International Journal of Engineering & Technology, 7 (4.10) (2018) 812-815

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

On Γ-TS-Acts Over Ternary Γ-Semigroups

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Abstract

We generalise the notion of acts over ternary semigroups to the Γ-TS-acts for a ternary Γ-semigroup T. Certain intrinsic notions of Γ-TS-acts are studied.

Keywords: *Ternary Γ-semigroup, Γ-TS-act, Γ-TS-congruence, Γ-TS-homomorphism, free Γ-TS-act.*

1. Introduction

Acts over semi group T, namely T-act, appeared and were used in a variety of applications such as algebraic automata theory, mathematical linguistics. We here generalize this notion to the Γ-TS-acts for a ternary Γ-semi group T. In the year 2008, Chinram. R and Thinpun. K.[1], investigated on isomorphism theorems for gamma semi groups. In 1991, Howie. J. M.[2], studied about Automata and Languages. In 2013, Hssin. Z.[3], investigated and studied about gamma modules with gamma rings of gamma endomorphism. In 2015, Vasantha. M and Madhusudhana Rao. D[4]. introduced the concept of ternary Γ-semi groups and they characterized the ternary Γ-semigroups.

2. Preliminaries

Definition 2.1[4]: Let P ≠ ∅ & Γ ≠ ∅ be two set. Then P is known as a ***Ternary* Γ*-semigroup*** if there exist a mapping from P×Γ×P×Γ×T to P which maps (g_1, α, g_2, β, g_3) $\rightarrow [g_1\alpha g_2\beta g_3]$ satisfying the condition :

$[[g_1\alpha g_2\beta g_3]\gamma g_4\delta g_5] = [g_1\alpha[g_2\beta g_3\gamma g_4]\delta g_5] =$
$[g_1\alpha g_2\beta[g_3\gamma g_4\delta g_5]]\ \forall\ g_i \in$ T, $1 \le i \le 5$ and $\alpha,\beta,\gamma,\delta \in \Gamma$.

Note 2.2[4]: For the convenience we write $r_1\alpha r_2\beta r_3$ instead of $[r_1\alpha r_2\beta r_3]$

For more preliminaries one can be go through the regerences.

3. Γ-TS-acts

Definition 3.1: Let T be a ternary Γ-semigroup as well as P ≠ ∅ with a mapping $\lambda : T\times\Gamma\times T\times\Gamma\times P \rightarrow P$ where $(s,\alpha,t,\beta,a) \rightarrow s\alpha t\beta a := \lambda(s,\alpha,t,\beta,a)$ is said to be a ***left Γ-TS-act***or a ***left Γ-TS-operand*** if $(p\alpha q\beta r)\delta s\gamma a = p\alpha(q\beta r\delta s)\gamma a$ $= p\alpha q\beta(r\delta s\gamma a)$ for all $p,q,r,s \in T, \alpha,\beta,\gamma,\delta \in \Gamma$. This is denoted by ${}_{\Gamma-TS}P$. Similarly, we can define ***lateral Γ-TS-act*** (demoted by $\underset{\Gamma-TS}{P}$) and ***right Γ-TS-act*** (denoted by $P_{\Gamma-TS}$).
Throughout this paper Γ-TS-act means left Γ-TS-act.

Note 3.2: If T has identity *e*, then $e\alpha e\beta a = a\ \forall\ a \in$ K.
Def 3.3: Let L be a Γ-TS-act. Then $l \in L$ is called to be ***zero*** of L if $l\alpha b\beta c = b\alpha l\beta c = b\alpha c\beta l = l\ \forall\ b,c \in$ T, $\alpha, \beta \in \Gamma$.

Definition 3.4: Let U beΓ-TS-act. A subset 'S ≠ ∅' is known as **Γ-*TS-sub-act*** of U if $a\alpha b\beta c \in$ S for all *a, b* ∈ T, *c*∈ S and *α, β*∈Γ.
Note 3.5: A non-empty subset S of aΓ-TS-act A is aΓ-TS-sub-act if and only if TΓTΓS ⊆ S. Clearly, T itself is a Γ-TS-act.

Note 3.6: A sub-act of the Γ-TS-act A is a left ternary Γ-ideal of the ternary Γ-semigroup T. A subset K ⊆ A is called a right ternary Γ-ideal of T if TΓTΓK ⊆ K, a two-sided ternary Γ-ideal of T if TΓTΓK ⊆ K and KΓTΓT⊆ K and a ternary Γ-ideal of T if it is two sided ternary Γ-ideal as well as TΓKΓT ⊆ K.

Def 3.7: An element *a* of a Γ-TS-act A is said to be a ***fixed*** or a ***zero*** element if *aαsβt* = *a*, for all *s, t*∈ T and *α, β*∈Γ.

Theorem 3.8: The non-empty intersection of any family ofΓ-TS-sub-acts of aΓ-TS-act ${}_{\Gamma-TS}A$ is a ternary Γ-TS-sub-act of ${}_{\Gamma-TS}A$.

proof: Let $\{S_\alpha\}_{\alpha\in\Delta}$ be a family ofΓ-TS-sub-acts of ${}_{\Gamma-TS}A$ and S= $\bigcap_{\alpha\in\Delta} S_\alpha$

Let *a,b* ∈ ${}_{\Gamma-TS}A$, *c* ∈ Sand *α, γ*∈ Γ.

$c \in$ S $\Rightarrow c \in \bigcap_{\alpha\in\Delta} S_\alpha \Rightarrow c \in S_\alpha$ for all $\alpha \in \Delta$

$c \in S_\alpha$ & $\alpha, \gamma \in \Gamma$, S_α is a Γ-TS-sub-act of ${}_{\Gamma-TS}A$

$\Rightarrow a\alpha b\gamma c \in S_\alpha$

$a\alpha b\gamma c \in S_\alpha$ for all $\alpha \in \Delta \Rightarrow a\alpha b\gamma c \in \bigcap_{\alpha\in\Delta} S_\alpha \Rightarrow a\alpha b\gamma c \in$ S.

Therefore, S is aΓ-TS-sub-act of ${}_{\Gamma-TS}A$.

Theorem 3.9: The union of any family of Γ-TS-sub-acts of aΓ-TS-act ${}_{\Gamma-TS}A$ is a Γ-TS-sub-act of ${}_{\Gamma-TS}A$.

Proof: Let $\{A_\alpha\}_{\alpha\in\Delta}$ be a family of Γ-TS-sub-acts of aΓ-TS-act ${}_{\Gamma-TS}A$.

Let A = $\bigcup_{\alpha\in\Delta} A_\alpha$. Let $a \in$ A; $b,c \in$ T, $\alpha, \beta \in \Gamma$. $a \in$ A

$\Rightarrow a \in \bigcup_{\alpha\in\Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$

$a \in A_\alpha$, $b, c \in {}_\Gamma A_T$, $\alpha, \beta \in \Gamma$, A_α is a Γ-TS-act of T

$\Rightarrow b\alpha c\beta a \in A_\alpha \subseteq \bigcup_{\alpha\in\Delta} A_\alpha$ = A $\Rightarrow b\alpha c\beta a \in$ A.

Therefore, A is a Γ-TS-sub-act of ${}_{\Gamma-TS}A$.

Definition 3.10: Let ${}_{\Gamma-TS}U$ and ${}_{\Gamma-TS}V$ are Γ-T-acts. A mapping $f: {}_{\Gamma-TS}U \to {}_{\Gamma-TS}V$ is said to be a ***Γ-TS-homomorphism*** provided $f(s\alpha t\beta a) = s\alpha t\beta f(a)$ for every $s, t \in$ T, $a \in$ U and $\alpha, \beta \in \Gamma$.

Definition 3.11: Let ${}_{\Gamma-TS}P$ and ${}_{\Gamma-TS}Q$ are Γ-TS-acts. A mapping $f: {}_{\Gamma-TS}P \to {}_{\Gamma-TS}Q$ is said to be a ***Γ-TS-monomorphism*** provided f is a one-one Γ-TS-homomorphism.

Definition 3.12: Let ${}_{\Gamma-TS}R$ and ${}_{\Gamma-TS}S$ be Γ-TS-acts. A mapping $f: {}_{\Gamma-TS}R \to {}_{\Gamma-TS}S$ is said to be a ***Γ-TS-epimorphism*** provided f is an onto Γ-TS-homomorphism.

Definition 3.13: Let ${}_{\Gamma-TS}Y$ and ${}_{\Gamma-TS}Z$ be Γ-TS-acts. A mapping $f: {}_{\Gamma-TS}Y \to {}_{\Gamma-TS}Z$ is said to be a ***Γ-TS-isomorphism*** provided f is a one-one Γ-TS-homomorphism as well as an onto Γ-TS-homomorphism.

Definition 3.14: AΓ-TS-act B containing (aΓ-TS-isomorphic copy of) a Γ-TS-act A as a subact is called an ***extension*** of A.

Example 3.15: As a very interesting example of acts, used in computer science as a convenient means of algebraic specification of process algebras, consider the ternary Γ-monoid (N^∞,Γ,[ ],∞), where N is the set of natural numbers, Γ is the any set and N^∞ = N ∪{∞} with n<∞,∀$n \in$ N and $[m\alpha n\beta p]$ = min $\{m, n, p\}$ for $m, n, p \in N^\infty$, $\alpha, \beta \in \Gamma$. Then a Γ-TN^∞-actis called a ***projection algebra***.

Th 3.16: Let T be a ternary Γ-semi group, ${}_{\Gamma-TS}K$ is a Γ-TS-act and *f*: K → T is a Γ-TS-homomorphism. Then A is a ternary Γ-semi group.

Proof: We have a mapping g: $K \times \Gamma \times K \times \Gamma \times K \to K$ where $(a,\alpha,a',\beta,a'') \to a\alpha a'\beta a'' := f(a)\alpha a'\beta a''$ for all $a, a', a'' \in A$ and $\alpha, \beta \in \Gamma$. Let $a,b,c,d,e \in A$ and $\alpha,\beta,\gamma,\delta \in \Gamma$. Then

$$\begin{aligned}(a\alpha b\beta c)\gamma d\delta e &= (f(a)\alpha b\beta c)\gamma d\delta e = f(f(a)\alpha b\beta c)\gamma d\delta e\\ &= f(a)\alpha f(b)\beta f(c)\gamma d\delta e = f(a)\alpha(f(b)\beta f(c)\gamma d)\delta e\\ &= a\alpha(b\beta c\gamma d)\delta e = a\alpha(f(b)\beta f(c)\gamma f(d))\delta e\\ &= a\alpha f(b)\beta(f(c)\gamma f(d)\delta f(e)) = a\alpha b\beta(c\gamma d\delta e)\end{aligned}$$

Therefore $(a\alpha b\beta c)\gamma d\delta e$ = $a\alpha(b\beta c\gamma d)\delta e$ = $a\alpha b\beta(c\gamma d\delta e)$ and hence A is a ternary Γ-semigroup.

Definition 3.17: Let ${}_{\Gamma-TS}U$ is a Γ-TS-act. An equivalence relation ϑ on ${}_{\Gamma-TS}U$ is said to be a ***Γ-TS-congruence*** of ${}_{\Gamma-TS}U$ provided for all $a, a' \in U$, $b,c, \in T, \alpha, \beta \in \Gamma$,

$a\rho a' \Rightarrow (a\alpha b\beta c)\rho(a'\alpha b\beta c), (b\alpha a\beta c)\rho(b\alpha a'\beta c), (b\alpha c\beta a)\rho(b\alpha c\beta a')$

Definition 3.18: The set ${}_{\Gamma-TS}K/\rho$ = $\{l_\rho : l \in {}_{\Gamma-TS}K\}$ with the Γ-action $s\alpha t\beta(l_\rho) = (s\alpha t\beta l)_\rho$ for all $s,t \in T$ and $\alpha,\beta \in \Gamma$ is known as a factor Γ-TS-act of ${}_{\Gamma-TS}K$ by ρ, and canonical surjection $\pi_\rho : {}_{\Gamma-TS}K \to {}_{\Gamma-TS}K/\rho$ where $l \to l_\rho$ is known as ***canonical Γ-TS-epimorphism.***

Definition 3.19: Let ${}_{\Gamma-TS}S$ and ${}_{\Gamma-TS}T$ be two Γ-TS-acts. A mapping $l: {}_{\Gamma-TS}S \to {}_{\Gamma-TS}T$ is a ***Γ***-TS-homomorphism, then the Γ-TS-congruence ρ = kernel l **(**simply *ker f*) on ${}_{\Gamma-TS}A$ where $a\rho a'$ iff $l(a) = l(a')$ for all $a, a' \in {}_\Gamma S_T$ is known as ***kernel Γ-TS-congruence*** of *l*.

Theorem 3.20: Let $k: {}_{\Gamma-TS}G \to {}_{\Gamma-TS}H$ is a Γ-TS-homomorphism as well as ρ be a Γ-TS-congruence on ${}_{\Gamma-TS}G$ ∃ $g\rho g' \Rightarrow$ k(a) = k(g'), i.e. $\rho \le \ker k$. Then $k': {}_{\Gamma-TS}G/\rho \to {}_{\Gamma-TS}H$ with $k'(g_\rho) := k(g)$, g $\in {}_{\Gamma-TS}G$, is the unique *Γ*-TS-homomorphism such that $k'\pi_\rho = g$. If $\rho = \ker k'$ is injective. Also if *k* is surjective, then so is k'.

Proof: The mapping k' is well-defined, because for all $g_\rho, g'_\rho \in {}_{\Gamma-TS}G,$

$g_\rho = g'_\rho \Leftrightarrow g\rho g' \Rightarrow k(g) = k(g') \Rightarrow k'(g_\rho) = k'(g'_\rho).$ For every $s,t \in T,\ \alpha,\beta \in \Gamma$ and g $\in G$,

$$\begin{aligned}k'(s\alpha t\beta g_\rho) &= k'(s\alpha t\beta g)_\rho = k(s\alpha t\beta g)\\ &= s\alpha t\beta k(g) = s\alpha t\beta k'(g_\rho)\end{aligned}$$

. Hence, k' is a *Γ*-TS-homomorphism. Also for every $g \in {}_{\Gamma-TS}G$, $(k'\pi_\rho)(g) = k'(\pi_\rho(g)) = k'(g_\rho) = k(g)$. Now we have to show k' is unique. Let there exists $k'': {}_{\Gamma-TS}G/\rho \to {}_{\Gamma-TS}H$ such that $k''\pi_\rho = k$. This implies that $k''\pi_\rho = k'\pi_\rho$. Since π_ρ is a *Γ*-TS-epimorphism, $k'' = k'$. The remainder is an easy for verification. This is called homomorphism theorem for **Γ**-TS-acts.

Corollary 3.22: Let $l: {}_\Gamma J_T \to {}_\Gamma K_T$ be a *Γ*-TS-epimorphism. Then ${}_{\Gamma-TS}J/\ker l \cong {}_{\Gamma-TS}K$.

4: Free Γ-TS-acts

Here, the notion of cyclic, free and indecomposable Γ-TS-acts are studied.

Definition 4.1: A non-empty subset P of a Γ-TS-act ${}_{\Gamma-TS}K$ is known as a generating set of ${}_{\Gamma-TS}K$ if every $k \in K$ can be expressed as $k = p\alpha q\beta u$ for some $p,q \in T$, $u \in P$ and $\alpha,\beta \in \Gamma$. In this case, we write ${}_\Gamma K_T$ = < P > = $T\Gamma T\Gamma P$, where $T\Gamma T\Gamma P = \{p\alpha q\beta u : p,q \in T, \alpha,\beta \in \Gamma, u \in P\}$. Also P is finitely generated Provided it has a finite generating set of elements. We say ${}_{\Gamma-TS}K$ a cyclic ${}_{\Gamma-TS}K$ provided ${}_{\Gamma-TS}K = < p > = T\Gamma T\Gamma p$ for some $p \in {}_{\Gamma-TS}K$.

Note 4.2: ${}_{\Gamma}L_T$ is always a generating set of itself. i.e. ${}_{\Gamma-TS}L = < L >$.

Theorem 4.3: If S is a nonempty sub set of a Γ-TS-act ${}_{\Gamma-TS}L$ & $l \in {}_{\Gamma-TS}L$. Then the following assertions hold:

(i) $K\Gamma K\Gamma l = K\alpha K\beta l$ for all $\alpha, \beta \in \Gamma$.

(ii) $K\alpha K\beta l = K\gamma K\delta l$ for all $\alpha, \beta, \gamma, \delta \in \Gamma$.

(iii) $K\Gamma K\Gamma P = K\alpha K\beta P = \{p\alpha q\beta u : p, q \in K, u \in P \text{ and } \alpha, \beta \in \Gamma\}$.

Proof: (i) Let $\alpha, \beta \in \Gamma$ and $l \in {}_{\Gamma-TS}L$. Clearly, $K\alpha K\beta l \subseteq K\Gamma K\Gamma l$. For the reverse inclusion, take $p, q \in K$ $p\alpha q\beta l = p\alpha q\beta(e\alpha e\beta l) = p\alpha(q\beta e\alpha e)\beta l \in K\alpha K\beta l$ which implies that $K\Gamma K\Gamma l = K\alpha K\beta l$ for all $\alpha, \beta \in \Gamma$. The remaining two assertions follows from (i).

This theorem express a simple characterization to generating sub sets of a Γ-TS-act.

Consider a cyclic Γ-TS-act ${}_{\Gamma-TS}L = < l >$as $T\alpha T\beta l$ for any $\alpha, \beta \in \Gamma$ and $l \in {}_{\Gamma-TS}L$, $p \in$ T. Then the map $\lambda_{s,a,\alpha,\beta} : {}_{\Gamma-TS}T \to {}_{\Gamma-TS}L$ defined by $\lambda_{s,a,\alpha,\beta}(q) = p\alpha q\beta l$ for all $q \in T$ is a Γ-TS-homomorphism. To see this, for every $u, v \in T$ and $\gamma, \delta \in \Gamma$ we have

$\lambda_{p,a,\alpha,\beta}(u\gamma v\delta t) = p\alpha(u\gamma v\delta t)\beta a = u\gamma v\delta p\alpha t\beta l = u\gamma v\delta\lambda_{p,a,\alpha,\beta}(q)$.

Now, we characterize cyclic Γ-TS-acts by means of factor Γ-TS-acts of ${}_{\Gamma-TS}T$.

Th 4.4: If a Γ-TS-act ${}_{\Gamma-TS}L$ is cyclic. Then there exists a Γ-TS-congruence ρ on ${}_{\Gamma-TS}T$ ∃ ${}_{\Gamma-TS}L \cong {}_{\Gamma-TS}T/\rho$ and the converse also hold if T is a ternary Γ-monoid.

Proof: Let ${}_{\Gamma-TS}L = < l >$as $T\alpha T\beta l$ for any $\alpha, \beta \in \Gamma$ and $l \in {}_{\Gamma-TS}L$, s∈ T. Then the Γ-TS-homomorphism $\lambda_{s,a,\alpha,\beta} : {}_{\Gamma-TS}T \to {}_{\Gamma-TS}L$ is obviously a Γ-TS-epimorphism. By using Corollary 3.22, we get ${}_{\Gamma-TS}L \cong {}_{\Gamma-TS}T / \ker \lambda_{s,a,\alpha,\beta}$. Then fix $\rho = \lambda_{s,a,\alpha,\beta}$, then we get the result.

Conversely, if ρ is a Γ-TS-congruence on a Γ-T-monoid ${}_{\Gamma-TS}T$ with unity *e*, then for all $t_\rho \in {}_{\Gamma-TS}T/\rho$ and $\alpha, \beta \in \Gamma$, $t_\rho = (t\alpha e\beta e)_\rho = t\alpha e_\rho\beta e_\rho$ which shows that ${}_{\Gamma-TS}T/\rho \cong < e_\rho >$.

Definition 4.5: A Γ-TS-act ${}_{\Gamma-TS}L$ is said to be decomposable if ∃ two Γ-TS-sub-acts ${}_{\Gamma-TS}M$ and ${}_{\Gamma-TS}N$ of ${}_{\Gamma-TS}L$ such that ${}_{\Gamma-TS}L = {}_{\Gamma-TS}M \cup {}_{\Gamma-TS}N$ and ${}_{\Gamma-TS}M \cap {}_{\Gamma-TS}N = \emptyset$. In this case, the disjoint union ${}_{\Gamma-TS}M \cup {}_{\Gamma-TS}N$ is known as a decomposition of ${}_{\Gamma-TS}L$. If not, ${}_{\Gamma-TS}L$ is known as in-decomposable. If we consider Γ-TS-acts with unique 0, then we have to change ∅ by {0} to define decomposable as well as in-decomposable Γ-TS-acts with unique 0.

Theorem 4.6: Every cyclic Γ-TS-act is in-decomposable.

Proof: Suppose that ${}_{\Gamma-TS}D = < d >$as $T\alpha T\beta d$ for any $\alpha, \beta \in \Gamma$ and $d \in {}_{\Gamma-TS}D$, s∈ T is cyclic and D = ${}_{\Gamma-TS}E \cup {}_{\Gamma-TS}F$ for some Γ-TS-sub-acts ${}_{\Gamma-TS}E$ and ${}_{\Gamma-TS}F$ of ${}_{\Gamma-TS}D$. Then $d = e\alpha e\beta d \in {}_{\Gamma}E_T$ say, then ${}_{\Gamma-TS}D = < d > \subseteq {}_{\Gamma-TS}E$ which is a contradiction.

Theorem 4.7: Let $A_i \subseteq {}_{\Gamma-TS}A, i \in \Delta$ be in-decomposable Γ-TS-sub-acts of a Γ-T-act ${}_{\Gamma-TS}A$ such as $\bigcap_{i\in I} A_i \neq \emptyset$. Then $\bigcup_{i\in I} A_i$ is an in-decomposable Γ-TS-sub-act of ${}_{\Gamma-TS}A$.

Proof: By theorem 3.7, $\bigcup_{i\in I} A_i$ is a Γ-TS-sub-act of ${}_{\Gamma-TS}A$.

Suppose there exists a decomposition $\bigcup_{i\in I} A_i = {}_{\Gamma-TS}B \cup {}_{\Gamma-TS}C$.

Take $a \in \bigcap_{i\in I} A_i$ with $a \in {}_{\Gamma}A_T$, say.

Then $a \in A_i \cap {}_{\Gamma-TS}B$ for all $i \in \Delta$.

Since $A_i = A_i \cap ({}_{\Gamma-TS}B \cup {}_{\Gamma-TS}C) = (A_i \cap {}_{\Gamma-TS}B) \cup (A_i \cap {}_{\Gamma-TS}C)$ and A_i is indecomposable, $A_i \cap {}_{\Gamma-TS}C = \emptyset$ for all $i \in I$.

Thus $\bigcup_{i\in I} A_i = {}_{\Gamma-TS}B$ It is a contradiction.

Th 4.8: Every Γ-TS-act ${}_{\Gamma-TS}A$ has a unique decomposition into in-decomposable Γ-TS-sub-acts.

Proof: Let ${}_{\Gamma-TS}A$. Than by th, 3.6, $T\alpha T\beta a$, $\alpha, \beta \in \Gamma$ is in-decomposable. Using th 4.7, we get $S_a = \bigcup\{{}_{\Gamma-TS}S \subseteq {}_{\Gamma-TS}A : {}_{\Gamma-TS}S$ is in-decomposable and $a \in {}_{\Gamma-TS}S\}$ is an in-decomposable Γ-TS-sub-act of ${}_{\Gamma-TS}A$.

For $p, q \in {}_{\Gamma-TS}L$ $V_a = V_b$ or $V_p \cap V_q = \emptyset$.

Indeed, $r \in V_p \cap V_q \Rightarrow V_p, V_q \subseteq V_r$.

Thus $p \in V_p \subseteq V_r, q \in V_q \subseteq V_r$, i.e. $V_r \subseteq V_p \cap V_q$.

Therefore, $V_p = V_q = V_r$. Denote by L' a representative subset of elements $p \in {}_{\Gamma-TS}L$ w.r.t the equivalence relation ~ defined by *p*~*q* iff $V_p = V_q$. Therefore, ${}_{\Gamma-TS}L = \bigcup_{p\in L'} V_p$ is the unique decomposition of ${}_{\Gamma-TS}L$ into in-decomposable Γ-TS-sub-acts.

Def 4.9: A set K of generating elements of a Γ-TS-act ${}_{\Gamma-TS}L$ is known as a ***basis*** of ${}_{\Gamma-TS}L$ provided every element $p \in {}_{\Gamma-TS}L$ can be uniquely expressed as $p = s\alpha t\beta u$ for some $s, t \in T, u \in K$ and $\alpha, \beta \in \Gamma$,

Theorem 4.10: Let $l : {}_{\Gamma-TS}K \to {}_{\Gamma-TS}B$ be a Γ-TS-homomorphism, then

(i) **If ${}_{\Gamma-TS}L$ is finitely generated then so is *h*(${}_{\Gamma-TS}L$).**

(ii) **If ${}_{\Gamma-TS}L$ = < P > and $i : {}_{\Gamma-TS}L \to {}_{\Gamma-TS}M$ is a Γ-TS-homomorphism, then *h*(*s*) = *i*(*s*) for every $s \in P$ implies *l* = *g*.**

(iii) **If *h* is a Γ-TS-epimorphism and ${}_{\Gamma-TS}L$ = < P >, then ${}_{\Gamma-TS}M$ = < *h*(P) >.**

(iv) **If *h* is a Γ-TS-isomorphism and ${}_{\Gamma-TS}L$ is a free Γ-TS-act, then so is ${}_{\Gamma-TS}M$.**

Proof: we just prove (iv), let P be a basis of ${}_{\Gamma-TS}L$ and then ${}_{\Gamma-TS}L$ = < P >. It follows from (iii) that ${}_{\Gamma-TS}M$ = < *h*(P) >, i.e. *h*(P) is a generating set of ${}_{\Gamma-TS}M$. Therefore, for all

$b \in {}_{\Gamma-TS}M$ there exist $s,t \in T, \alpha, \beta \in \Gamma$ and $u \in P$ such that $b = s\alpha t\beta f(u)$. Suppose that $b = s'\alpha' t'\beta' f(u')$, for $s',t' \in T, \alpha', \beta' \in \Gamma$ and $u' \in P$. Then $b = s\alpha t\beta f(u)$ = $s'\alpha' t'\beta' f(u')$. This implies that $h(s\alpha t\beta u) = h(s'\alpha' t'\beta' u')$ and hence $s\alpha t\beta u$ = $s'\alpha' t'\beta' u'$ because h is one-one. Since S is a basis. Therefore $s = s', t = t', \alpha = \alpha', \beta = \beta', h(u) = h(u')$. Hence, h(P) is a basis of ${}_{\Gamma-TS}M$.

Th 4.11: If ${}_{\Gamma-TS}K$ is a free Γ-TS-act, then $|\Gamma| = 1$.

***Proof*:** Let ${}_{\Gamma-TS}K$ is a free Γ-TS-act with a basis P.

Consider $\alpha, \beta, \alpha', \beta' \in \Gamma, s, t \in T$ and $u \in S$ By using theorem 3.3(ii), $s\alpha t\beta u \in T\Gamma T\Gamma u$ and then $s\alpha t\beta u = s'\alpha' t'\beta' u'$ for some $s',t' \in T, \alpha', \beta' \in \Gamma$ and $u, u' \in S$. Since P is a basis, $\alpha = \alpha', \beta = \beta'$.

5. Conclusion

This type of ternary structures and their generalizations, the so called Γ-TS-act rise certain hopes in view of their possible applications in Organic Chemistry. the well- known generalization of ternary semi group T is ternary γ-semi group.

Acknowledgement

Our thanks to all who supported to us for preparation of the paper.

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