



On WS Closed and WS Open Maps in Topological Spaces

Veerasha A Sajjanar^{1*}, Basavaraj M. Ittanagi²

¹Department of Mathematics, Sri Krishna Institute of Technology,, Bangalore, Karnataka State, India

²Department of Mathematics, Siddaganga Institute of Technology, Tumakuru, Karnataka State, India

*Corresponding author E-mail: veereshas@rediffmail.com

Abstract

New type of closed and open maps called ws-closed and ws-open maps in topological spaces are introduced and studied. During this process some of their properties have been investigated. Also we introduce ws*-closed and ws*-open maps in topological spaces and discuss some properties.

Keywords: : ws-Closed maps, ws*-Closed maps and ws-open maps, ws*-open maps.

1. Introduction

In 1982, the concept of generalized closed maps is introduced and studied by S. R, Malghan [4], After regular, rw, arw-closed maps are elaborated by Long [3] Benchalli et. al [2] and R S Wali et. al [6] respectively. Intention of paper is to illustrate and study ws-closed and ws-open maps, ws*-closed and ws*-open maps in topological spaces, Also some of their properties have been investigated.

2. Preliminaries

Here P or (P, τ) and Q or (Q, σ) indicate topological spaces for which, separation axioms are not assumed. For, a subset D of a topological space P , $cl(A)$, $int(A)$, $P - D$ or D^c represent closure of D , interior of D and complement of D in P respectively. CLD refers short form of Closed and (CLD)¹ refers short form of Open

Definition 2.1: A map h is said, to be

- i. ws- Continuous [1] if $h^{-1}(V)$ is ws-CLD in P , \forall CLD subset V of Q .
- ii. Strongly ws-continuous [1], if $h^{-1}(V)$ is CLD set in P \forall ws-CLD set V in Q .

Definition 2.2: A map h is said to be

- i. ws- irresolute [1] if $h^{-1}(V)$ is ws-CLD in P \forall ws-CLD subset V of Q .

Results 2.3: [1]

- i. A subset D of a space (P, τ) is called ws-CLD set if $scl(D) \subseteq U$, whenever $D \subseteq U$ and U is w-open in (P, τ) .
- ii. Every CLD (respectively, semi-CLD, α , $g\#^*g\alpha$, $g\zeta^*$, \ddot{g} , regular, rb, $g\#s$ -CLD) set is ws-CLD set in P .
- iii. Every ws-CLD set is gspr-CLD (respectively, gsp, rgb-CLD) set in P .

Results 2.4: [1] Aysubset D of astopologicalspace P , and

- i. If D is regular open and gspr-CLD set then D is ws-CLD set in P .
- ii. If D is regular open and rgb-CLD set then D is ws-CLD set in P .
- iii. If D is semiopen and swg*-CLD then D is ws-CLD set, in P .
- iv. If D is semiopen and swg-CLD then D is ws-CLD set, in P .
- v. If D is semiopen and sg-CLD then D is ws-CLD set, in P .
- vi. If D is semiopen and sgb-CLD then D is ws-CLD set, in P .
- vii. If D is semiopen and ags-CLD then D is ws-CLD set, in P .
- viii. If D is β -open and βwg^* -CLD then D is ws-CLD set, in P .
- ix. If D is both open and g-CLD then D is ws-CLD set, in P .
- x. If D is regular semiopen and rw-CLD then D is ws-CLD set, in P .
- xii. If D is regular semiopen and R^* -CLD then D is ws-CLD set, in P .
- xiii. If D is regular semiopen and gprw-CLD then D is ws-CLD set, in P .
- xiv. If D is regular semiopen and rgw-CLD then D is ws-CLD set in P .

Definition. 2.5: Topological space P is called

- i. $T_{1/2}$ space [4] if every semi-CLD set is CLD.
- ii. T_{ws} space [1] if every ws-CLD set is CLD.

Definition. 2.6: A map h is, said to, be

- i. w-open map [5] if $f(U)$ is w-open in Q for every open set U of P ,

3. ws-CLD Maps and ws-(CLD)¹ Maps

Definition 3.1: A map h is, said to, be, weakly, semi-CLD (briefly ws-CLD) map,, if the, image of \forall CLD set in (P, τ) is ws-CLD in (Q, σ) .

Example 3.2.: Let $P = Q = \{v_a, v_b, v_c\}$, $\tau = \{P, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}\}$ be a topology on P. $\sigma = \{Q, \phi, \{v_a\}, \{v_b, v_c\}\}$ be a topology on Q and $wsC(Q) = \{Q, \phi, \{v_b\}, \{v_c\}, \{v_a, v_c\}, \{v_b, v_c\}\}$. Let h defined by identity map, then h is ws-CLD map.

Theorem, 3.3.: Every CLD map is ws-CLD map, but, reverse is not true..

Proof.: Let h be CLD map and, V, be any CLD set in P. Then h(V) is CLD set in Q, since every CLD set is ws-CLD set. Hence h(V) is ws-CLD set in Q. Therefore h is ws-CLD.

Example 3.4: Let $P = Q = \{v_a, v_b, v_c\}$ $\tau = \{P, \phi, \{v_b\}, \{v_c\}, \{v_a, v_c\}, \{v_b, v_c\}\}$ be a topology on P. $\sigma = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}\}$ be a topology on Q and $wsC(Q) = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_c\}, \{v_a, v_c\}, \{v_b, v_c\}\}$. Let h defined by identity map, then h is ws-CLD map but, not CLD as image, of CLD set {b}, in P is {b}, which is not CLD set in Q.

Theorem 3.5: If h is a CLD map then the following holds.

- i. Every, semi-CLD is ws-CLD but, reverse is not true.
- ii. $\forall \alpha$ -CLD is ws-CLD but, reverse is not true.
- iii. $\forall g^\#$ -CLD is ws-CLD but, reverse is not true.
- iv. $\forall *g\alpha$ -CLD is ws-CLD but, reverse is not true.
- v. $\forall g\xi^*$ -CLD is ws-CLD but, reverse is not true.
- vi. $\forall \alpha gp$ -CLD is ws-CLD but, reverse is not true.
- vii. $\forall \ddot{g}$ -CLD is ws-CLD but, reverse is not true.
- viii. \forall regular-CLD is ws-CLD but, reverse is not true.
- ix. \forall rb-CLD is ws-CLD but, reverse is not true.

Proof.: The proof, based on fact that \forall semi CLD set (respectively, $\alpha, g^\#, *g\alpha, g\xi^*, \ddot{g}$, regular, rb-CLD set) is ws-CLD set.

The reverse, of above Theorem 3.5 is not true as elaborated in Example 3.6.

Example 3.6: Let $P = Q = \{v_a, v_b, v_c\}$ $\tau = \{P, \phi, \{v_b\}, \{v_c\}, \{v_a, v_c\}, \{v_b, v_c\}\}$ be a topology on P and $\sigma = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}\}$ be a topology on Q and $wsC(Q) = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_c\}, \{v_a, v_c\}, \{v_b, v_c\}\}$. Let h defined by identity map, then h is ws-CLD map but not CLD as ,image, of CLD set {v_b} in P is {v_b}, which is not semi-CLD set (respectively $\alpha, g^\#, *g\alpha, g\xi^*, \ddot{g}$, regular, rb-CLD set) in Q.

Theorem3.7: Every $g^\#$ -s-CLD map is ws-CLD map, but reverse is not true.

Proof: The, proof based on the fact that every $g^\#$ -s-CLD set is ws-CLD set.

Example 3.8: Let $P, = Q, = \{v_a, v_b, v_c, v_d\}$. Let $\tau = \{\phi, P, \{v_a, v_b\}, \{v_c, v_d\}\}$ be a topology on P and $\sigma = \{\phi, Q, \{v_a\}, \{v_b\}, \{v_a, v_b\}, \{v_a, v_b, v_c\}\}$ be a topology on Q and $wsC(Q) = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_c\}, \{d\}, \{v_a, v_c\}, \{v_a, v_d\}, \{v_b, v_c\}, \{v_b, v_d\}, \{v_c, v_d\}, \{v_a, v_b, v_d\}, \{v_a, v_c, v_d\}, \{v_b, v_c, v_d\}\}$. $g^\#sC(Q) = \{Q, \phi, \{v_b\}, \{v_c\}, \{v_d\}, \{v_b, v_c\}, \{v_b, v_d\}, \{v_c, v_d\}, \{v_b, v_c, v_d\}\}$. Let $h: (P, \tau) \rightarrow (Q, \sigma)$ be a function defined by $h(v_a) = v_a, h(v_b) = v_b, h(v_c) = v_c, h(d) = d$, is ws-CLD map but not CLD as image, of CLD set {v_a, v_b} in P is {v_a, v_d}, is not $g^\#$ -s-CLD set in Q.

Theorem 3.9: If h is a CLD map then the following holds.

- i. Every ws-CLD map is gsp-CLD map but reverse is not true.
- ii. Every ws-CLD map is gsp-CLD map but reverse is not true.

- iii. Every ws-CLD map is rgb-CLD map but reverse is not true.

Proof: The proof based on the fact that every ws-CLD set is gsp-CLD set (respectively, rgb, gsp-CLD set).

Example 3.10: Let $P = Q = \{v_a, v_b, v_c, v_d\}$. Let $\tau = \{\phi, P, \{v_a\}, \{v_b\}, \{v_a, v_b\}, \{v_a, v_b, v_c\}\}$ be a topology on P and $\sigma = \{\phi, Q, \{v_a, v_b\}, \{v_c, v_d\}\}$ be a topology on Q and $wsC(Q) = \{Q, \phi, \{v_a, v_b\}, \{v_c, v_d\}\}$, $gspC(Q) = gspC(Q) = rgbC(Q) = \{P(Q)\}$. Let h defined by $f(v_a) = v_a, f(v_b) = v_b, f(v_c) = v_c, f(v_d) = v_d$, then h is gsp-CLD map (respectively, rgb, gsp-CLD) but not ws-CLD as image of CLD set {v_d} in P is {v_d}, is, not ws-CLD set in Q.

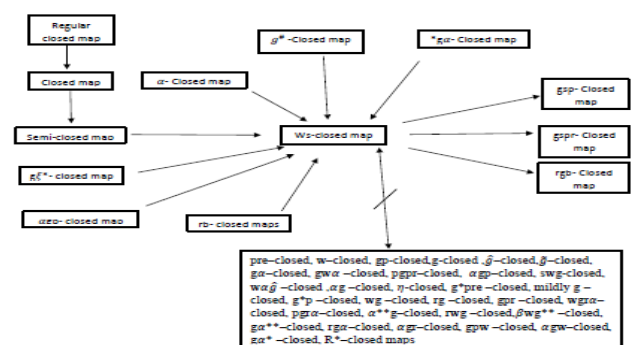
Remark 3.11: The following Examples 3.12 and 3.13, shows that ws-CLD maps are independent of pre-CLD, w, gp, g, \hat{g} , \tilde{g} , $g\alpha$,

$gw\alpha, pgpr, \alpha gp, swg, w\alpha \hat{g}, \alpha g, \eta, g^*pre$, mildly g, $g^*p, wg, rg, gpr, wgr\alpha, pgr\alpha, \alpha^*g, rwg, \beta wg^{**}, g\alpha^{**}, rg\alpha, \alpha gr, gpw, \alpha gw, g\alpha^*, R^*-CLD$.

Example 3.12: Let $P = Q = \{v_{ai}, v_{bi}, v_{ci}, v_{di}\}$. Let $\tau = \{\phi, P, \{v_a\}, \{v_a, v_b\}, \{v_a, v_b, v_c\}\}$ be a topology on P and $\sigma = \{\phi, Q, \{v_a, v_b\}, \{v_c, v_d\}\}$ be a topology on Q and $wsC(Q) = \{Q, \phi, \{v_a, v_b\}, \{v_c, v_d\}\}$. Let h defined by, $h(v_a) = v_a, h(v_b) = v_b, h(v_c) = v_c, h(v_d) = v_d$, then h is of pre-CLD, w, gp, g, \hat{g} , \tilde{g} , $g\alpha$, $gw\alpha, pgpr, \alpha gp, swg, w\alpha \hat{g}, \alpha g, \eta, g^*pre$, mildly g, $g^*p, wg, rg, gpr, wgr\alpha, pgr\alpha, \alpha^*g, rwg, \beta wg^{**}, g\alpha^{**}, rg\alpha, \alpha gr, gpw, \alpha gw, g\alpha^*, R^*-CLD$ maps. But h is not ws-CLD map, as CLD set {v_d} in P is {v_d}, is not ws-CLD set in Q.

Example 3.13: Let $P = Q = \{v_{ai}, v_{bi}, v_{ci}, v_{di}\}$. Let $\tau = \{\phi, P, \{v_a, v_b\}, \{v_c, v_d\}\}$ be a topology on P and $\sigma = \{\phi, Q, \{v_a\}, \{v_b\}, \{v_a, v_b\}, \{v_a, v_b, v_c\}\}$ be a topology on Q and $wsC(Q) = \{\phi, Q, \{v_a\}, \{v_b\}, \{v_c\}, \{v_d\}, \{v_a, v_c\}, \{v_a, v_d\}, \{v_b, v_c\}, \{v_b, v_d\}, \{v_c, v_d\}, \{v_a, v_b, v_d\}, \{v_a, v_c, v_d\}, \{v_b, v_c, v_d\}\}$. Let $h: (P, \tau) \rightarrow (Q, \sigma)$ defined by, $f(v_a) = v_a, f(v_b) = v_a, f(v_c) = v_c, f(v_d) = v_d$, then h is ws-CLD, but not a pre-CLD (respectively, w, gp, g, \hat{g} , \tilde{g} , $g\alpha$, $gw\alpha, pgpr, \alpha gp, swg, w\alpha \hat{g}, \alpha g, \eta, g^*pre$, mildly g, $g^*p, wg, rg, gpr, wgr\alpha, pgr\alpha, \alpha^*g, rwg, \beta wg^{**}, g\alpha^{**}, rg\alpha, \alpha gr, gpw, \alpha gw, g\alpha^*, R^*-CLD$) maps as CLD set {v_a, v_b} in P is {v_a}, is not pre-CLD (respectively, w, gp, g, \hat{g} , \tilde{g} , $g\alpha$, $gw\alpha, pgpr, \alpha gp, swg, w\alpha \hat{g}, \alpha g, \eta, g^*pre$, mildly g, $g^*p, wg, rg, gpr, wgr\alpha, pgr\alpha, \alpha^*g, rwg, \beta wg^{**}, g\alpha^{**}, rg\alpha, \alpha gr, s gpw, \alpha gw, g\alpha^*, R^*-CLD$) set in Q.

Remark 3.14: By the known facts, the relation between ws-CLD map and some existing CLD maps in topological space is shown in the following figure.



Theorem 3.15: If a map h is contra regular CLD and gspr-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P , Then $h(U)$ is open and gspr-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.16: A map h is contra CLD and rgb-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is open and rgb-CLD set in Q . By results 2.4 $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.17: A map h is contra CLD and αg -CLD map then h is ws-CLD map.

Proof: Let U be any CLD set in P . Then $h(U)$ is open and αg -CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.18: If a map h is contra CLD and rgb-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is open and rgb-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.19: If a map h is contra semi CLD and swg*-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P Then $h(U)$ is semi open and swg*-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.20: If a map h is contra semi CLD and swg-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is semi open and swg-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.21: If a map h is contra semi CLD and sg-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is semi open and sg-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.22: If a map h is contra semi CLD and sgb-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is semi open and sgb-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.23: A map h is contra semi CLD and $\alpha g s$ -CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is semi open and $\alpha g s$ -CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.24: If h is contra β -CLD and $\beta w g^*$ -CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is β -open and $\beta w g^*$ -CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.25: If h is contra is both CLD and g-CLD map then f is ws-CLD map in P .

Proof: Let V be any CLD set in P . Then $f(U)$ is open and g-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Theorem 3.26: If h is contra is regular semi CLD and rw-CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is semi open and rw-CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore f is ws-CLD map.

Theorem 3.27: If f is contra is regular semi CLD and R^* -CLD map then h is ws-CLD map in P .

Proof: Let U be any CLD set in P . Then $h(U)$ is semi open and R^* -CLD set in Q . By results 2.4, $h(U)$ is ws-CLD in Q . Therefore h is ws-CLD map.

Remarkt 3.28: The composition of two ws-CLD maps not required to be ws-CLD map in general and this is shown by the following example 3.29.

Example 3.29: Let $P = Q = R = \{v_a, v_b, v_c\}$, $\tau = \{P, \phi, \{v_a\}, \{v_b, v_c\}\}$ be a topology on P , $\sigma = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}\}$ be a topology on Q and $\eta = \{R, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}, \{v_a, v_c\}\}$ be a topology on R . Define $h: P \rightarrow Q$, defined by $f(v_a) = v_a$, $f(v_b) = v_b$, $f(v_c) = v_c$ and $g: Q \rightarrow R$ are the identity maps. Then h and g are ws-CLD maps, but their composition $g \circ h: P \rightarrow R$ is not ws-CLD map, because $E = \{v_a\}$ is CLD in P but $g \circ h(E) = g \circ h(\{v_a\}) = g(h(\{v_a\})) = g(\{v_a\}) = \{v_a\}$ is not ws-CLD in R .

Theorem 3.30: If h is CLD map, $g: Q \rightarrow R$ is ws-CLD map, then the composition $g \circ h: P \rightarrow R$ is ws-CLD.

Proof: Let E be any CLD set in P . Since h is CLD, $h(E)$ is CLD set in Q . g is ws-CLD, $g(h(E))$ is ws-CLD set in R . That is $g \circ h(E) = g(h(E))$ is ws-CLD and hence $g \circ h$ is ws-CLD.

Theorem 3.31: If $h: P \rightarrow Q$ and $g: Q \rightarrow R$ is ws-CLD maps and Q be T_{ws} -space then $g \circ h: P \rightarrow R$ is ws-CLD map.

Proof: Let E be a CLD set of P . h is ws-CLD, $h(E)$ is ws-CLD in (Q, σ) . Then by hypothesis, $h(E)$ is CLD. Since g is ws-CLD, $g(h(E))$ is ws-CLD in R and $g(h(E)) = g \circ h(E)$. Hence $g \circ h$ is ws-CLD.

Theorem 3.32: If $h: P \rightarrow Q$ is g-CLD, $g: Q \rightarrow R$ be ws-CLD and Q is $T_{1/2}$ -space then their composition $g \circ h: P \rightarrow R$ is ws-CLD map.

Proof: Let E be a CLD set of P . Since h is g-CLD, $h(E)$ is g-CLD in Q . Since Q is $T_{1/2}$ -space, $h(E)$ is CLD in Q . Since g is ws-CLD, $g(h(E))$ is ws-CLD in R and $g(h(E)) = g \circ h(E)$. Therefore $g \circ h$ is ws-CLD map.

Definition 3.33: A map h is ws-open map if the image $h(E)$ is ws-open in Q for each open set E in P . From the definitions we have the following results.

Theorem 3.34:

- i. \forall (CLD)¹ map is ws-(CLD)¹ but reverse is not true.
- ii. \forall semi-(CLD)¹ map is ws-(CLD)¹ but reverse is not true.
- iii. $\forall \alpha$ -(CLD)¹ map is ws-(CLD)¹ but reverse is not true.
- iv. $\forall g\#$ -(CLD)¹ map is ws-(CLD)¹ but reverse is not true.

- v. $\forall *g \alpha$ $-(CLD)^1$ map is $ws-(CLD)^1$ but reverse is not true.
- vi. $\forall g \xi^*$ $-(CLD)^1$ map is $ws-(CLD)^1$ but reverse is not true.
- vii. $\forall \alpha_{gp}$ $-(CLD)^1$ map is $ws-(CLD)^1$ but reverse is not true.
- viii. $\forall \ddot{g}$ $-(CLD)^1$ map is $ws-(CLD)^1$ but reverse is not true.
- ix. \forall rb-regular $(CLD)^1$ map is $ws-(CLD)^1$ but reverse is not true.
- x. $\forall g\#s$ $(CLD)^1$ map is $ws-(CLD)^1$ but reverse is not true.
- xi. \forall $ws-(CLD)^1$ map is $gspr-(CLD)^1$ but reverse is not true
- xii. \forall $ws-(CLD)^1$ map is $gsp-(CLD)^1$ but reverse is not true.
- xiii. \forall $ws-(CLD)^1$ map is $rgb-(CLD)^1$ but reverse is not true.

Theorem 3.35: Bijection map h the Conditions below are equivalent:

- i) h^{-1} is ws -continuous.
- ii) h is $ws-(CLD)^1$ map and
- iii) h is ws -CLD map.

Proof:

- (i) \Rightarrow (ii) Let M is $(CLD)^1$ gset, of (P, τ) . By consideration, $(h^{-1})^{-1}(M) = h(M)$ is ws -open in (Q, σ) and so h is $ws-(CLD)^1$.
- (ii) \Rightarrow (iii) Let N is a CLD set of (P, τ) . Then N^c is open set in (P, τ) . By assumption, $h(N^c)$ is ws -open in (Q, σ) . That is $h(N^c) = h(N)^c$ is ws -open in (Q, σ) and therefore $h(N)$ is ws -CLD in Q . Hence h is ws -CLD.
- (iii) \Rightarrow (i) Let N is a CLD set of P . By consideration, $h(N)$ is ws -CLD in Q . But $h(N) = (h^{-1})^{-1}(N)$ and therefore h^{-1} is continuous.

Theorem 3.36: If h is ws -open, then $h(\text{int}(B)) \subseteq ws\text{-int}(h(B))$ for every subset A of P .

Proof: Let h be an open map and B be any subset of P . Then $\text{int}(B)$ is open in P and so $h(\text{int}(B))$ is ws -open in (Q, σ) . We have $h(\text{int}(B)) \subseteq h(B)$. Therefore $h(\text{int}(B)) \subseteq ws\text{-int}(h(B))$.

Definition 3.37: If h is said to be ws^* -CLD map if the image $h(B)$ is ws -CLD in Q for every ws -CLD set B in P .

Theorem 3.38: Every ws^* -CLD map is ws -CLD map but reverse is not true.

Proof: The proof follows from the definitions and fact that every CLD set is ws -CLD.

Example 3.39: Let $P = Q = \{v_a, v_b, v_c\}$, $\tau = \{P, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}\}$ be a topology on P and $\sigma = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}, \{v_a, v_c\}\}$ be a topology on Q and h be the identity map. Then h is ws -CLD map but not ws^* -CLD map. Since $\{a\}$ is ws -CLD set in P . but its image under h is $\{v_a\}$, which is not ws -CLD in Q .

Theorem 3.40: If $h: P \rightarrow Q$ and $g: Q \rightarrow R$ are ws^* -CLD maps, then their composition $g \circ f: P \rightarrow R$ is also ws^* -CLD.

Proof: Let E be a ws -CLD set in P . Since h is ws^* -CLD map, $h(E)$ is ws -CLD set in Q . Since ws^* -CLD map, $g(h(E))$ is ws -CLD set in R . Therefore $g \circ h$ is ws^* -CLD map. Analogous to ws^* -CLD map, we define another new class of maps called ws^* -open maps which are stronger than $ws-(CLD)^1$ maps.

Definition 3.41: A map $h: P \rightarrow Q$ is said to be ws^* -open map if the image $h(E)$ is $ws-(CLD)^1$ set in Q for every ws -open set E in P .

Theorem 3.42: Every ws^* - $(CLD)^1$ map is ws -open map but reverse is not true.

Proof: Proof is similar to the Theorem 3.40, fact that every open set is a $ws-(CLD)^1$ set.

Example 3.43: Let $P = Q = \{v, v_b, v_c\}$, $\tau = \{P, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}\}$ and $\sigma = \{Q, \phi, \{v_a\}, \{v_b\}, \{v_a, v_b\}, \{v_a, v_c\}\}$ and h be the identity map. Then h is ws -open map but not ws^* -open map. Since $\{v_b, v_c\}$ is ws -open set in P , but its image under h is $\{v_b, v_c\}$, is not $ws-(CLD)^1$ in Q .

Theorem 3.44: If $h: P \rightarrow Q$ and $g: Q \rightarrow R$ are ws^* -open maps, their composition $g \circ h: P \rightarrow R$ is also ws^* - $(CLD)^1$.

Proof: Proof is similar to the Theorem 3.35.

Theorem 3.45: Bijection map h the conditions below are equivalent:

1. h^{-1} is ws irresolute.
2. h is ws^* - $(CLD)^1$ map
3. h is ws^* -CLD map.

Proof: Proof is similar to that of Theorem 3.40.

References

- [1] Basavaraj M. Ittanagi and Veerasha A Sajjanar, On ws continuous and ws irresolute Maps in Topological Spaces, **communicated**.
- [2] Benchalli S.S, Wali R.S., on rw - Closed sets in Topological Spaces, Bull, Malays, Math, sci, soc30, 2007, 99-110.
- [3] Long PE, Herington LL. Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo 1978; 27:20- 28.
- [4] Malghan SR. Generalized Closed Maps, J Karnat Univ. Sci., 1982; 27:82-88.
- [5] Sundaram P, Sheik John M. On w -closed sets in topology, Acta Ciencia Indica 2000; 4:389-39
- [6] Wali R.S., Mandalgeri P.S. On rw -Continuous and rw -Irresolute Maps in Topological Spaces, IOSR-JM 2014; 10(6):14-24.