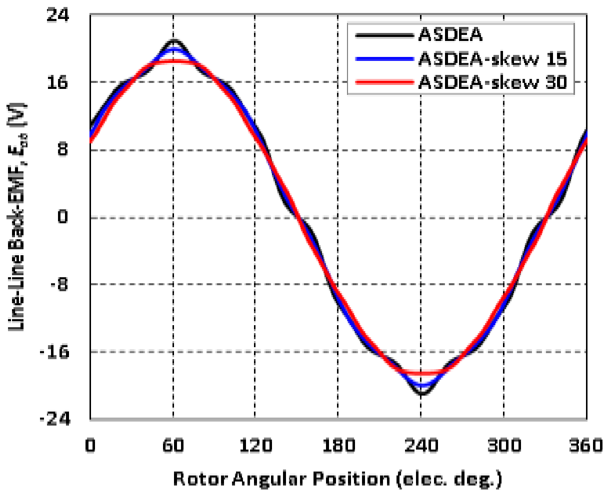



**Fig. 8:** Line-line back-EMF of PMSM before and after stator skewing

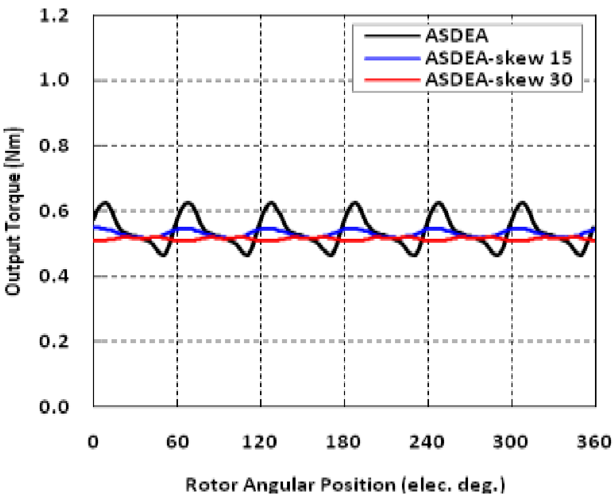



**Fig. 9:** Output torque of PMSM before and after stator skewing

## 5. Conclusion

This paper has shown that the combination of Analytical Subdomain with Differential Evolution Algorithm (ASDEA) is able to predict the motor performance very accurately. Its results have been validated with that of 2D FEA. Further comparison is also made among ASDEA, ASGA and ASPSO. The accuracy of ASDEA is as good as ASGA, but its computational time is 50% faster than ASDGA. ASPSO is the fastest but it lacks accuracy. The 6-slot/4-pole PSMSM was used in this optimization studies. Five motor parameters were chosen to be optimized i.e. magnet thickness, airgap length, slot-opening width, magnet arc, and stator inner radius with four objective functions i.e. to have lowest total harmonic distortions in the induced back-emf, lowest cogging torque, highest output torque and highest efficiency. This research work illustrates that the optimization technique such as ASDEA can confidently be used in predicting the motor performance. While repetitive changes in motor parameters in FEM could be avoided.

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