# Application of Measurement Signal Reduction to Improve Measurement Accuracy 

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#### Abstract

Most modern technical tasks require high precision measurements. To do this, it is necessary to analyze the causes of errors and take measures to reduce their influence on the accuracy of measurements. The causes of errors are very diverse and cannot always be identified. However, some systematic components of the measurement error can be described and calculated mathematically. In this case, the task of reducing the signal at the output of a measuring device to the form it would have when using an "ideal" device is reduced to calculating a certain linear operator which product to the measured signal allows obtaining the minimum systematic error. In this paper, the application of the reduction method is given by the example of a measuring instrument for the degree of polarization of light radiation which comprises three measuring channels for measuring the intensity of linearly polarized radiation. Each channel is built with the use of three operational amplifiers. The main errors of a measuring channel that can be described and determined are the errors of the operational amplifiers associated with the bias voltages and temperature drift. In real measuring systems there are much larger of such components. However, the use of computer equipment for modeling systems and processes, as well as measurements, removes all restrictions on the possibilities of processing the obtained data in a software way. With the help of computer technology it is possible to reduce the influence of perturbing effects and systematic errors, and also to eliminate gross errors. The random component of an error can be reduced by increasing the number of measurements and carrying out statistical data processing.


Keywords: measurements, modeling of the measuring channel, reduction of the measuring signal, improvement of measurement accuracy.

## 1. Introduction

Technical means of measurement and control are an integral part of almost any sphere of human activity. They control the quality of products being part of control systems for various purposes; they ensure objectivity of measurements, etc. Current requirements for the quality of products cannot be met without the use of high-precision measurements [1-3]. Therefore, high metrological and operational requirements are imposed on measuring instruments. Improvement of the accuracy of measurements, as a result, is quite an urgent task.
A measurement error is the result of many different perturbing effects. To reduce errors, it is necessary to analyze the causes of their occurrence and take measures to reduce their impact on the accuracy of measurements. The causes of error are very diverse. Some disturbances can be identified and compensated; the sources of others are not always possible to determine, much less to eliminate. By the nature of the manifestation of errors they can be divided into two groups: random and systematic. There is no any pattern in the appearance of random errors. Random errors can occur in the case of simultaneous actions of a set of independent factors, each of which separately does not have a significant effect on the measurement result. Random errors are almost impossible to exclude, they are always present as a result of measurement. However, the effect of random errors can be reduced by statistical processing of the measurement result. A systematic error is a component of measurement error that is constant or varies according to a certain law, repeating itself when measuring the
same parameter. Systematic errors are the result of a combination of non-random perturbing effects, the composition of which depends on the construction, technological and physical features of the measuring instruments used and the conditions of their application. Systematic error cannot be eliminated by repeated measurements. As a rule, it is eliminated either by introducing amendments, or by improving the methodology or research design.
The limiting characteristics of the measuring circuit depend on the physical phenomena that determine the measuring process. If we obtain a fairly complete mathematical model of the physical phenomena that form the measurement process, then it is possible to fully or partially compensate for their disturbing effects [4].The implementation of analog circuits to compensate for the systematic error of the measuring channels is very difficult, although, in some cases, successfully used. The use of computer technology to compensate for such errors allows us to simplify this process and make it more flexible. With the help of calculations one can perform mathematical modeling of what is not directly observable. The use of computer technology makes it possible to correct the mathematically described error programmatically and, thus, to improve the accuracy of measurements.

## 2. Methods

Measurement reduction methods are considered in the theory of measuring and computing systems created at the physics department of Moscow State University named after M.V.

Lomonosov under the guidance of Professor Yu.P. Pytyev. This theory involves obtaining measurement signals for physical quantities and subsequent mathematical processing of those signals using computer technology. According to the proposed theory, the use of a measuring channel equipped with a computing device operating in accordance with a given program, taking into account the essence of the physical phenomena of the measurement process, will overcome the fundamental limitations that impose a limit on the improvement of measuring instruments only through technical solutions, without using calculations. The main goal of this method is to obtain the most accurate model and parameters of the object under study in a state unperturbed by measurement. To achieve this, it is necessary to have a mathematical model that takes into account the interaction of all the components of the measuring system [5].
We consider a measuring channel, the block diagram of which is shown in Figure 1.
Let $A$ be some linear operator defining a mathematical model of the device. Then the association of the output signal $\xi$ and the measured signal $f$ at the input of the device is described by the relation:
$\xi=A \cdot f+v$,
Where $v$-measurement error;
$A f$ is the measurement result without perturbing effects.


Figure 1: Structural diagram of the measuring channel
The signal $\xi$ is the $v$ noise-distorted measurement result of the signal $A f$ at the output of the measuring channel $A$, which input is the signal $f$ from the object and medium under study, disturbed by the measurement.
The task of reducing the output signal $\xi$ to the form that it would have at the output of the "ideal" device is set as follows:
It is necessary to find a linear operator $R$ for which:
$E\|R \xi-U f\|^{2}=\inf \left\{E\left\|R^{\prime} \xi-U f\right\|^{2}, E R \xi=U f\right\}$,
Where $E\left\|\|^{2}\right.$ is an expected value;
U - an operator describing the "ideal" measuring channel.
If the problem has a solution, then $E R \xi=R A f=U f$ and signal $R \xi=R A f+R v=U f+R v$ may be interpreted as noise - distorted $R V \square$ output signal of the device $U$ to which input the signal $f$ is fed, or as the output signal of the calculator, if the vector $R \quad \xi$ was calculated using a computer connected with a measuring channel $A$. The signal $R \xi$ is called the measurement reduction $\xi$ to $U f[6,7]$.
Thus, the task of increasing the accuracy of a measuring device is reduced to finding such an operator $R$ for which the product $R v$ is minimal.

## 3. Results and discussion

For example, we consider the main systematic errors that occur when measuring the degree of polarization of electromagnetic radiation using a system of three photodetectors [8-10]. This metering device contains three polarization photodetectors, the
polarization axes of which are 0,45 and $90^{\circ}$. To receive signals from photodetectors, it is necessary to convert the current into voltage, to filter and to amplify the signal, to convert it to a digital form and feed it to the input of the computer system. To reduce the dynamic error, the measuring system must either use sampling and storage schemes or carry out parallel analog-digital conversion [4].
The block diagram of the polarization metering device is shown in Figure 2.


Figure 2: Block diagram of the polarization degree metering device
The measurement is carried out as follows: linearly polarized radiation with the help of the optical system is focused on three photodetectors $1,2,3$. The radiation is converted there into electrical signals which are converted, amplified, filtered and detected via measuring channels $4,5,6$. Then the signals are fed to the inputs of analog-to-digital converters 7, 8, 9. The digital code of analog signals obtained using analog-to-digital converters, is fed to the input of the computer system 10 , where the polarization degree is calculated according to the mathematical model.
The metering device contains three identical channels for measuring the intensity of linearly polarized radiation $4,5,6$. The electrical circuit of one of the three channels is shown in Figure 3.


Figure 3: Channel for measuring the light radiation intensity
Each measuring channel contains a VD photodetector connected in series with a current-to-voltage converter D1, a band-pass filter D2, an amplitude detector D3, and an analog-to-digital converter ADC. Current-to-voltage converter, filter and detector are based on operational amplifiers. Let us describe some errors of the measuring path for a given electrical circuit. The errors of the cascades of the measuring channel are determined by the inaccuracy of the elements used, for example, the values of the resistances and their temperature coefficients, and the difference between the real operational amplifier and the ideal amplifier.
The photodetector measurement error is determined by the dark current of the photodiode. However, with the following cascade (band-pass filter), the operational amplifier output is connected without DC coupling; therefore this error can be ignored.
The operational amplifier error caused by temperature change is determined by the expression:
$\gamma_{T}=\frac{T K e}{U_{M A X}} \cdot \Delta t \cdot K_{T}$,
Where $T K e$ is the temperature drift of the bias voltage,
$U_{\text {MAX }}$ - the maximum voltage,
$\Delta t$ is the temperature increment
$K_{T}$ - gain.
The errors of the operational amplifiers caused by temperature changes are correlated, additive and have a normal distribution. The standard deviation is determined by:
$\sigma_{T i}=\frac{\gamma_{T i}}{3}$,
Where $\sigma_{T i}$ is the error of the i-th operational amplifier caused by temperature change.

$$
\sigma_{T}=\sum_{i=1}^{m} \sigma_{T i}
$$

Where $\sigma_{T}$ is the total error of the operational amplifiers caused by temperature change.
Quantization error $\sigma_{A D C}$ of the analog-to-digital converter is additive and uniformly distributed.
The errors caused by the bias voltage of the operational amplifiers are evenly distributed. They are not correlated, and the total error is determined by:

$$
\sigma_{\delta}=\sqrt{\sum_{i=1}^{n} \sigma_{\delta i}^{2}},
$$

Where $\sigma_{\delta i}$ is the error caused by the bias voltage of the i-th operational amplifier;
$\sigma_{\delta}$ is the total error caused by the bias voltage of the operational amplifiers.
$\sigma=\sqrt{\sigma_{\delta}^{2}+\sigma_{T}^{2}+\sigma_{\mathrm{ADC}}^{2}}$,
Where $\sigma_{A D C}$ is the quantization error of the analog-to-digital converter;
$\sigma$ - total error of the measuring channel.
Thus, to improve the measurement accuracy, we can make a software adjustment of the bias voltage of the operational amplifiers and use temperature compensation.
The current $I$ through the photodetector is determined by the expression:
$I=k \cdot \Psi \cdot S_{I}+I_{T} \approx k \cdot \Psi \cdot S_{I}$,
where $\Psi$ is the radiation flux;
$S_{I}$-integrated current sensitivity;
$k$-coefficient which characterizes the attenuation of the luminous flux when passing through a polarizer;
$I_{T}$ is the dark current of the photodiode.
The voltage $U_{D I}$ at the output of the current transducer through the photodetector into the voltage (taking into account the bias voltage of the operational amplifier and temperature drift) is determined by:
$U_{\mathrm{D} 1}=R \cdot I+U_{\mathrm{D} 1}^{T}+U_{\mathrm{D} 1}^{\delta}=R \cdot k \cdot \Psi \cdot S_{I}+U_{\mathrm{D} 1}^{T}+U_{\mathrm{D} 1}^{\delta}$,
Where $R$ is the resistance in the feedback circuit of the operational amplifier;
$U^{\delta}$ is the bias voltage of the operational amplifier;
$U^{T}$ is the bias voltage of the operational amplifier caused by a change in temperature.
At the output of the bandpass filter (for a signal with a frequency in the filter bandwidth) we get:
$U_{\mathrm{D} 2}=k_{\mathrm{D} 2} \cdot U_{\mathrm{D} 1}+U_{\mathrm{D} 2}^{T}+U_{\mathrm{D} 2}^{\delta}=k_{\mathrm{D} 2} \cdot k \cdot R \cdot \Psi \cdot S_{I}+k_{\mathrm{D} 2} \cdot\left(U_{\mathrm{D} 1}^{T}+U_{\mathrm{D} 1}^{\delta}\right)+U_{\mathrm{D} 2}^{T}+U_{\mathrm{D} 2}^{\delta}$
where $k_{D 2}$ is the filter gain.
The voltage at the output of the amplitude detector is determined by:
$U_{\mathrm{D} 3}=\frac{k_{\mathrm{D} 2} \cdot k \cdot R \cdot \Psi \cdot S_{I}}{\sqrt{2}}+\frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 2} \cdot\left(U_{\mathrm{D} 1}^{T}+U_{\mathrm{Dl}}^{\delta}\right)+U_{\mathrm{D} 2}^{T}+U_{\mathrm{D} 2}^{\delta}\right)+U_{\mathrm{D} 3}^{T}+U_{\mathrm{D} 3}^{\delta}$
Taking into account the three measuring channels, operator $A$ and noise $v$ can be recorded as follows:

$$
\begin{gathered}
A=\left(\begin{array}{l}
\frac{k_{\mathrm{D} 2} \cdot k_{1} \cdot R_{1} \cdot S_{I}}{\sqrt{2}} \\
\frac{k_{\mathrm{D} 22} \cdot k_{2} \cdot R_{2} \cdot S_{I}}{\sqrt{2}} \\
\frac{k_{\mathrm{D} 23} \cdot k_{3} \cdot R_{3} \cdot S_{I}}{\sqrt{2}}
\end{array}\right) \\
v=\left(\begin{array}{l}
\frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 21} \cdot\left(U_{\mathrm{D} 11}^{T}+U_{\mathrm{D} 11}^{\delta}\right)+U_{\mathrm{D} 21}^{T}+U_{\mathrm{D} 21}^{\delta}\right)+U_{\mathrm{D} 31}^{T}+U_{\mathrm{D} 31}^{\delta} \\
\frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 22} \cdot\left(U_{\mathrm{D} 12}^{T}+U_{\mathrm{D} 12}^{\delta}\right)+U_{\mathrm{D} 22}^{T}+U_{\mathrm{D} 22}^{\delta}\right)+U_{\mathrm{D} 32}^{T}+U_{\mathrm{D} 32}^{\delta} \\
\frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 23} \cdot\left(U_{\mathrm{D} 13}^{T}+U_{\mathrm{D} 13}^{\delta}\right)+U_{\mathrm{D} 23}^{T}+U_{\mathrm{D} 23}^{\delta}\right)+U_{\mathrm{D} 33}^{T}+U_{\mathrm{D} 33}^{\delta}
\end{array}\right)
\end{gathered}
$$

From where the signal $\xi$ at the input of the computing system is determined in such a manner:
$\left(\begin{array}{l}\xi_{1} \\ \xi_{2} \\ \xi_{3}\end{array}\right)=\Psi \cdot\left(\begin{array}{l}\frac{k_{\mathrm{D} 21} \cdot k_{1} \cdot R_{1} \cdot S_{I}}{\sqrt{2}} \\ \frac{k_{\mathrm{D} 22} \cdot k_{2} \cdot R_{2} \cdot S_{I}}{\sqrt{2}} \\ \frac{k_{\mathrm{D} 23} \cdot k_{3} \cdot R_{3} \cdot S_{I}}{\sqrt{2}}\end{array}\right)+\left(\begin{array}{l}\frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 21} \cdot\left(U_{\mathrm{D} 11}^{T}+U_{\mathrm{D} 11}^{\delta}\right)+U_{\mathrm{D} 21}^{T}+U_{\mathrm{D} 21}^{\delta}\right)+U_{\mathrm{D} 31}^{T}+U_{\mathrm{D} 31}^{\delta} \\ \frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 22} \cdot\left(U_{\mathrm{D} 12}^{T}+U_{\mathrm{D} 12}^{\delta}\right)+U_{\mathrm{D} 22}^{T}+U_{\mathrm{D} 22}^{\delta}\right)+U_{\mathrm{D} 32}^{T}+U_{\mathrm{D} 32}^{\delta} \\ \frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 23} \cdot\left(U_{\mathrm{D} 13}^{T}+U_{\mathrm{D} 13}^{\delta}\right)+U_{\mathrm{D} 23}^{T}+U_{\mathrm{D} 23}^{\delta}\right)+U_{\mathrm{D} 33}^{T}+U_{\mathrm{D} 33}^{\delta}\end{array}\right)$

As was shown earlier, it is necessary to find the operator $R$ to improve the accuracy of the measuring channel, at which the product $R v$ will be minimal. For the case of $R v=0$, we obtain the following operator:
$R=\left(\begin{array}{ccc}\frac{1}{v_{1}} & -\frac{1}{v_{2}} & O \\ 0 & \frac{1}{v_{2}} & -\frac{1}{v_{3}} \\ -\frac{1}{v_{1}} & 0 & \frac{1}{v_{3}}\end{array}\right)$,

Where
$v_{i}=\frac{1}{\sqrt{2}} \cdot\left(k_{\mathrm{D} 2 i} \cdot\left(U_{\mathrm{D} 1 i}^{T}+U_{\mathrm{D} 1 i}^{\delta}\right)+U_{\mathrm{D} 2 i}^{T}+U_{\mathrm{D} 2 i}^{\delta}\right)+U_{\mathrm{D} 3 i}^{T}+U_{\mathrm{D} 3 i}^{\delta}$
, $i=1$... 3 .
Thus, the increase in accuracy can be achieved by determining the bias voltages and temperature drift of the operational amplifiers and calculation:

$$
R \xi=\left(\begin{array}{ccc}
\frac{1}{v_{1}} & -\frac{1}{v_{2}} & 0 \\
0 & \frac{1}{v_{2}} & -\frac{1}{v_{3}} \\
-\frac{1}{v_{1}} & 0 & \frac{1}{v_{3}}
\end{array}\right) \cdot\left(\begin{array}{l}
\xi_{1} \\
\xi_{2} \\
\xi_{3}
\end{array}\right)
$$

Where $\xi 1, \xi 2, \xi_{3}$ - the output signals from the measuring channels.

## 4. Summary

Using the measuring signal reduction method allows us to reduce the systematic measurement error. Reducing the random component of the measurement error can only be achieved by increasing the number of measurements and carrying out an appropriate statistical processing of the research results. In this case, the random error with an increase in the number of measurements n decreases in $\sqrt{n}$ times.

## 5. Conclusions

Thus, systematic errors are compensated by measuring perturbing effects, describing the mathematical model of the measuring channel and calculating the output signal using computer technology. The application of the measurement signal reduction method allows us to increase instrumentation accuracy. At the same time, random error components are minimized by increasing the number of measurements.

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