International Journal of Engineering & Technology, 7 (4.42) (2018) 130-133

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Shape Parameter of Extended Uniform Cubic B-Spline in Designing Three Dimensional Objects

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Abstract

In Computer Aided Geometric Design (CAGD), B-spline curves are piecewise polynomial parametric curves that play an important role. CAGD which has been widely used, brings the good impact of computers to industries such as automobile. To meet engineering requirements, Extended Cubic Uniform B-Spline is proposed to be applied in creating new objects. Furthermore, three dimensional objects such as rod, bottle and others can be generated from Extended Cubic Uniform B-Spline curves by using translation technique of sweep surface method. In this research, the three-dimensional objects are formed by transforming Extended Cubic Uniform B-Spline with degree 4 by using translation technique. The advantage of using Extended Cubic Uniform B-Spline is the curve can be modified by changing the value of shape parameter. Various shapes of three dimensional objects can be formed by using different shape parameters. The smoothness of three dimensional objects is analyzed by shape parameter value from $\lambda = 0$ to $\lambda = 1$. The result shows object with $\lambda = 0, 0.1, 0.5, 0.8$, and $\lambda = 1$ are smooth.

***Keywords**: B-spline curve, Extended Cubic B-spline curve, Sweep Surface, Translation*

1. Introduction

Extended Cubic Uniform B-Spline curve was proposed to overcome the disadvantage of Cubic Uniform B-Spline which is a traditional global interpolation method. The basis function of Extended Cubic Uniform B-Spline is constructed by linear combination of the Cubic Uniform B-spline basis functions. Extended Cubic Uniform B-Spline has a shape parameter and is introduced within the basis function. The shape of curve can be modified by changing the shape parameter value λ to meet the needs [5]. Thus, changes to any data point will not require solving all the linear system again. The user can modify the curve globally by adjusting the shape parameter λ. If $\lambda = 0$, Extended Cubic Uniform B-Spline basis will degenerate into cubic B-spline basis [5]. Meanwhile, Extended Cubic Uniform B-Spline shares the same properties such as local support, nonnegativity, partition of unity and C^2 continuity when $\lambda \in [-8,1]$ [6].

The three-dimensional objects can be formed by using Sweep Surface method. Sweep Surface is an important method and is power to determine the object in three dimensional [1,2,3,5,7,9,10,11,12]. Rod, bottle, vase and others can be formed by using sweep surface method [1]. Translation is one of the techniques in Sweep Surface method; it is also known as one of the simple Sweep Surface method. Translational technique is a sweep along a straight line and sweep of the two-dimensional object along the normal direction [1]. In other words, translation surface is a transition of sweeping where the contour moves from one end of the line segment to another line segment perpendicularly [9].

The main objective of this paper is to study on an Extended Cubic Uniform B-Spline degree 4 curves with shape parameter value from $\lambda = 0$ to $\lambda = 1$. Then, Extended Cubic Uniform B-Spline curves were used to generate the three-dimensional objects by using Translation technique of Sweep Surface method. The shapes obtained were analyzed based on their smoothness.

2. Methodology

2.1. Extended Cubic B-Spline Curve

Extended Cubic Uniform B-Spline was used to design curve and generate the object. The basis functions of Extended Cubic Uniform B-Spline degree 4 for $t \in [0,1]$ are as follows [5].

$$\begin{cases} b_0^4(t) = \dfrac{1}{24}(4-\lambda-3\lambda t)(1-t)^3 \\ b_1^4(t) = \dfrac{1}{24}\left[16+2\lambda-12(2+\lambda)t^2+12(1+\lambda)t^3-3\lambda t^4\right] \\ b_2^4(t) = \dfrac{1}{24}\left[4-\lambda+12t+6(2+\lambda)t^2-12t^3-3\lambda t^4\right] \\ b_3^4(t) = \dfrac{1}{24}\left[4(1-\lambda)+3\lambda t\right]t^3 \end{cases}$$

The basis functions satisfy the properties as in Theorem 1.

Theorem 1: The basis functions $b_i^k(t)$, where k=4,5,6, and $i = 0,1,2,3$, satisfy

i. $\sum_{i=0}^{3} b_i^k(t) = 1$ (convex hull)

ii. $b_i^k(t) = b_{3-i}^k(1-t)$ (symmetric)

iii. When $-k(k-2) \le \lambda \le 1$, $b_i^k(t) \ge 0$, $t \in [0,1]$. (positivity)

The graphs of basis functions for different values of λ are shown in Figure 1.

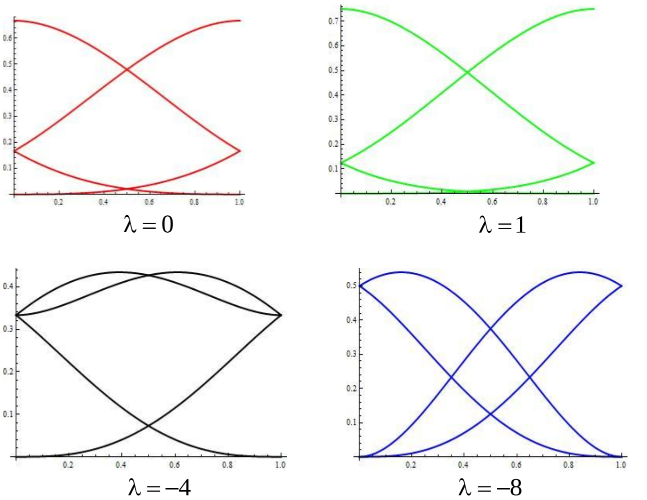


Fig. 1: Graph basis function of Extended Cubic Uniform B-Spline degree 4 with different value of shape parameter λ .

Fig 1 above shows that the graph basis function of Extended Cubic Uniform B-Spline with different value of shape parameter, λ for degree 4. Different basis will form different shape of curve. It can be observed that higher value of λ will tense the curve shape without changing its original shape. However, when λ value becomes lesser (towards -8), it changes the shape of the curve and has more attractive effect.

The polynomial curve segments for

$u \in [u_i, u_{i+1}]$, $i = 3,4,\ldots\ldots,n$ are defined as follows [5].

$$C_{j,k}(\lambda : t) = \sum_{i=0}^{3} b_i^k(t) P_{j+i-3}, \quad k = 4,5,6$$

Where, $P_i \in R^d$ $(d = 2,3, i = 0,1,2,\ldots.n)$ is control point and knots is $u_1 < u_2 < \ldots.. < u_{n+1}$.

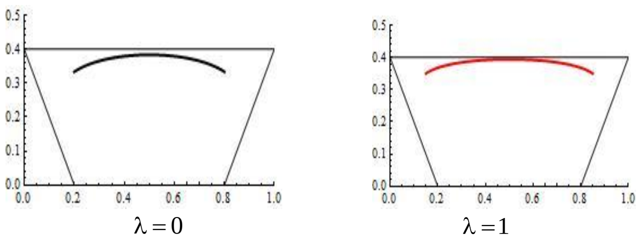
The polynomial curve is defined as follows [5].

$$C_k(\lambda; u) = C_{i,k}\left(\lambda; \frac{u-u_i}{h_i}\right), \quad u \in [u_i, u_{i+1}]$$

Where,

$$t = \frac{u-u_i}{h_i}, h_i = u_{i+1} - u_i .$$

Curve segments with different values of λ are shown in Figure 2.



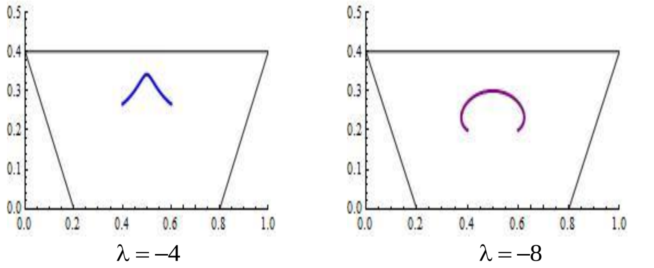


Fig. 2: Extended cubic uniform B-spline degree 4 with different value of shape parameter λ .

The curves of Extended Cubic Uniform B-Spline for degree 4 with the different value of shape parameter, λ are shown in Fig 2 above. The curves obtained does not touch the end control points. It is called as close curve. The distant between close curve and control polygon is changed based on the values of shape parameter, λ and the degree of Extended Cubic Uniform B-spline curve. As discussed for Fig. 1 before, positive values of λ will tense the curve towards its control polygon. However, negative values of λ give different effect on curve shape.

2.2. Translation Technique

In addition, the Frenet Frame method is usually considered a natural coordinate system for analyzing curves [3]. The use of this method is also to determine the relationship between the cross section and the trajectory in which this coordinate system relies only on trajectory local traits and has tangent units T, normal N and binormal B for the trajectory [8].

In this study, the parameter along the trajectory is represented by the letter v and the parameters along the cross section are represented by the letter u. The two-dimensional cross-sectional curve is represented by g (u) and the trajectory curve r (v) [10].

$$g(u) = [g_1(u), g_2(u)]^T \quad u \in [u_{0,} u_n]$$

$$r(v) = [r_x(v), r_y(v), r_z(v)]^T \quad v \in [v_{0,} v_n]$$

The formula used in to obtain the value of the vector tangent T, normal N and binormal B are as follows [10];

Tangent T,

$$T(v) = \frac{r'(v)}{\|r'(v)\|}$$

Normal N,

$$N(v) = \frac{T'(v)}{\|T'(v)\|}$$

Binormal B,

$$B(v) = T(v) \times N(v)$$

The positions of T, N and B are illustrated in Figure 3.

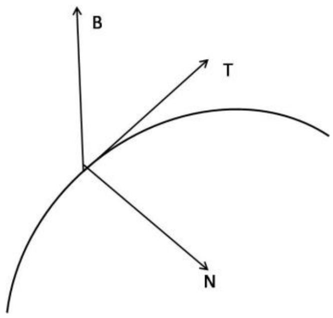


Fig 3: Position of tangent line, normal and binormal at a point

The tangent vector will always be normal to the flat surface of the cross section. While normal and binormal vector define the direction of coordinate axes on the flat surface of the cross section [8].

However, the Fernet Frame method does not specify binary and normal vector at a point where the trajectory continuity value is empty [3]. In addition, this curvature also affects the continuity of a point, the orientation of the cross section and the direction of the normal and binormal line [8]. Subsequently, using normal and binormal vector, translation technique can be defined as follows [2];

$$S(u,v) = r(v) + g_1(u)N(v) + g_2(u)B(v)$$

Where,

$r(v) =$ Trajectory curve.

$g(u) =$ Cross section curve.

$N(v) =$ Normal vector.

$B(v) =$ Binormal Vector.

3. Result and Discussion

3.1. Designing Three-Dimensional Object

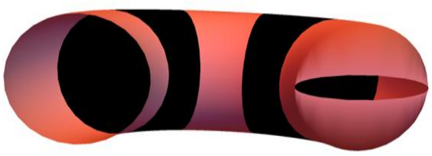
The extended cubic uniform B-spline curves were transformed into three dimensional objects. A circle was translated along the extended cubic uniform B-spline curve degree 4 with value shape parameter value from $\lambda = 0$ to $\lambda = 1$ to form a rod in three dimensional.

$\lambda = 0$

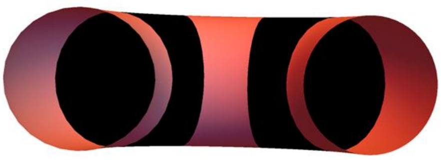
$\lambda = 0.1$

$\lambda = 0.2$

$\lambda = 0.3$

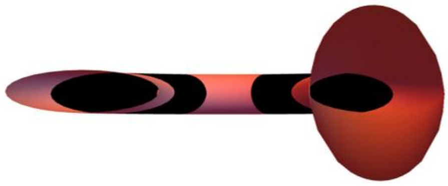


$\lambda = 0.4$



$\lambda = 0.5$

$\lambda = 0.6$



$\lambda = 0.7$

$\lambda = 0.8$

$\lambda = 0.9$

$\lambda = 1.0$

Fig 4: Rod in three dimensional with different value of shape parameter.

Result in Fig 4 shows that various designs of rod were obtained by using different values of shape parameter. Rod with $\lambda = 0, \lambda = 0.1, \lambda = 0.5, \lambda = 0.8$ and $\lambda = 1.0$ were smooth at the end of the rod compared to $\lambda = 0.2, \lambda = 0.3, \lambda = 0.4, \lambda = 0.6, \lambda = 0.7$ and $\lambda = 0.9$ which gave uncertain shapes and did not meet the needs at all.

4. Conclusion

Two dimensional curves can be generated by using extended cubic uniform B-spline curve degree 4. Different value of shape parameter can produce different shape of curve. Thus, it is cost-saving as all linear systems do not have to be solved many times to change the curve. The curves can be analyzed by consider the distant between the curve and the control polygon. The curve with $\lambda = 1$ will approach the control polygon and more distant when the value of $\lambda = -8$. Therefore, curve with $\lambda = 1$ will give more accurate result.

Next, various shapes of three dimensional objects can be formed by transforming extended cubic uniform B-spline curve degree 4 by using translation technique. In this research a rod in three dimensional was formed to test the ability of translation technique. The changes value of shape parameter λ will yield small changes in the shapes of curve but contribute more changes in the shapes of three dimensional objects. $\lambda = 0, \lambda = 0.1, \lambda = 0.5, \lambda = 0.8$ and $\lambda = 1.0$ produced smooth result at the end of the translation compared to $\lambda = 0.2, \lambda = 0.3, \lambda = 0.4, \lambda = 0.6, \lambda = 0.7$ and $\lambda = 0.9$ which result uncertain shapes and did not meet the needs at all. However, it can be an alternative to the design the three-dimensional object. $\lambda = 0, \lambda = 0.1, \lambda = 0.5, \lambda = 0.8$ and $\lambda = 1.0$ can be used to produce three-dimensional objects which are similar to the model and others value of λ if something unique and creative are required.

Acknowledgement

The authors would like to acknowledge Universiti Teknologi MARA for the financial support (Pair to Publish Grant). All the objects were generated using Mathematica 11 software.

References

[1] Al-Enzi AMJ. (2008). Studying curve interpolator for CNC System Master Thesis, University of Technology.
[2] Ali MJ (2005). Permukaan sapuan translasi dan putaran lengkung kuartik serupa bezier. *Simposium Sains Kebangsaan Matematik ke XIII*. pp. 495-499.
[3] Azernikov S. (2008). Sweeping solids on manifolds. *Proceeding of the 2008 ACM Symposium on Solid and Physical Modelling*, pp. 249-256.
[4] Elber G. (1997). Global error bounds and amelioration of sweep surfaces. *Computer-Aided Design*, 29 (6), 441-447.
[5] Gang X. & Zhao WG. (2008). Extended cubic uniform b-spline and a -b-spline. *Acta Automatica Sinica*. 34(8).
[6] Hamid ANN, Majid AA, & Ismail AI (2010). Extended cubic b-spline interpolation method applied to linear two-point boundary value problems. *World Academy of Science, Engineering and Technology*, 4(2), 276-278.
[7] Jung HB & Kim K. (2011). The redefinition of B-spline curve. *International Journal of Advanced Manufacturing Technology*, 57(1), 265-270.
[8] Marhl M, Guid N, Oblonsek C, & Horvat M. (1996). Extensions of sweep surface constructions. *Comput & Graphics*, 20(6), 893-903.
[9] Pocock L & Rosebush J. (2002). *The computer animator's technical handbook*. Morgan Kauffman Publisher, Burlington, USA.
[10] Salomon D. (2007). Curves and surfaces for computer graphics. Curves and Surfaces for Computer Graphics. Springer Science & Business Media.
[11] Tai CL & Loe KF (1996). Surface design via deformation of periodically swept surfaces. *The Visual Computer*, 12(10), 475-483.
[12] Wang X & Qin J. (2017). Surface editing using sweep surface 3D models. Journal on Image and Video Processing, (57), 1-11.