International Journal of Engineering & Technology, 7 (4.10) (2018) 816-817

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

R*-I-Lc-Continuous Functions in an Ideal Space

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Abstract

Some new sets like of "R^*-LC-sets", "R^*- *I*-LC-sets" are introduced using R^*-topology. Also defined the notions of "R^*- *I*-LC-continuous maps", "I^*_R- continuous maps", "g-R^*- continuous map"s and represent "*-continuous function" as decomposition two newly defined functions.

Keywords: *R^*-LC-sets; R^*-I-LC-sets; I^*_R-closed sets; R_t-sets; I^*-R_t-sets; g-R^*- sets;*

1. Introduction

The triplet (X,τ, ***I***) usually represents an ideal topological space (ITS) where X≠Φ is a set having a topology τ on X and an ideal ***I*** ⊆ X which satisfies (i) P(A) ∈I when ever A ∈I and (ii) Union of two members of I is also a member of I.. For all A of X, A^*(***I***) of A is the set of all elements of X having the property that O ∩ A ∉ I for each of its neighborhoods O corresponds to the ideal ***I*** and the topology τ. We write just A^* in the place of A^*(***I***) if we are clear about I. From [12], $cl^*(A)=A\cup A^*$, which generates the topology $\tau^*(\boldsymbol{I})$ which is finer than τ . Manoharan *et.al* [10] studied that if $A^*\setminus A \in \boldsymbol{I}$ for any set A of an ITS with ideal *I* then A is called as R^*-perfect set . An R^*-open set is a member of an R^*-topology, which is formed by the collection of R^*-perfect sets as basis. Complement of an R^*-open set is known as an R^*-closed set . For a topological space we use TS and for Ideal topological space we use ITS .In the place of (X,τ),we use τ(X) and for (X,τ, ***I***) we use $\tau(X)_I$.

Definition 1.1

Let τ(X) be a TS and S ⊆ X then S is called

(1) Locally closed (or LC) if S is intersection of a member of τ and a closed set [3].

(2) t-set if *int*(S) = *int*(*cl*(S)) [11].

Definition 1.2

Let $\tau(X)_I$ be an ITS and S ⊆ X then

(1) S is a " t-*I*-set" if *int*(S)= *int* (*cl**(S)), [4]

(2) S is a "α^*-*I*-set if *int* (*cl**(*int*(S)))= *int*(S)",[4]

(3) S is a "*I*-LC set" if S=P∩Q, where P∈τ and Q is *-perfect, [2]

(4) S is a "Weakly- *I*-LC set" if S is intersection of a open set and *-closed set.[8].

Definition 1.3

[7] A *-continuous function f is defined as a function from an ITS to a TS namely (Y ,σ) such that $f^{-1}(S)$ is *-closed in $\tau(X)_I$ where S is closed in (Y ,σ) .

2. New Results

In this section, we introduce and study about "R^*-LC sets", "R^*_t-sets", "R^*-C_I sets" , "R^**I*-LC sets", "I^*_R-closed sets" and "I^*-R_t sets" and the relation between these sets.

Definition 2.1

Consider an ITS $\tau(X)_I$ and S ⊆ X then

(1)R^*-LC set if S is the intersection of a member of R*-topology and a closed set.

(2) R^*_t-set if S is the intersection of a member of R*-topology and a t-set.

(3) R^*-C_I-set if S is the intersection of a member of R*-topology and a α^*-*I*-set.

The set of all R^*-LC(R^*_t-sets, R^*-C_I-sets) sets are given by R^*-LC(X)(R^*_t(X), R^*-C_I(X)).

Definition 2.2

An R^*-LC-continuous function(respectively R^*_t-continuous) f is defined as a function from an ITS $\tau(X)_I$ to a TS (Y ,σ) such that $f^{-1}(S)$ is R^*-LC-set (respectively R^*_t-set) in $\tau(X)_I$ for any σ- closed set S in Y .

Definition 2.3

Consider a subset S of an ITS $\tau(X)_I$. The set S is called an "R^**I*-LC-se"t if

(i) B=C∩D, with C is a member of R*-topology and D is a *-closed set."R^**I*-LC(X)" will denote all the "R^**I*-LC-sets";

(ii) I^*_R-closed if $S^*\subset U$ whenever S ⊂ U and U is R^*-open in X.

(iii) I^*-R_t-set if S is the intersection of R^*-open set and t-*I*-set. Set of all I^*-R_t-sets of X are denoted by I^*-R_t(X).

Proposition 2.4

Let $\tau(X)_I$ be an ITS and S⊂X. Then,

(1) Whenever S is a member of R*-topology , then S ∈ R^**I*-LC(X);

(2) If S is "*-closed", then S $\in$ "R^{*}-*I*-LC(X)";
(3) If S is "weakly-*I*-LC-set", then S $\in$ "R^{*}-*I*-LC(X)";
(4) Also If S $\in$ R^{*}-*I*-LC(X) . Then
(i)For given "*-closed set" T , S$\cap$T $\in$ R^{*}-*I*-LC(X);
(ii) For any R^{*}-open set T , S $\cap$T $\in$ "R^{*}-*I*-LC(X)" ;
(iii) For any "R^{*}-*I*-LC-set" T , S $\cap$T $\in$ "R^{*}-*I*-LC(X)".
(5) If S be an "I^{*}-R_t-subset" of X. Then
(i) For any "t-*I*-set" T, S $\cap$ T$\in$ "I^{*}-R_t (X)";
(ii) For any R^{*}-open set T, S $\cap$ T$\in$ "I^{*}-R_t (X)" ;
(iii) For any "I^{*}-R_t-set" T, S $\cap$ T$\in$ "I^{*}-R_t (X)" .

Proof
Directly we can obtain through definitions.

Proposition 2.5
Let $\tau(X)_I$ be an ITS and S be a subset of X. (i) If S is an "R^{*}-*I*-LC-set", then S is an "I^{*}- R_t-set" (ii) If S is an I^{*}-R_t-set, then S is a R^{*}- C_I –set.

Proof
(i) Let S be an R^{*}-*I*-LC-set. Then S is the intersection of R^{*}-open set and a *-closed set. Since every *-closed set is a t-*I*-set, S is the intersection of R^{*}-open set and a t-*I*-set. Hence S is an I^{*}-R_t-set.(ii) Can prove using Definitions.

Theorem 2.6
Let $\tau(X)_I$ be an ITS and S be subset of X. Then the properties mentioned below are equivalent:
(1) S is "*-closed";
(2) S is "weakly-*I*-LC-set" and an "I^{*}_{R}-closed set";
(3) S is an "R^{*}-*I*-LC-set" and an "I^{*}_{R}-closed set".

Proof
Since every "*-closed set" is an "R^{*}-*I*-LC-set", it is easy to prove (1) $\Rightarrow$ (2).
(2) $\Rightarrow$ (3): Since every *-closed set is a "weakly-*I*-LC-set" and t-*I* set, (3) follows from (2).
(3) $\Rightarrow$ (1): Since every open set is an R^{*}-open set, every "weakly-*I*-LC-set" is an "R^{*}-*I*-LC-set".

Definition 2.7
Consider a function f: $\tau(X)_I \to \sigma(Y)$. Then f is an "I^{*}_{R}- continuous (respectively R^{*}-*I*-LC- continuous, I^{*}-R_t- continuous") if $f^{-1}(V)$ is "I^{*}_{R}-closed (respectively R^{*}-*I*-LC- set, I^{*}-R_t- set)" in $\tau(X)_I$ for every closed set V in $\sigma(Y)$.

Remark 2.8
(1) Every "*-continuous function" is "weakly-*I*-LC-continuous".
(2) Every "weakly-*I*-LC-continuous function" is "R^{*}-*I*-LC- continuous".

Proposition 2.9
(1) Let f: $\tau(X)_I \to \sigma(Y)_J$ is said to be "I^{*}_{R}- continuous" and g: $\sigma(Y)_J \to \eta$ (Z) be continuous. Then g_0 f: $\tau(X)_I \to \eta$ (Z) is "I^{*}_{R}-continuous".
(2) Let f: $\tau(X)_I \to \sigma(Y)_J$ be "I^{*}_{R}- continuous" and g: $\sigma(Y)_J \to \eta$ (Z) be "*-continuous". Then g_0 f: $\tau(X)_I \to \eta$ (Z) is "I^{*}_{R}- continuous".

Theorem 2.10
For any function g from an ITS to a TS the following are equivalent:
(1) g is "continuous";
(2) g is "weakly-*I*-LC-continuity" and an "I^{*}_{R}-continuity";
(3) g is "R^{*}-*I*-LC- continuous" and "I^{*}_{R}- continuous".
Proof
Immediately follows from theorem already proved for the corresponding sets.

3. Conclusion

(1) Continuous functions have significant role in pure and applied mathematics. In this paper we introduced and studied about some new set like R^{*}-LC sets ,R^{*}-*I*-LC- sets, I^{*}_{R}-closed set, R_t-sets, I^{*}-R_t-sets in ideal topological spaces Also using this sets we obtain a decomposition of continuity. Further that can be extended to soft ideal topological spaces.

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