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Research paper

Parametric Study on Dynamic Instability of Fully Anisotropic Composite Plates

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Abstract

Laminated fibre reinforced composite can be subjected to periodic loading that can cause parametric instability in structures. This paper is to report on the parametric instability analysis of unsymmetric laminated composite plate. Using finite element method, the Mathieu-Hill equation that describes the parametric instability behavior of composite plate was derived based on the higher order shear deformation theory of composite and the equation was solved using the Bolotin's method. Compared to similar symmetric composite, the anisotropic nature of the composite has caused the composite to be more unstable by shifting the instability region to the left while narrowing the region at the same time. However, the effects of parameters such as the orientation angle and the aspect ratio are similar to the effect of the same parameters on the parametric instability of symmetric composites.

Keywords: *Unsymmetric composite; Mathieu-Hill; Bolotin's method; parametric instability*

1. Introduction

Laminated composite is known to give advantages of low density and high in strength per weight ratio and as such it has been widely used in industries that require low weight high strength material such as aerospace, marine and automotive industries [1-2]. To save further weight, the composite structures are designed to be thin and slender and consequently failures such as buckling, vibration and parametric instability have to be considered in the design. Parametric instability is a result of having periodic loading such as periodic compressive load that will cause time-varying (periodic) change of parameter such as stiffness in a structure. Consequently, the structure may experience failure of parametric resonance at the dynamic load amplitude that is even lower than the static buckling load of the structure [3].

The study in the area of dynamic instability of composite structures has been intensive in the last decades. Bolotin [4] can be said to be the first to comprehensively review the parametric instability of several structures that include bar, beam, plate, shell and cylinder. A study was conducted on the parametric instability of orthotropic plates that were given non-uniform parabolic in-plane load [5]. It was found that the instability region can be predicted accurately using the first order approximation. Wang and Dave [9] studied the dynamic instability of laminated composite plates and prismatic plate structures. To obtain high quality results, the sturm sequence method and the multi-level substructuring technique were applied into the B-spline finite strip method while the degree of instability of the structure was measured using a dynamic instability index. Samukham et al. [11] derived the Mathiue-Hill equation for variable angle tow (VAT) composite laminate based on the first order shear deformation theory. The study found that the improvement made in the parametric resonance of the VAT composite laminate was due to the increase in the natural frequency and the critical load of the VAT laminates. Parametric instability of laminated composite curved and flat panels subjected to periodic non-uniform in-plane compressive loading was studied using two finite strip methods, namely the sa-FSM and sp-FSM [12]. The effect of area delamination and hygrothermal environment were considered by Panda et al. [13] in the study on parametric instability of bidirectional composite panels. The intensive results given in forms of tabular values and instability charts can be treated as a guideline for parametric instability, safety and structural health monitoring of bi-directional composite structures.

The reviewed works above involve the dynamic instability of orthotropic composite plates where the dynamic instability analysis of fully anistropic composite plates [14] has rarely been found. In this study, parametric resonance analysis of unsymmetric composite plate is conducted. Several unsymmetric configurations are considered. The Mathieu-Hill equation are derived based on the higher order shear deformation theory (HSDT) using finite element method (FEM). The results are compared with those of the symmetric composite and those of the first order shear deformation theory (FSDT). Furthermore the effects of several parameters on the dynamic instability of the unsymmetric composite plates are studied.

2. Material and Method

The properties and geometries of the anisotropic composite plates and the FEM formulation are explained in the following.

2.1. Material

The unsymmetric 12 layer laminated composite plate applied here is $[0_6/\theta_6]$ where θ can be 15°, 30°, 45°, 75° and 90°. For comparison purpose, the study was also conducted on symmetric composite plate with configuration of $[0_3/0_3]_s$. Rectangular plate is used where length is a = b = 100 mm. The ratio of side length to thickness is taken as 100. The material properties are $E_1/E_2 = 40$, $v_{12} = v_{13} = 0.25$, $G_{12}/E_2 = 0.6$, $G_{23}/E_2 = 0.5$, $G_{23} = G_{12}$.

2.2. Formulation

This study uses FEM to investigate the parametric resonance problem of unsymmetric composite plates exerted with periodic axial loading. The finite element formulations developed here is based on the third (higher) order shear deformation theory. The results will be compared with those of the first order shear deformation theory determined from past research [15]. From the governing FEM formulation, the Mathieu's – Hill equation was derived and then applying the Bolotin's method, the parametric resonance equation of the unsymmetric plates which gives the sought instability chart can be derived. The displacement field used in this study is based on the following equations [16]:

$$u(x,y,z,t) = u_0(x,y,t) + z\theta_x(x,y,t) + z^3\xi_x$$
$$v(x,y,z,t) = v_0(x,y,t) + z\theta_y(x,y,t) + z^3\xi_y \qquad (1)$$
$$w(x,y,z,t) = w_0(x,y,t)$$

where the generalized displacements on the plates in the x, y and z- directions, respectively are u, v and w. The displacements at any points on the mid-plane of the plates in the x, y and z directions respectively are u_0, v_0 and w_0 while θ_x and θ_y are the rotations in the x-z and the y-z planes respectively and the warping functions in the x-z and the y-z planes respectively are represented by ξ_x and ξ_y. In this study, isoparametric quadrilateral elements are used. Each element contains 8 node while there are 7 degrees of freedom per node. The governing equation can be derived following the standard FEM derivation procedures and in accordance to the Hamilton's principle [17] such as:

$$[M]\{\ddot{q}\} + ([K_L] + [K_s])\{q\} + P(t)[K_G]\{q\} = 0 \qquad (2)$$

where $[M]$ is the mass matrix, $[K_L]$ is the linear stiffness matrix, $[K_s]$ is the shear stiffness matrix and $[K_G]$ is the geometric stiffness matrix. $\{q\}$ is the generalized displacement vector while $P(t)$ is the periodic compressive load. Assuming the compressive load, $P(t)$ consists of static and dynamic parts such as:

$$P(t) = P_s + P_t cos\omega t \qquad (3)$$

where P_s and P_t represent the static and dynamic parts of the load, $P(t)$ while the loading's frequency is ω. Furthermore, both static and dynamic components of load are parts of the critical load of the plate, P_{cr} such as:

$$P_s = \alpha P_{cr}\,.\quad P_t = \beta P_{cr} \qquad (4)$$

where the static and dynamic load factors here are α and β, respectively. Inserting into equation (2), we have

$$[M]\{\ddot{q}\} + ([K_L] + [K_s])\{q\} -$$
$$(\alpha P_{cr}[K_G] - \beta P_{cr}[K_G]cos\omega t)\{q\} = 0 \qquad (5)$$

Equation (5) is the dynamic instability problem of the SMA composite plate or the so called the Mathieu-Hill equation. Based on the Bolotin's method [4], the periodic solutions exist at the period of T and 2T where T = 2π/ω. Assuming the solution of $\{q\}$(t) is in the form of the trigonometric series for the period of T such as

$$\{q\} = \frac{1}{2}\{b_o\} + \sum_{i=2,4..}^{\alpha}[\{a\}_i \sin\left(\frac{i\omega t}{2}\right) + \{b\}_i cos\left(\frac{i\omega t}{2}\right)] \qquad (6)$$

Inserting equation (6) into equation (5) and equating coefficients for sin (ωt) and cos(ωt), we obtain

$$\left[[K] - \alpha P_{cr}[K_G] \pm \frac{1}{2}\beta P_{cr}[K_G] - \frac{\omega^2}{4}[M]\right]\{q\} = 0 \qquad (7)$$

Equation (7) is a two parts of an eigen-value problem due to the $\pm$ symbols correspond to the upper and lower stability boundaries of the dynamic instability region. Thus given the input values of α, β and P_{cr}, the instability charts can be determined. This is possible by developing FEM codes in FORTRAN environment to solve equation (7).

3. Results and Discussion

This section validates the developed formulation and the FEM codes and this is followed by giving several results on parametric studies conducted.

3.1. Validation

The formulation and codes are validated using the following symmetric composite since the results on the fully anisotropic composite is not available in the literature. The validation study used symmetric composite plate with orientation of $[0/90]_s$. The material properties are: the ratio of modulus is $E_1/E_2 = 40$, the shear modulus is $G_{12} = G_{13} = 0.6\ E_{22}$ and $G_{23} = 0.5\ E_{22,}$ the Poisson's ratio is $v_{12} = 0.25$ and the density is $\rho = 1.0$ kg/m^3. The applied rectangular plate has the length, a = b = 500 mm while the thickness follows the length to thickness ratio of a/t = 25.

The load factors, α and β are 0 and 0.3, respectively. The instablity frequency, taken in a non-dimensional form such as $\Omega = \Omega_e a^2(\rho t/E_2)$ where Ω_e is the actual dynamic instability frequency. The results in **Table 1** shows that both upper and lower frequencies obtained in this study are very close to values taken from past literatures [9].

Table 1. Validation on the parametric instability of symmetric composite plate

Ω	Present FEM HSDT	Analytical HSDT [9]
Ω^U	154.943	155.03
Ω^L	133.209	133.29

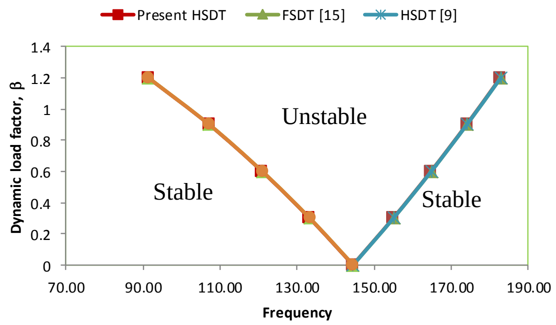


Fig.1: The validation on the dynamic instability of composite plate

At the same time, **Figure 1** shows the parametric instability charts determined using the FSDT and HSDT as compared to the chart taken from Wang and Dave [9] who used analytical method in their parametric instability study of composite plate. The chart shows the typical dynamic instability chart that gives the information on the stable and unstable regions, the frequency centre i.e. the frequency value at β = 0.0 (i.e $\Omega = 144.57$ for present HSDT case) and the width of the instability region. The finding shows that the formulation and codes in this study provides results that are in closed agreement to findings of past research.

Table 2 shows the results of dynamic instability correspond to the HSDT developed in this study and those of the FSDT of composite plate [15]. It shows that the results of the dynamic instability based on to the HSDT that does not require the shear correction factor agree excellently with the results correspond to the FSDT.

Table 2. Comparison between the HSDT and FSDT on the dynamic instability of laminated composite plate

□	□ = 0				□ = 0.2				□ = 0.4			
	FSDT		HSDT		FSDT		HSDT		FSDT		HSDT	
	□U	□L	□U	□L	□U	□L	□U	□L	□U	□L	□U	□L
0	23.20	23.20	23.20	23.20	20.82	20.82	20.82	20.82	18.10	18.10	18.10	18.10
0.2	24.29	22.05	24.30	22.05	22.05	19.51	22.05	19.51	19.51	16.55	19.51	16.55
0.4	25.33	20.82	25.33	20.82	23.20	18.10	23.20	18.10	20.82	14.83	20.82	14.83
0.6	26.32	19.51	26.33	19.51	24.29	16.55	24.30	16.55	22.05	12.87	22.05	12.87
0.8	27.28	18.10	27.28	18.10	25.33	14.83	25.33	14.83	23.20	10.53	23.20	10.53
1	28.19	16.55	28.19	16.55	26.32	12.87	26.33	12.87	24.29	7.46	24.30	7.46
1.2	29.07	14.83	29.07	14.83	27.28	10.53	27.28	10.53	25.33	-	25.33	-
1.4	29.92	12.87	29.92	12.87	28.19	7.46	28.19	7.46	-	-	-	-
1.6	30.74	10.53	30.74	10.53	29.07	-	29.07	-	-	-	-	-
1.8	31.54	7.46	31.54	7.46	-	-	-	-	-	-	-	-

3.2. The static load factor effect

The effect of the static load factor, α on the parametric instability of unsymmetric composite is investigated in this section. **Figure 2** shows that as the value of α is increased, the frequency centre is decreased i.e. the instability chart move to the left. At the same time, the instability charts are seen to widen as the value of α is increased. These show instability of the composite plate is increased that as α is increased. The pattern of results here is similar to those of the previous works on symmetric composite plates [17].

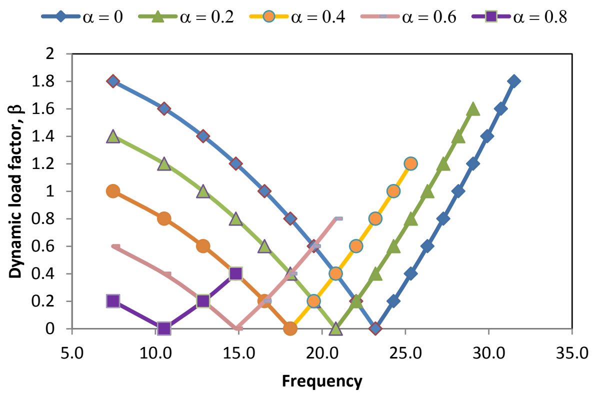


Fig. 2: The effect of the static load factor

3.3. The mechanical coupling effect

It is known that the difference between the symmetric and unsymmetric composite is the presence of elastic couplings in the unsymmetric composite which can affect the structural behavior of the composite. **Table 3** shows the types of mechanical coupling that exist in the unsymmetric cross-ply (UCP) composite with $[0_6/\theta_6]$ configuration as compared to the symmetric cross-ply (SCP) composite with $[0_3/\theta_6/0_3]$ orientation that contains none of the couplings. The two composites in fact have similar A_{12}, A_{22}, A_{16} and A_{26} couplings but vary greatly in B_{11}, B_{22}, D_{11} and D_{22} couplings. **Figure 3** shows the instability charts for both UCP and SCP. It shows that the existence of mechanical couplings in the unsymmetric composite has greatly decreased the frequency center of the instability chart. For example at α = 0.2, the symmetric composite has the frequency center of 33.697 while for unsymmetric composite, the frequency centre has been reduced to 20.823. However the reduction is lowered as α is increased. The width of instability region is however decreased as we moved from symmetric composite to unsymmetric composite.

Table 3. The mechanical couplings in symmetric and unsymmetric composite [18]

Laminates	Ext.-shear	Ext.-bend	Ext-Twist	Shear-twist	Bend-Twist
$[0_3/\theta_6/0_3]$	-	-	-	-	-
$[0_6/\theta_6]$	√	√	√	√	√

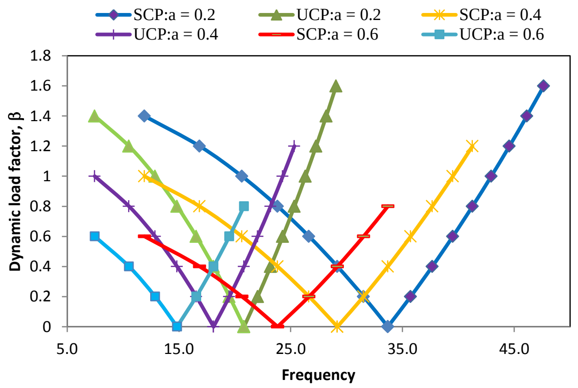
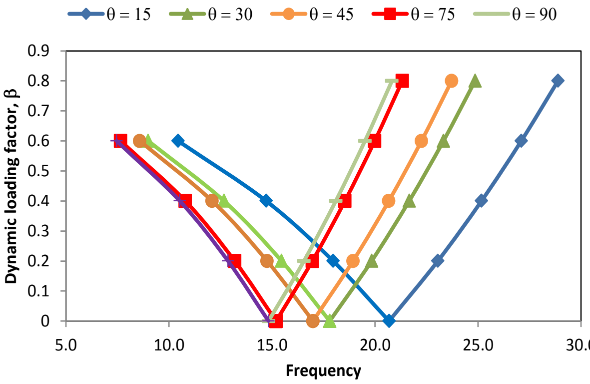


Fig.3: The effect of the anisotropic factor on the dynamic instability of composite (a = α)

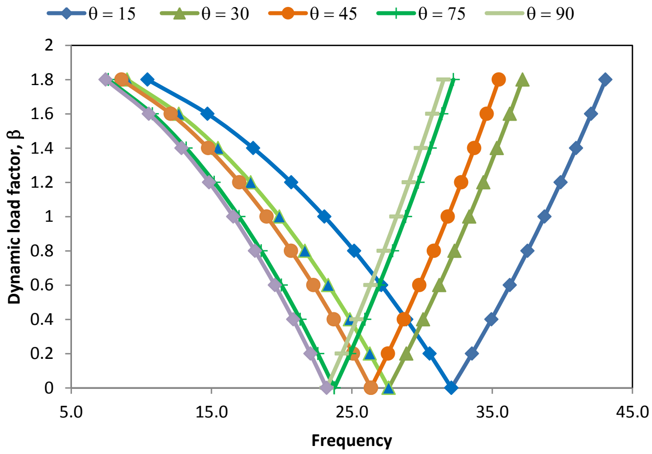
3.4. The angle of orientation effect

In this section, the value of θ in the unsymmetric configuration of $[0_6/\theta_6]$ is varied from $= 15^0$ to $\theta = 90^0$. **Figure 4** (a) – (c) shows the instability charts correspond to different values of the angle of orientation, θ for α values of 0, 0.4 and 0.6 respectively.

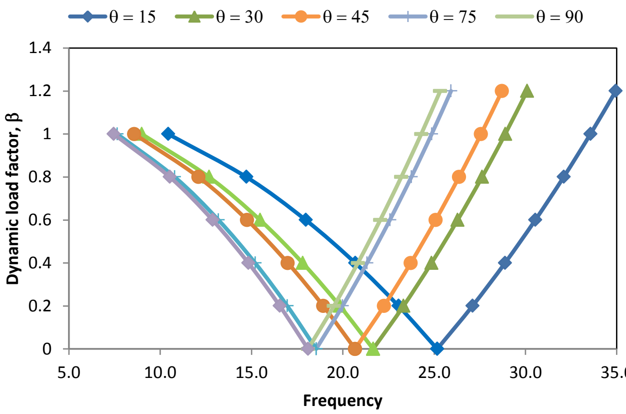
All figures show that as θ is increased, the frequency center moves to the left and this increases the instability of the composite plate. The reduction of the frequency center to the left seems to be lowered when the angle changes from $\theta = 30^0$ to $\theta = 45^0$ and again when the angle changes from $\theta = 75^0$ to $\theta = 90^0$. Furthermore, the width of the instability chart seems to be reduced as the angle of orientation, θ is increased.



(a)



(b)

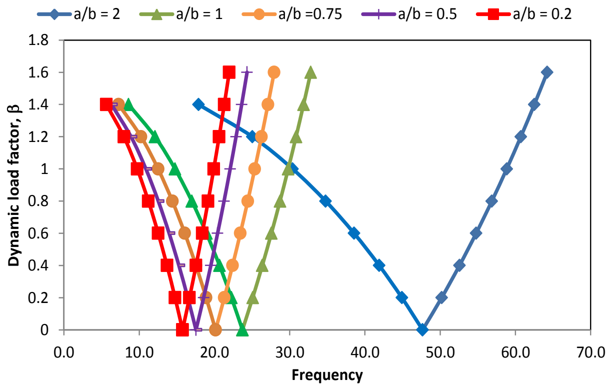


(c)

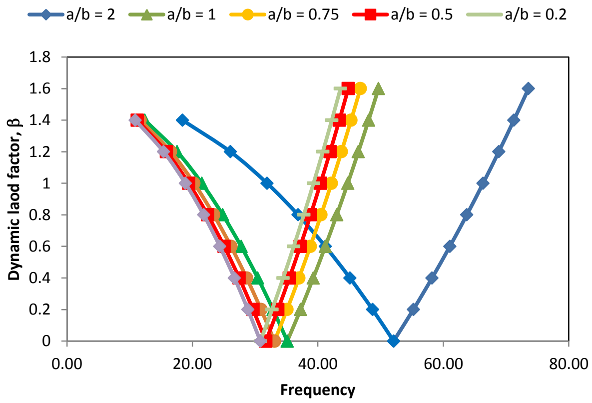
Fig.4: The effect of the angle of orientation on the dynamic instability of unsymmetric composite plate for the cases of (a) $\alpha = 0$ (b) $\alpha = 0.4$ (c) $\alpha = 0.6$

3.5. The length to thickness ratio effect

The study on the effect of length to thickness ratio on the parametric instability of composite plate is conducted in this section. For comparison purpose, **Figure 5** shows the dynamic instability charts for both unsymmetric and symmetric composite plates. It shows that the effect of mechanical couplings in the unsymmetric composites has shifted all charts to the left. For example, the frequency center in the case of a/b = 2 has moved from 52.073 to 47.644. Other than that, it can be seen that the reduction of the a/b ratio gives the same effect for symmetric and unsymmetric composite i.e as the a/b ratio is decreased, the instability charts move to the left even though the width of the instability chart is now narrower.



(a)



(b)

Fig.5: The effect of the length to thickness ratio on the dynamic instability of composite for (a) unsymmetric composite and (b) symmetric composite

4. Conclusion

A study on the parametric instability of unsymmetric composite plate has been conducted. The higher order shear deformation theory was used to derive the Mathieu-Hill equation. The derived formulation was validated with reference to past results. It can be concluded that the existence of mechanical coupling in composite plate has shifted the instability region to the left while reducing the width of the instability region. However, the patterns of the effect of several parameters such as the angle of orientation and the aspect ratio on the parametric instability of composite plates are similar to the effect of the same parameters on the symmetric composite plates.

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References

[1] Mohamed M, Din MFM, Hashim FR, Amini MHM, Masri MN, Bakar MBA, Razab MKAA and Rizman ZI (2018) Comparison between carbon diree reinforced, plastic, aluminum alloy and alloy steel for car hood application for impact test by using finite element analysis *J. Fund. Applied Sci.*, 10(2S).

[2] Isa MFM, Risby MS, Norazman MN, Khalis S, Hafizi MN and Arif S (2017) Simulation on the shock attenuation behavior of cou-

pled RHA and sandwich composite panel under blast loading. *J. Fund. Applied Sci.* 9(3S).
[3] Fazilati J (2017) Stability analysis of variable stiffness composite laminated plates with delamination using Spline-FSM *Latin America J. Solid Struct.* 528-543, doi: 10.1590/1679-78253562.
[4] Bolotin V (1964) The dynamic stability of elastic systems. San Franscisco: Holden-Day.
[5] Ramachandra LS and Panda SK (2012) Dynamic instability of composite plates subjected to non-uniform in-plane loads *J. Sound and Vib.* 331(1) 53–65.
[6] Srinivasan RS and Chellapandi P (1986) Dynamic stability of rectangular laminated composite plates *Compu. and Struc.* 24 233-238.
[7] Kwon YW (1991) Finite element analysis of dynamic instability of layered composite plates using a high order bending theory *Composite Struc.* 38 57-62.
[8] Balamurugan V, Ganapathi M and Varadan TK (1996) Nonlinear dynamic instability of laminated composite plates using an efficient finite element model *Comp. Struc.* 60 123-130.
[9] Wang S and Dawe DJ (2002) Dynamic instability of composite laminated rectangular plates and prismatic plate structures *Comp. Method in App. Mech. Eng.* 191 1791-1826.
[10] Dey P and Singha MK Dynamic stability analysis of composite skew plates subjected to periodic in-plane load *Thin Wall Struc.* 44 937-942.
[11] Samukham S, Raju G and Vyasarayani CP (2017) Parametric instabilities of variable angle tow composite laminate under axial compression *Comp. Struc.* 166 229–238.
[12] Ovesy HR and Fazilati J (2014) Parametric instability analysis of laminated composite curved shells subjected to non-uniform in-plane load *Comp. Struc.* 108 449–455.
[13] Panda HS, Sahu SK and Parhi PK (2015) Hygrothermal response on parametric instability of delaminated bidirectional composite flat panels *European Journal of Mechanics A/Solids* 53 268-281.
[14] Birman V (1985) Dynamic stability of unsymmetrically laminated rectangular plates *Mech. Research Comm.*12(2) 81-86.
[15] Yusof Z and Rasid ZA (2017) Parametric Instability of Laminated Composite Plates Using Third Order Shear Deformation Theory *Adv. Sci. Letters* 23 (5) 4434-4438.
[16] Zabaras N and Pervez T (1990) Viscous damping approximation of laminated anistropic composite plates using the finite element method *Comp. methods in App. Mech. Eng.* 81 291-316.
[17] Reddy JN (2002) Mechanics of composite plates and shells: theory and analysis, New York: CRC Press.
[18] Lagace PA, Jensen DW and Finch DC (1986) Buckling of Unsymmetric Composite Laminates *Compo. Struct.*15 101-123.