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Research paper

On a Pseudo Smarandache Ideals of BH-algebra

Husein Hadi Abbass[1*], Ahmed Abdulhussein Jabbar[2]

[1]Department of Mathematics, Faculty of Computer Science and Mathematics, Iraq.
[2]Department of Mathematics, Faculty of Computer Science and Mathematics, Iraq.

Abstract

In this paper the notion of a pseudo Samarandache BH-algebra, a pseudo Samarandache ideal, a pseudo Smarandache closed ideal and a pseudo Smarandache completely closed ideal of a pseudo Smarandache BH-algebra are defined. There notion are stadied. The relationships among these types of ideals are discussed.

Keywords: *BCK-algebra, BH-algebra, ideal of BH-algebra, a Smarandache of BH-algebra, ,a pseudo BH-algebra, apseudo ideal of a pseudo BH-algebra, a pseudo closed ideal of a pseudo BH-algebra, a pseudo completely closed ideal of a pseudo BH-algebra, a pseudo Smarandache ideal of BH –algebra, a pseudo Smarandache closed ideal of BH-algebra, a pseudo Smarandache completely closed ideal of BH –algebra.*

1. Introduction

In 1966 by Y .Imai and K.Iseki introduceed the notion of BCK-algebra[8], In 1998, Y. B. Jun, E. H. Rogh and H. S. Kim introduceed the notion of a BH- algebra, and the notion of ideal of a BH- algebra[6]. In 2005, Y. B. Jun introduceed the notion Q-Smarandache of $\mathcal{B}$CH- algebra and Q-Smarandache ideal of $\mathcal{B}$C$\mathcal{H}$- algebr[5]. In 2012, H. H. Abbass and H. A. Dahham introduced the notion of completely closed ideal of a BH-algebr[2]. In 2013 H. H. Ab bass and S.J. Mohammed introduced the notion of Q-Smarandache closed ideal and Q-Smarandache completely closed ideal of a Smarandache BH-algebra[4]. In 2015, Y. B. Jun.and S.S. Ahn introduceed the notion of a pseudo BH- algebra and a pseudo ideal of a pseudo $\mathcal{BH}$ –algebr[7]. , In 2017, H. H. Abbass and A. H.Nouri introduceed the notion of a pseudo completely closed ideal of a pseudo BH-algebra[1]
In this paper, we define the concepts of a pseudo Smarandache completely closed ideal and a pseudo Smarandache closed ideal of a pseudo Smarandache BH-algebra. We stated and proved some theorems which determine the relationships between these notions and some types of a pseudo Smarandache ideals of a Smarandache BH-algebra.

2. Materials and Methods

In this section, some basic concepts about a BCK-algebra, a BH-algebra,apseudoBH-algebra, pseudo ideal and a pseudo closed ideal of a pseudo BH- algebra are given.

Definition *(1.1) [8]*A BCk-algebra is an algebra (X,*,0), where X is a nonempty set, "*" is a binary operation And 0 is a constant, satisfying the following axioms:

i. $((x*y)*(x*z))*(z*y)=0\,, \quad \forall x,y,z\in X$

ii. $(x*(x*y))*y=0\ ,\ \forall x,y,z\in X$. iii. $x*x=0\ ,\ \forall x\in X$.

iv. $x*y=0\ \ and\ \ y*x=0 \Longrightarrow x=y,\ \ \forall x,y\in X$

v. $0*x=0\ ,\ \forall x\in X$

Definition (1.2) [6] A BH-algebrais a nonempty set Xwith constant 0 and a binaryoperationconditio
i.x∗x=0, ∀x∈X.ii. x∗0=x, ∀x∈X.iii. $x*y=0$ andy $*\,x=0 \Rightarrow x=y, \forall x,y\in X$.

Definition (1.3**)** [4]A Smarandache BH-algebra is defined to be a BH-algebra X in which there exists a proper subset Q of X such that .i. $0\in Q$ and $|Q|\geq 2$
ii. Q is a BCK – algebra under the operation of X.

Definition (1.4)[6]Let I be a nonempty subset of a BH-algebra X and I ($\neq\emptyset$)⊆X .Then I is called an ideal of X if it is satisfies: i. $0\in I$. ii. $x*y\in$ and $y\in I$ $imply$ $x\in I,\ \forall x,y\in X$

Now, we define the a Smarandache ideal of X to the Smarandache BH algebra X.

Definition (1.5) [4] A nonempty subset I of a Smarandache BH –algebra X is called a Smarandache ideal of X if:

i. $0\in I$

ii. $x*y\in I\ and\ y\in I \Longrightarrow x\in I\ ,\ \forall x\ \in Q$

Proposition (1.6) [4]Every ideal of a Smarandache BH-algebra X is a Smarandache ideal of X .

Definition (1.7**)**[3]An ideal I of a BH-algebra X is called a closed ideal of X if and only if$0*x\in I$ $for\ all$ $x\in I$

Now, we define the Smarandache closed ideal of X to the Smarandache BH algebra X.

Definition (1.8) [4]A Smarandache ideal I of a Smarandache BH –algebra X is called a Smarandache closed ideal of X if:
$for\ all\ x\in I, 0*x\in I$

Proposition (1.9) [4] Every closed ideal of a Smarandache BH-algebra X is a Smarandache closed ideal of X.

Definition (1.10) [2] An ideal I of a BH-algebra X is called a completely closed ideal of X if it is Satisfies: $x*y\in I,\ \forall x,y\in I$

Remark (1.11) [2] Every a completely closed ideal of BH-algebra X is closed ideal of X.

Now, we define the Smarandache a completely closed ideal of X to the Smarandache BH algebra X.

Definition (1.12) [4] A Smarandache ideal I of a Smarandache BH-algebra X is called a Smarandache completely closed ideal of X if: $x*y\in I,\ \forall x,y\in I$.

Proposition (1.13) [4] Every completely closed ideal of a BH-algebra X is a Smarandache completely closed ideal of X

Remarks (1.14) [4] Every a Smarandache completely closed ideal of Smarandache BH-algebra X is a Smarandache closed ideal of X.

Definition (1.15)[7]

A pseudoBH algebrais a nonempty set X with aconstant 0and two binary operations "*"and "#" satisfying the following conditioni. $x * x = x\#x = \forall x \in$ X.ii. $x * 0 = x\# 0 = x, \forall x \in$ X. iii. $x *$ y = y# x =0 ⇒ x = y, ∀x,y ∈X

Definition (1.16)[7] Let (X, ∗ ,#, 0) be a pseudoBH − algebra , Then I is called pseudo ideal of 𝔛if it satisfies:

i. 0 ∈ I. ii. $x * y$, $x \# y$ ∈I,y ∈I⇒ x ∈ , ∀x, y ∈X.

Definition (1.17) [7] A pseudoideaII of a pseudoBH-algebraX is called a pseudo closed idealof X, if for every x∈I,we have 0*x, 0 #x∈I .

Definition (1.18) [1] A pseudoideaII of a pseudoBH -algebra X .is called a pseudo completely closed idealof X, if satisfies: $x * y, x\#y \in I,\ for\ all\ x, y \in I$

Remarks (1.19) [1] Every a pseudo completely closed ideal of a pseudo BH-algebra X is a pseudo closed ideal of X.

3. Main Results

In this section, the concepts a pseudo Smarandache BH-algebra, a pseudo Smarandache ideal, a pseudo Smarandache closed ideals and a pseudo Smarandache compeletly closed ideals of a pseudoSmarandache BH-algebra are given.

Definition (2.1) A peusdo Smarandache BH-algebra (X, *, #, 0) is defined to be a pseudo BH-algebra in which there exists a proper subset Q of X such that

i.$0 \in Q$ and $|Q| \geq 2$

ii. Q is BCK – algebra under the operations "*" and "#" of X.

Example (2.2) the a pseudo BH- algebra X = {0,1 ,2 ,3 ,4 } with constant 0 and binary operations" *" and" #" defined the following tables and Q={0,1,2} is a pseudo Smarandache BH-algebra.

*	0	1	2	3	4
0	0	0	0	0	4
1	1	0	0	2	4
2	2	1	0	2	4
3	3	2	0	0	4
4	4	2	1	0	0

#	0	1	2	3	4
0	0	0	0	2	4
1	1	0	2	3	1
2	2	2	0	1	0
3	3	0	0	0	2
4	4	1	1	1	0

Definition (2.3) let X be a pseudo Smarandache BH- algebra An on empty subset I of X is called a pseudo Smarandache ideal of X related to Q (or briefly , a pseudo Smarandache ideal of X if.

i. 0 $\in I$

ii. $\forall\, y \in I,\ \ x * y\ ,\ \ x\#y\ \in I\ \ imply\ \ x \in I,\ \forall\, x \in Q$

Example(2.4) Consider the pseudo Smarandache BH- algebra X={0,1,2,3,4} with the binary Operations "*" and " # " defined by the tables.

*	0	1	2	3	4
0	0	0	0	3	4
1	1	0	0	2	3
2	2	1	0	2	4
3	3	3	2	0	4
4	4	2	1	0	0

#	0	1	2	3	4
0	0	0	0	3	4
1	1	0	0	2	1
2	2	2	0	1	0
3	3	3	2	0	2
4	4	1	2	1	0

And Q={0,1,2} the subset I ={0,1,3} is a pseudo Smarandache ideal of X .

Proposition (2.5)Let X be a pseudo Smarandache – BH algebra .Then every a pseudo ideal of X is a pseudo Smarandache ideal of X .

Proof: It is clear

Remark (2.6) The following example shows that convers of proposition is not correct in general.

Example (2.7)Consider the a pseudo Smarandache BH- algebra X={0,1,2,3} with binary operations "*" and "#" defined by the following tables.

*	0	1	2	3
0	0	0	2	3
1	1	0	1	2
2	2	2	0	1
3	3	3	2	0

#	0	1	2	3
0	0	0	2	3
1	1	0	0	2
2	2	2	0	1
3	3	3	2	0

And Q ={0,1}. The subset I ={0,2} is a pseudo Smarandache ideal of X but it is not a pseudo ideal of X

since $3 * 2 = 2 \in I\ \ and\ 3\#2 = 2 \in I\ but\ 3\ \ \ \notin I$

Theorem (2.8)Let X be a pseudo Smarandache BH- algebra and I be a pseudo Smarandache ideal such that $x * y, x\#y \notin I\ for\ all\ x \notin I\ and\ \ y \in I, then\ I\ is\ a\ pseudo\ ideal\ of\ X.$

Proof:

Let I be a pseudo Smarandache ideal of X ,$x \in X, and\ y \in$ I. $0 \in I ii.\ let\ x * y, x\#y \in I\ and\ y \in I$

Then we have two cases. Case 1 if $x \in Q\ \ \Longrightarrow x \in I$

Case 2 if $x \notin Q$,$either\ x \in I, or\ x \notin I$

If $x \in I \Longrightarrow I\ is\ a\ pseudo\ ideal$if $x \notin I, \Longrightarrow x * y, x\#y\ \ \notin I$

And this contradiction since x * y , x # y ∈ I

Therefore , I is a pseudo ideal

Definition (2.9)A pseudo Smarandache ideal of a pseudo Smarandache BH-algebra is called a pseudo Smarandache closed ideal of X if $0 * x\ , 0\#x\ \in I\ , \forall x \in I$

Example(2.10) the a pseudo Smaranadche ideal I={0,1,3} of X in example (2 ,4)is a pseudo Smaranadche closed ideal of X.

Definition (2.11) A pseudo Smarandache ideal of a pseudo Smarandache BH-algebra is called a pseudo Smarandache completly closed ideal of X if $x * y\ and\ x\#y\ \in I\ , \forall x\ , y \in I$

Example (2.12) the a pseudo Smaranadche ideal I={0,4} of X in example (2.4) is a pseudo Smaranadche completely closed ideal of X.

Proposition (2.13) Let X is a pseudo Smarandache BH –algebra .Then every a pseudo Smarandache completely closed ideal of X is a pseudo Smarandache closed ideal of X.

Proof: It is clear

Remark(2.14) The following example shows that convers of proposition is not correct in ganeral.

Example(2.15) Consider the a pseudo Smarandache BH-algebra X={0,1,2,3} with binary operation"*" "#" defined by the following tables.

*	0	1	2	3
0	0	0	1	3
1	1	0	3	1
2	2	3	0	2
3	3	2	1	0

#	0	1	2	3
0	0	0	2	3
1	1	0	3	1
2	2	3	0	3
3	3	3	1	0

And Q={0,1} , Then X a pseudo Smarandache BH- algebra where The pseudo Smarandache ideal I ={0,1,2} is a pseudo Smarandache closed ideal of X.
But is not a pseudo Smarandache completely closed ideal of X. Since,$1 * 2 = 3 \notin I$,$1\#2 = 3 \notin I$ *and* $1 , 2 \in I$

Remark (2.16)let X a pseudo Smarandache BH- algebra and I be a pseudo completely closed ideal of X then I is a pseudo Smarandache completely closed ideal of X .

Proposition (2.17)Let X be a pseudo Smarandache BH- algebra and I be apseudo Smarandache closed ideal such that x*y , x#y∉ I for all x∉I and y∈I, then I is a pseudo closed ideal of X.

Proof: Let I be a pseudo Smarandache closed ideal of X
⟹I is a pseudo Smarandache ideal of X
By theorem 1 I is a pseudo Smarandache closed ideal of X
Implies that $0 * x, 0 \# x \in I$
Therefore, I is a pseudo closed ideal of X

Proposition (2.18) Let X be a pseudo Smarandache BH- algebra and I be a pseudo Smarandache completely closed ideal such that x * y, x # y ∉ I for all x∉I and y∈I, then I is pseudo completely closed ideal of X

Proof: Let I be a pseudo Smarandache completely closed ideal of X. This yield
I is a pseudo Smarandache ideal of X. by theorem (2.8) we have
I is a pseudo ideal of X [since I a pseudo Smarandache completely closed ideal of X]
It followos $x * y$ *and* x # y ∈I, for all x, y∈I
Hence, I is a pseudo completely closed ideal of X

Proposition (2.19) Let { $I_i, i \in \lambda$ } be a family of A Pseudo Smarandache ideal of pseudo Smarandache BH – algebra. Then $\bigcap_{i\in\lambda} I_i$ is a pseudo Smarandache ideal of X .

Proof: *i. since* $0 \in I_i$,$\forall i \in \lambda \Rightarrow 0 \in \bigcap_{i\in\lambda} I_i$

ii. let $x * y$,$x\#y \in \bigcap_{i\in\lambda} I_i$, $y \in \bigcap_{i\in\lambda} I_i$

$\Rightarrow x * y , x\#y \in I_I$ *and* $y \in I_i$,$\forall i \in \lambda$

$\Rightarrow x \in I_i$ $\forall i \in \lambda$ [since I_i is a pseudo Smarandache ideal of X ,$\forall i \in \lambda$]

$\Rightarrow x \in \bigcap_{i\in\lambda} I_i \Rightarrow \bigcap_{i\in\lambda} I_i$ is a pseudo Smarandache ideal of X.

Proposition (2.20) Let { $I_i, i \in \lambda$ } be a family of a pseudo Smarandache closed ideal of a pseudo Smarandache BH – algebra . Then $\bigcap_{i\in\lambda} I_i$ is a pseudo Smarandache closed ideal of X.

Proof: Since I_i is a pseudo Smarandache closed ideal of X , $\forall i \in \lambda$

$\Rightarrow I_i$ is a pseudo Smarandache ideal of X, $\forall i \in \lambda$. [From propos

$\bigcap_{i\in\lambda} I_i$ is a pseudo Smarandache ideal of X. Now, let $x \in \bigcap_{i\in\lambda} I_i \Rightarrow x \in I_i, \forall i \in \lambda \Rightarrow 0 * x, 0\#x \in I_i$

$\Rightarrow 0 * x, 0\#x \in \bigcap_{i\in\lambda} I_i$

So, $\bigcap_{i\in\lambda} I_i$ is a pseudo Smarandache closed ideal of X.

Proposition (2.21) Let { $I_i, i \in \lambda$ } be a family of a pseudo Smarandache completely closed ideal of a pseudo Smarandache BH – algebra . Then $\bigcap_{i\in\lambda} I_i$ is a pseudo Smarandache completely closed ideal of X.

Proof Since I_i is a pseudo Smarandache completely closed ideal of X , $\forall i \in \lambda$ I_i is a pseudo Smarandache ideal of X , $\forall i \in \lambda$.[From proposition (2.9)] we get
$\bigcap_{i\in\lambda} I_i$ a pseudo Smarandache ideal of X, Now ,let $x, y \in \bigcap_{i\in\lambda} I_i \Rightarrow x, y \in I_i$, $\forall i \in \lambda$.

Then $x * y, x\#y \in I_i$, $\forall i \in \lambda$. [since I_i is a pseudo Smarandache completely closed ideal of X. Hence, $x * y, x\#y \in \bigcap I_i$, $\forall i \in \lambda$

Therefore, $\bigcap_{i\in\lambda} I_i$ a pseudo Smarandache completely closed ideal of X.

Remark (2.22) Let { $I_i, i \in \lambda$ }be a family of a pseudo Smarandache ideal of a pseudo Smarandache BH-algebra X. $\bigcup_{i\in\lambda} I_i$ may not be a pseudo Smarandache ideal of X.

Example(2.23)Consider the a pseudo Smarandache BH- algebra X={0,1,2,3,4} with the binary operation "*" and "#"defined by the following tables.

*	0	1	2	3	4
0	0	0	2	3	4
1	1	0	3	2	1
2	2	2	0	1	4
3	3	3	2	0	2
4	4	4	1	2	0

#	0	1	2	3	4
0	0	0	0	0	1
1	1	0	2	3	4
2	2	1	0	2	2
3	3	2	3	0	1
4	4	4	1	2	0

and Q ={0,1} the subsets I_1 ={0,2} and I_2 ={0,3} are a pseudo Smarandache ideal of X.
But $I_1 \cup I_2$ ={0,2,3} is not a pseudo Smarandache ideals of X.

Since$1 * 2 = 3 \in I_1 \cup I_2$,$1\#2 = 2 \in I_1 \cup I_2$ *but* $1 \notin I_1 \cup I_2$

Proposition (2.24) Let $\{ I_i, i \in \lambda \}$ be a chain of a pseudo Smarandache ideal of a pseudo Smarandache BH –algebra X. Then $\bigcup_{i\in\lambda} I_i$ is a pseudo Smarandache ideal of X.

Proof

i. $0 \in I_i$, $\forall i \in \lambda$ [since each I_i is a pseudo Smarandache ideal 0f X, $\forall i \in \lambda$] $\Rightarrow 0 \in \bigcup_{i\in\lambda} I_i$

ii. let $x * y, x\#y \in \bigcup_{i\in\lambda} I_i$ and $y \in \bigcup_{i\in\lambda} I_i$. There exists $I_k \in \{I_i\}i \in \lambda$ such that $x * y, x\#y \in I_k$ and $y \in I_i$ since $\{I_i\}i \in \lambda$ a chain. So, $x \in I_i$ since I_k is a pseudo Smarandache ideal 0f X. Therefore $x \in \bigcup_{i\in\lambda} I_i$

$\bigcup_{i\in\lambda} I_i$ a pseudo Smarandache ideal 0f X

4. Conclusion

In this paper, the notions of a pseudo Smarandache BH-algebra, a pseudo Smarandache ideal of BH-algebra, a pseudo Smarandache closed ideal of BH-algebra, a pseudo Smarandache completely closed ideal of a BH-algebra are introduced. Furthermore, the results are examined in terms of the relationships between a pseudo Smarandache closed ideal of BH-algebra, a pseudo Smarandache completely closed ideal of BH-algebra

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