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Research paper

Mean Square Cordial Labeling of Some Cycle Related Graphs

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Abstract

In this paper we investigate the Mean square cordial labeling for some cyclic graphs like Helm graph H_p, Closed helm graph CH_p, Gear graph G_p, Sunlet graph SL_p, Fan graph $F_{1,p}$ and $C_p ʘ nK_1$.

Keywords*: Mean square cordial labeling, Closed helm graph, Gear graph, Sunlet graph.*

1. Introduction

Many graph labeling [1] techniques have been discussed by different researchers and it is still getting enrichment due to its broad range of applications in various fields like electrical circuit theory, social psychology, addressing communication network Systems, finding optimal circuit layouts, channel assignment process etc..For basics terms and notations we follow Harary[2]. Cahit initiated the Cordial labeling [3] and Ponraj and et al[4] paved the way to mean cordial labeling of a graph .Mean square cordial labeling (MSCL)was first introduced by A.Nellai murugan et al and this labeling technique is applied for few classes of graphs[5].In addition to that they discussed that some path ,tree and cycle related graph admits MSCL [6],[7]. Along with that MSCL of some acyclic graphs and upper approximations [8] were studied by Dhanlakshmi et al . In this paper we investigate the MSCL for some cyclic graphs like Helm graph H_p, Closed helm graph CH_p, Gear graph G_p, Sunlet graph SL_p, Fan graph $F_{1,p}$ and $C_p ʘ nK_1$.

2. Preliminaries

A.Nellai murugan and et al defined "A Mean Square Cordial labeling(MSCL) of a Graph G(V,E) with p vertices and q edges is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u)^2 + f(v)^2)/2 \rceil)$ where $\lceil x \rceil$ (ceil(x)) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled 0 and the number of edges labeled with 1 differ by at most 1".

3. Main results

Theorem: 3.1 MSCL of a Helm graph H_p when p is odd $p \geq 3$.

Proof: Consider a helm graph H_p be G.
Let the vertex set and edge set as V (G) = {u0, v_i, ui ,:1≤i≤p}

and E(G) ={[(u_0u_i):1≤i≤ p]∪ [(u_iv_i:1≤i≤ p] ∪[(u_iu_{i+1}):1≤i≤p-1] ∪ [(u_1u_p):1≤i≤p]} where u_0 ,u_i and v_i be the centre vertex ,vertices on a cycle and pendent vertices respectively.
Consider the elements of the vertex set maps either 0 or 1.

$$f(u_0) = 0$$

$$f(u_i) = \begin{cases} 0, 1 \leq i \leq \frac{p+1}{2} \\ 1, \frac{p+3}{2} < i \leq p \end{cases}$$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p+1}{2} < i \leq p \end{cases}$$

The edge labeling pattern of the above vertex labeling is as follows

$$f(u_0u_i) = \begin{cases} 0, 1 \leq i \leq \frac{p+1}{2} \\ 1, \frac{p+3}{2} < i \leq p \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p+1}{2} < i \leq p \end{cases}$$

$$f(u_iv_i) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p+1}{2} < i \leq p \end{cases}$$

$$f(u_1u_p) = 1,$$

It is very clear that the above labeling pattern proved that the

difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.
Hence helm graph H_p admits MSCL when m is odd $\forall p \geq 3$.

Example 3.1: MSCL of a Helm graph H_5 is shown in the Fig 1

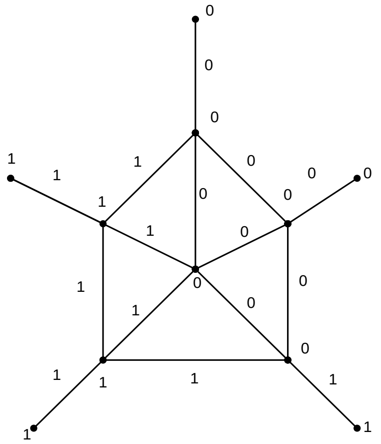

Fig 1:

Theorem: 3.2 MSCL of a closed helm graph CH_p $\forall p \geq 2$.

Proof: Consider a closed helm graph $P_p \Theta K_1$ be G.
Let V(G) = { u_0, v_i,u_i :1≤i≤p} and E(G) ={[(u_0u_i):1≤i≤ p]∪ [(u_iv_i:1≤i≤ p] ∪ [(v_iv_{i+1}):1≤i≤p-1] ∪ [(u_iu_{i+1}):1≤i≤p-1]∪ [(v_1v_p):1≤i≤p]∪ [(u_1u_p):1≤i≤p]} where u_0 ,u_i and v_i be the centre vertex vertices on a inner cycle and vertices on a outer cycle respectively.
Consider the elements of the vertex set maps either 0 or 1.
$f(u_0) = 0$,
$f(u_i) = 0$, 1≤i≤p
$f(v_i) = 1$, 1≤i≤p
The edge labeling pattern of the above vertex labeling is as follows
$f(u_0u_i) = 0$, 1≤i≤p
$f((u_iu_{i+1}) = 0$, 1≤i≤p-1
$f(u_iv_i) = 1$, 1≤i≤p
$f((v_iv_{1+1}) = 1$, 1≤i≤p-1
$f(u_pu_1) = 0$
$f(v_pv_1) = 1$

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.
Hence closed helm graph CH_p admits MSCL when $\forall p \geq 3$.

Example 3.2: MSCL of a closed helm graph CH_6 shown in the Fig 2

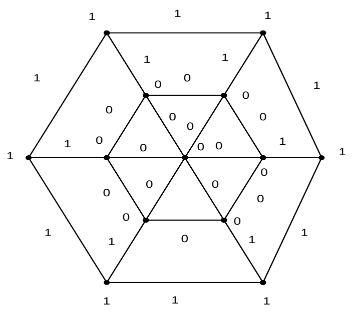

Fig 2

Theorem: 3.3 MSCL of a Gear graph G_p when p is odd $\forall p \geq 3$.
Proof: Consider a Gear graph G_p be G
Let V(G) = { u_0, v_i,w_i :1≤i≤p} and E(G) ={[(u_0u_i):1≤i≤ p]∪ [(v_iw_i:1≤i≤ p] ∪ [(v_iw_{i+1}):1≤i≤p-1]∪ [(w_pu_1)]} where u_0 ,v_i and w_i be the centre vertex ,vertices on a cycle and the centre vertices of a edge of a cycle respectively.

Consider the elements of the vertex set maps either 0 or 1.

$$f(u_0) = 0$$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq \frac{p+1}{2} \\ 1, \frac{p+3}{2} < i \leq p \end{cases}$$

$$f(w_i) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p+1}{2} < i \leq p \end{cases}$$

The edge labeling pattern of the above vertex labeling is as follows

$$f(u_0u_i) = \begin{cases} 0, 1 \leq i \leq \frac{p+1}{2} \\ 1, \frac{p+3}{2} < i \leq p \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p+1}{2} < i \leq p \end{cases}$$

$$f(u_iv_i) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p+1}{2} < i \leq p \end{cases}$$

$$f(u_1u_p) = 1$$

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.

Hence Gear graph G_p admits MSCL when m is odd $\forall p \geq 3$.

Example 3.3: MSCL of a Gear graph G_5 is shown in the Fig 3.

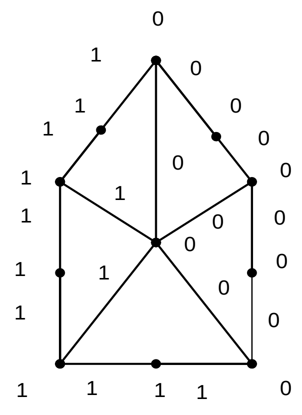

Fig 3

Theorem: 3.4 MSCL of a sunlet graph S_p $\forall p \geq 2$.
Proof: Consider a sunlet graph S_p be G.
Consider V(G) = { v_i,u_i :1≤i≤p} and E(G) ={ [(u_iu_{i+1}:1≤i≤ p-1]

$\cup$ $[(u_iv_i):1\leq i\leq p]$]$\cup$ $[(u_1u_p):1\leq i\leq p]\}$ where u_i and v_i be the vertices on a cycle and pendent vertices respectively.
Consider the elements of the vertex set maps either 0 or 1.
$f(u_i) = 0$, $1\leq i\leq p$
$f(v_i) = 1$, $1\leq i\leq p$

The edge labeling pattern of the above vertex labeling is as follows
$f((u_iu_{i+1}) = 0$, $1\leq i\leq p-1$
$f(u_iv_i) = 1$, $1\leq i\leq p$
$f(u_nu_1) = 0$

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.
Hence sunlet graph admits MSCL when $\forall\, p \geq 3$.

Example 3.4 MSCL of a Sunlet graph SL_8 shown in the Fig 4

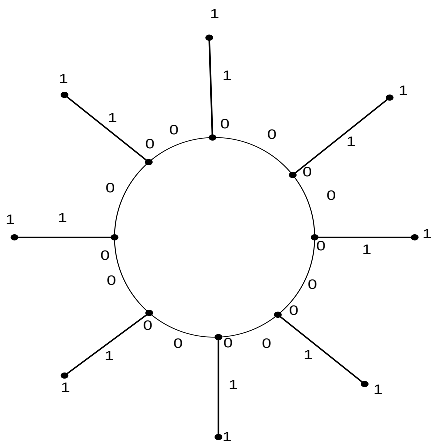

Fig 4

Theorem: 3.5 MSCL of a Fan graph $F_{1,p}$ when p is even $\forall\, p \geq 2$.
Proof: Consider a Fan graph $F_{1,n}$ be G
Let $V(G) = \{ u_0,u_i:1\leq i\leq p\}$ and $E(G) =\{[(u_0u_i):1\leq i\leq p]\cup [(u_iu_{i+1}):1\leq i\leq p-1]\cup [(u_pu_1)]\}$ where u_0 and u_i be the apex vertex and the remaining vertices other than apex respectively.

Consider the elements of the vertex set maps either 0 or 1.

$$f(u_0) = 0$$

$$f(v_i) = \begin{cases} 0, 1 \leq i \leq \frac{p}{2} \\ 1, \frac{p}{2}+1 \leq i \leq p \end{cases}$$

The edge labeling pattern of the above vertex labeling is as follows

$$f(u_0u_i) = \begin{cases} 0, 1 \leq i \leq \frac{p}{2} \\ 1, \frac{p}{2}+1 \leq i \leq p \end{cases}$$

$$f(u_iu_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{p-1}{2} \\ 1, \frac{p-1}{2} \leq i \leq p \end{cases}$$

It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.
Hence Fan graph $F_{1,p}$ admits MSCL when p is even $\forall\, p \geq 2$.

Example 3.5 MSCL of a Fan graph $F_{1,6}$ shown in the Fig 5

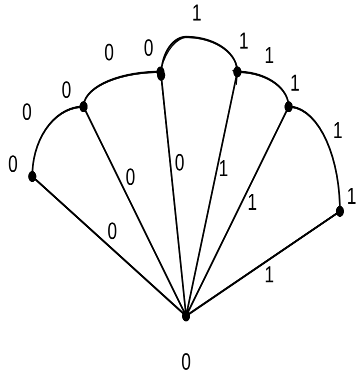

Fig 5

Theorem: 3.6 MSCL of a graph $C_p \Theta nK_1$ when n is odd $\forall\, p$.
Proof: Consider a graph $C_p\Theta nK_1$ be G.
Let $V(G) = \{ u_i,v_j,:1\leq i\leq p, 1\leq j\leq pn \}$ and $E(G) =\{ [(u_iu_{i+1}:1\leq i\leq p-1] \cup [(u_iv_j):1\leq i\leq p, 1\leq j\leq pn] \}$ where u_i and v_j be the vertices on a cycle and pendent vertices respectively.
Consider the elements of the vertex set maps either 0 or 1.
$f(u_i) = 0$, $1\leq i\leq p$

$$f(v_j) = \begin{cases} 1, j \equiv 1 \bmod 2 \\ 0, j \equiv 0 \bmod 2, \end{cases}$$

The edge labeling pattern of the above vertex labeling is as follows
$f((u_iu_{i+1}) = 0$, $1\leq i\leq p-1$
$f(u_iv_j) = 1$, $1\leq i\leq p$, j is odd
$f(u_iv_j) = 0$, $1\leq i\leq p$, j is even
It is very clear that the above labeling pattern proved that the difference between the vertices and edges labeled with 0 and 1 differ atmost by 1.
Hence sun let graph admits MSCL $\forall\, p$

Example 3.6 MSCL of a graph $C_p\Theta nK_1$ shown in the Fig 6

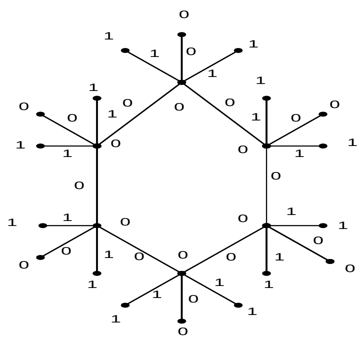

Fig 6

5. Conclusion

Here we investigated the MSCL for different cyclic graphs. Further it is an open to all the researchers in this domain to discuss the same labeling technique for various types of graphs.

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