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Signless Laplacian Energy of Operations on Intuitionistic Fuzzy Graphs

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Abstract

After determining the Signless Laplacian energy of an Intuitionistic fuzzy graphs and the study of lower and upper boundaries of Signless Laplacian energy of an Intuitionistic fuzzy graphs, then we planned to search Signless Laplacian energy of an Intuitionistic fuzzy graphs on some operations such as Union and Join of the Intuitionistic Fuzzy Graphs and also examine the difference between two Intuitionistic fuzzy graphs.

Keywords: Intuitionistic fuzzy graphs, Union operation, Join operation of two Intuitionistic fuzzy graphs.

1. Introduction

Fuzzy set has developed as a probable area of inter corrective study, and fuzzy graph theory is also of recent interest. The thought of a fuzzy graph relation was demarcated by Zadeh [6], and it has found solicitations in the analysis of cluster patterns. Rosenfeld [4] measured the fuzzy relations on fuzzy sets and developed the construction of fuzzy graphs.

In this paper we are disturbed with simple graphs. Let G be a graph with n vertices and m edges, and we say this G is a (n,m) graph. Let di be the degree of *i*th vertex of G,i=1,2,..., n. The spectrum of the graph G, consisting of the numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$, is the spectrum of its adjacency matrix [5]. The Laplacian spectrum of the graph G, containing of the numbers $\mu_1, \mu_2, \ldots, \mu_n$, is the spectrum of its Laplacian matrix.

In this paper we introduce the concept of Signless Laplacian energy of operations on intuitionistic fuzzy graphs. Section 2 consists of different operations such as complement of an Intuitionistic fuzzy graph , union and join of two intuitionistic fuzzy graphs definition and we present the Signless Laplacian energy of union and join of two intuitionistic fuzzy graphs . We give the conclusion in the last section.

2. Results

Signless -Laplacian Energy of Some Operations on Intuitionistic Fuzzy Graphs

2.1 . Signless -Laplacian Energy of Complement of an Intuitionistic Fuzzy Graphs

Definition: - The complement of an Intuitionistic fuzzy graph G=(V,E) is an Intuitionistic fuzzy graph, $\overline{G} = (\overline{V}, \overline{E})$ where

$$\overline{V} = V \quad , \quad \mu_{1i} = \mu_{i1} \quad \text{and} \quad \gamma_{1i} = \gamma_{i1} \quad \text{for all } i=1,2,...,n,$$

$$\overline{\mu}_{2ij} = \min(\mu_{i1},\mu_{1i}) - \mu_{2ij} \quad \overline{\gamma}_{2ij} = \min(\gamma_{i1},\gamma_{1i}) - \gamma_{2ij}$$

for all i,j=,2,..,n

Example 2.1.2: Let $V = \{V_1, V_2, V_3, V_4\}$



Fig. 1: Intuitionistic fuzzy graph G and \widetilde{G}

The adjacency matrices of an intuitionistic fuzzy graph G and complement of this intuitionistic fuzzy graph \overline{G} are

$$A(IFG) = \begin{bmatrix} 0 & (0.3, 0.8) & 0 & (0.0, 0.8) \\ (0.3, 0.8) & 0 & (0.6, 0.1) & 0 \\ 0 & (0.6, 0.1) & 0 & (0.0, 0.9) \\ (0.0, 1.0) & 0 & (0.0, 0.9) & 0 \end{bmatrix}$$
$$A(IF\tilde{G}) = \begin{bmatrix} 0 & 0 & (0.3, 0.6) & (0.0, 0.2) \\ 0 & 0 & (0.1, 0.1) & (0.0, 1.0) \\ (0.3, 0.6) & (0.1, 0.1) & 0 & (0.0, 0.1) \\ (0.0, 0.2) & (0.0, 1.0) & (0.0, 0.1) & 0 \end{bmatrix}$$

$$A(\mu_{ij}) = \begin{bmatrix} 0 & 0.3 & 0 & 0 \\ 0.3 & 0 & 0.6 & 0 \\ 0 & 0.6 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$$A(\gamma_{ij}) = \begin{bmatrix} 0 & 0.8 & 0 & 0.8 \\ 0.8 & 0 & 0.1 & 0 \\ 0 & 0.1 & 0 & 0.9 \\ 1.0 & 0 & 0.9 & 0 \end{bmatrix}$$

Signless Laplacian matrix of the membership and non- membership value of an intuitionistic fuzzy graph G are

$$Q(\mu_{ij}) = \begin{bmatrix} 0.3 & 0.3 & 0 & 0 \\ 0.3 & 0.9 & 0.6 & 0 \\ 0 & 0.6 & 0.6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$Q(\gamma_{ij}) = \begin{bmatrix} 1.6 & 0.8 & 0 & 0.8 \\ 0.8 & 0.9 & 0.1 & 0 \\ 0 & 0.1 & 1.0 & 0.9 \\ 1.0 & 0 & 0.9 & 1.7 \end{bmatrix}$$

$$spec(Q(\mu_{ij}(IFG))) = \{-0.0, 0, 0.3804, 1.4196\}$$

$$spec(Q(\gamma_{ij}(IFG))) = \{3.0147, 1.7060, 0, 0.6794\}$$

$$sLe(\mu_{ij}(IFG)) = \left|0 - \frac{2(1.8)}{4}\right| + \left|0 - \frac{2(1.8)}{4}\right| + \left|0.3804 - \frac{2(1.8)}{4}\right| + \left|1.4196 - \frac{2(1.8)}{4}\right|$$

$$= 1.9392$$

$$sLe(\gamma_{ij}(IFG)) = \left|3.0147 - \frac{2(5.4)}{4}\right| + \left|1.7060 - \frac{2(5.4)}{4}\right| + \left|0 - \frac{2(5.4)}{4}\right| + \left|0.6794 - \frac{2(5.4)}{4}\right|$$

$$= 4.0412$$

The Signless laplacian energy of complement of an intuitionistic fuzzy graph \widetilde{G} are

$$A(\tilde{\mu}_{ij}) \begin{bmatrix} 0 & 0 & 0.3 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0.3 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$A(\tilde{\gamma}_{ij}) = \begin{bmatrix} 0 & 0 & 0.6 & 0.2 \\ 0 & 0 & 0.1 & 1.0 \\ 0.6 & 0.1 & 0 & 0.1 \\ 0.2 & 1.0 & 0.1 & 0 \end{bmatrix}$$

Signless Laplacian matrix of the membership and non - membership value of a complement of intuitionistic fuzzy graph \widetilde{G} are

$$Q(\tilde{\mu}_{ij}) = \begin{bmatrix} 0.3 & 0 & 0.3 & 0 \\ 0 & 0.1 & 0.1 & 0 \\ 0.3 & 0.1 & 0.4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$Q(\tilde{\gamma}_{ij}) = \begin{bmatrix} 0.8 & 0 & 0.6 & 0.2 \\ 0 & 1.1 & 0.1 & 1.0 \\ 0.6 & 0.1 & 0.8 & 0.1 \\ 0.2 & 1.0 & 0.1 & 1.3 \end{bmatrix}$$
$$spec(Q(\mu_{ij}(IF\tilde{G}))) = \{-0.0, 0, 0.1\}$$

 $spec\left(Q\left(\mu_{ij}\left(IF\tilde{G}\right)\right)\right) = \{-0.0, 0, 0.1354, 0.6646\}$ $spec\left(Q\left(\gamma_{ij}\left(IF\tilde{G}\right)\right)\right) = \{0.0943, 0.2939, 1.3575, 2.2543\}$ $A(G_{2}) = \begin{bmatrix} 0 & (0.1, 0.7) \\ (0.1, 0.7) & 0 \end{bmatrix}$ $sLe\left(\mu_{ij}\left(IF\tilde{G}\right)\right) = \begin{vmatrix} -0.0 - \frac{2(0.4)}{4} \end{vmatrix} + \begin{vmatrix} 0.1354 - \frac{2(0.4)}{4} \end{vmatrix} + \begin{vmatrix} 0.6646 - \frac{2(0.4)}{4} \end{vmatrix}$ The membership and non- membership values of G_{1}

$$SLE\left(\tilde{\gamma}_{ij}\left(IF\tilde{G}\right)\right) = \left|0.0943 - \frac{2(2)}{4}\right| + \left|0.2939 - \frac{2(2)}{4}\right| + \left|1.3575 - \frac{2(2)}{4}\right| + \left|2.2543 - \frac{$$

= 3.2237

2.2: - Signless Laplacian energy of Union of two intuitionistic fuzzy graphs:

Definition 2.2.1 :- Union of intuitionistic fuzzy graph

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be any two intuitionistic fuzzy graphs with $V_1 \cap V_2 = \phi$ and $G = G_1 \cap G_2 = (V_1 \cap V_2, E_1 \cap E_2)$. Now the union of intuitionistic fuzzy graphs G_1 and G_2 is also an intuitionistic fuzzy graph defined by

$$(\mu_{1} \cup \mu_{1}')(v) = \begin{cases} \mu_{1}(v) \text{ if } v \in V_{1} - V_{2} \\ \mu_{1}'(v) \text{ if } v \in V_{2} - V_{1} \end{cases}$$

$$(\gamma_{1} \cup \gamma_{1}')(v) = \begin{cases} \gamma_{1}(v) \text{ if } v \in V_{1} - V_{2} \\ \gamma_{1}'(v) \text{ if } v \in V_{2} - V_{1} \end{cases}$$

$$(\mu_{1} \cup \mu_{1}')(v_{i}v_{j}) = \begin{cases} \mu_{2ij} \text{ if } e_{ij} \in E_{1} - E_{2} \\ \mu_{2ij}' \text{ if } e_{ij} \in E_{2} - E_{1} \end{cases}$$

$$(\gamma_{1} \cup \gamma_{1}')(v_{i}v) = \begin{cases} \gamma_{2ij} \text{ if } e_{ij} \in E_{1} - E_{2} \\ \gamma_{2ij}' \text{ if } e_{ij} \in E_{2} - E_{1} \end{cases}$$

Where (μ_1, γ_1) and (μ'_1, γ'_1) refer the vertex membership and non-membership of G_1 and G_2 respectively; (μ_2, γ_2) and (μ'_2, γ'_2) refer the edge membership and non-membership of G_1 and G_2 respectively;





Adjacency matrices of $\,G_1\,$ and $\,G_2\,$ are given below

$$A(G_1) = \begin{bmatrix} 0 & (0.1, 0.6) & (0.2, 0.8) \\ (0.1, 0.6) & 0 & (0.5, 0.2) \\ (0.2, 0.8) & (0.5, 0.2) & 0 \end{bmatrix}$$
$$A(G_2) = \begin{bmatrix} 0 & (0.1, 0.7) \\ (0.1, 0.7) & 0 \end{bmatrix}$$

= 0.9292

$$\begin{split} \mu_{ij} [G_1 \cup G_2] = \begin{bmatrix} 0 & 0.1 & 0.2 & 0 & 0 \\ 0.1 & 0 & 0.5 & 0 & 0 \\ 0.2 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 0.7 \\ 0 & 0 & 0 & 0.7 & 0.7 \\ 0 & 0 & 0 &$$

2.3: - Signless Laplacian energy of join of two intuitionistic fuzzy graphs:

Definition 2.3.1:- The join of two intuitionistic fuzzy graph's G_1 and G_2 is an intuitionistic fuzzy graph

$$G_{1} + G_{2} - (V_{1} \cup V_{2}, E_{1} \cup E_{2}) \text{ defined by} (\mu_{1} + \mu_{1}')(v) = (\mu_{1} \cup \mu_{1}')(v) \text{ if } v \in V_{1} \cup V_{2} (\gamma_{1} + \gamma_{1}')(v) = (\gamma_{1} \cup \gamma_{1}')(v) \text{ if } v \in V_{1} \cup V_{2} (\mu_{2} + \mu_{2}')(v_{i}v_{j}) = (\mu_{2} \cup \mu_{2}')(v_{i}v_{j}) \text{ if } v_{i}v_{j} \in E_{1} \cup E_{2} \text{ and} = \min(\mu_{1}(v_{i}), \mu_{1}'(v_{j}))\text{ if } v_{i}v_{j} \in E' (\gamma_{2} + \gamma_{2}')(v_{i}v_{j}) = (\gamma_{2} \cup \gamma_{2}')(v_{i}v_{j}) \text{ if } v_{i}v_{j} \in E_{1} \cup E_{2} = \max(\gamma_{1}(v_{i}), \mu_{1}'(v_{j}))\text{ if } v_{i}v_{j} \in E'$$



Fig. 3: Intuitionistic Fuzzy Graphs...G₁, G₂ and G₁+G₂

$$A(\mu_{ij}(G_{1})) = \begin{bmatrix} 0 & 0.1 & 0.2 \\ 0.1 & 0 & 0.5 \\ 0.2 & 0.5 & 0 \end{bmatrix}$$

$$A(\gamma_{ij}(G_{1})) = \begin{bmatrix} 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.5 \\ 0.8 & 0.2 & 0 \end{bmatrix}$$
and
$$A[\mu_{ij}(G_{2})] = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}$$

$$A[\gamma_{ij}(G_{2})] = \begin{bmatrix} 0 & 0.7 \\ 0.7 & 0 \end{bmatrix}$$

$$Q[\mu_{ij}(G_{1})] = \begin{bmatrix} 0.3 & 0.1 & 0.2 \\ 0.1 & 0.6 & 0.5 \\ 0.2 & 0.5 & 0.7 \end{bmatrix}$$

$$Q[\gamma_{ij}(G_{1})] = \begin{bmatrix} 1.4 & 0.6 & 0.8 \\ 0.6 & 0.8 & 0.5 \\ 0.8 & 0.2 & 1.0 \end{bmatrix}$$

$$spec[Q(\mu_{ij}(G_{1}))] = \{0.1207, 0.2753, 1.2040\}$$

$$spec[Q(\gamma_{ij}(G_{1}))] = \{2.4247, 0.4330, 0.6393\}$$

$$SLE[\mu_{ij}(G_{1})] = |0.1207 - \frac{2(0.8)}{3}| + |0.2753 - \frac{2(0.8)}{3}| + |1.2040 - \frac{2(0.8)}{3}|$$

$$= 1.3413$$

$$SLE[\gamma_{ij}(G_{1})] = |2.4277 - \frac{2(1.75)}{3}| + |0.4330 - \frac{2(1.75)}{3}| + |0.6393 - \frac{2(1.75)}{3}|$$

Similarly the membership and non –membership values of G_2

$$A(\mu_{ij}(G_{2})) = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} \text{ and } A(\gamma_{ij}(G_{2})) = \begin{bmatrix} 0 & 0.7 \\ 0.7 & 0 \end{bmatrix}$$

$$Q(\mu_{ij}(G_{2})) = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0,1 \end{bmatrix} \text{ and } Q(\gamma_{ij}(G_{2})) = \begin{bmatrix} 0.7 & 0.7 \\ 0.7 & 0,7 \end{bmatrix}$$

$$Spec \left[Q(\mu_{ij}(G_{2})) \right] = \{0,0.2\}$$

$$Spec \left[Q(\gamma_{ij}(G_{2})) \right] = \{0,1.4\}$$

$$SLE \left[\mu_{ij}(G_{2}) \right] = \left| 0 - \frac{2(0.1)}{2} \right| + \left| 0.2 - \frac{2(0.1)}{2} \right| = 0.1 + 0.1 = 0.2$$

$$SLE \left[\gamma_{ij}(G_{2}) \right] = \left| 0 - \frac{2(0.7)}{2} \right| + \left| 1.4 - \frac{2(0.7)}{2} \right| = 0.7 + 0.7 = 1.4$$
Adjacency matrix of $G_{1} \cup G_{2}$ is given below

 $A[G_1 \cup G_2] = \begin{bmatrix} 0 & (0.1,0.6) & (0.2,0.8) & 0 & 0 \\ (0.1,0.6) & 0 & (0.5,0.2) & 0 & 0 \\ (0.2,0.8) & (0.5,0.2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (0.1,0.7) \\ 0 & 0 & 0 & (0.1,0.7) & 0 \end{bmatrix}$

The membership matrix of $\,G_1 \cup G_2\,{
m is}\,$

In the previous section we already find the $SLE(G_1) = (1.3413, 2.5221)$ $SLE(G_2) = (0.2, 1.4)$

Now we will verify the laplacian energy of join of two intuitionistic fuzzy graph $G = G_1 + G_2$

$$\begin{split} A(G_1+G_2) &= \begin{bmatrix} 0 & (0.1,0.6) & (0.2,0.8) & (0.3,0.6) & (0.2,0.8) \\ (0.1,0.6) & 0 & (0.5,0.0) & (0.2,0.8) & (0.2,0.8) \\ (0.2,0.8) & (0.2,0.8) & (0.6,0.4) & 0 & (0.1,0.7) \\ (0.2,0.8) & (0.2,0.8) & (0.2,0.8) & (0.1,0.7) & 0 \end{bmatrix} \\ = \begin{bmatrix} 0 & 0.1 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0 & 0.5 & 0.2 & 0.2 \\ 0.2 & 0.5 & 0 & 0.6 & 0.2 \\ 0.3 & 0.2 & 0.6 & 0 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.1 & 0 \end{bmatrix} \\ \mathcal{Q}\Big[\mu_{ij}\left((G_1+G_2)\right)\Big] = \begin{bmatrix} 0 & 0.1 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0 & 0.5 & 0.2 & 0.2 \\ 0.2 & 0.5 & 0 & 0.6 & 0.2 \\ 0.3 & 0.2 & 0.6 & 0 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.1 & 0 \end{bmatrix} \\ \mathcal{Q}\Big[\mu_{ij}\left((G_1+G_2)\right)\Big] = \begin{bmatrix} 0.8 & 0.1 & 0.2 & 0.3 & 0.2 \\ 0.8 & 0.1 & 0.2 & 0.3 & 0.2 \\ 0.1 & 1.0 & 0.5 & 0.2 & 0.2 \\ 0.2 & 0.5 & 1.5 & 0.6 & 0.2 \\ 0.3 & 0.2 & 0.6 & 1.2 & 0.1 \\ 0.2 & 0.2 & 0.2 & 0.1 & 0.7 \end{bmatrix} \\ Spec\Big[\mathcal{Q}\big(\mu_{ij}(G_1+G_2)\big)\Big] = \begin{bmatrix} 0.4883 - \frac{2(2.6)}{5} \\ + \left| 0.5910 - \frac{2(2.6)}{5} \\ + \left| 0.8830, 0.5910, 0.8259, 0.9537, 2.3411 \\ \end{bmatrix} \\ SLE\Big[\mu_{ij}\left((G_1+G_2)\right)\Big] = \begin{bmatrix} 0 & 0.6 & 0.8 & 0.6 & 0.8 \\ 0.6 & 0 & 0 & 0.8 & 0.8 \\ 0.6 & 0 & 0 & 0.8 & 0.8 \\ 0.6 & 0 & 0 & 0.8 & 0.8 \\ 0.6 & 0.8 & 0.4 & 0 & 0.7 \\ 0.8 & 0.8 & 0.8 & 0.7 & 0 \end{bmatrix} \\ \mathcal{Q}\Big[\gamma_{ij}\left((G_1+G_2)\right)\Big] = \begin{bmatrix} 2.8 & 0.6 & 0.8 & 0.6 & 0.8 \\ 0.6 & 0.8 & 0.4 & 0.5 & 0.7 \\ 0.8 & 0.8 & 0.8 & 0.7 & 3.1 \end{bmatrix} \\ Spec\Big[\mathcal{Q}\big(\gamma_{ij}(G_1+G_2)\big)\Big] = \{1.1361, 1.7443, 2.1315, 2.3775, 5.2107\} \\ SLE\big[\gamma_{ij}\left(G_1+G_2\right)\Big] = \big[1.1361, \frac{2(6.3)}{5} \Big] + \big| 1.7443 - \frac{2(6.3)}{5} \Big] + \big| 2.1315 - \frac{2(6.3)}{5} \Big] + \big| 2.3775 - \frac{2(6.3)}{5} \Big] + \big| 5.2107 - \frac{2(6.3)}{5} \Big] \\ = 5.3814 \end{aligned}$$

We observed here that

 $SLE\left[\mu_{ij}\left(G_{1}\right)\right]+SLE\left[\mu_{ij}\left(G_{2}\right)\right]\leq SLE\left[\mu_{ij}\left(G_{1}+G_{2}\right)\right] \text{ and}$ also $SLE\left[\gamma_{ij}\left(G_{1}\right)\right]+SLE\left[\gamma_{ij}\left(G_{2}\right)\right]\leq SLE\left[\gamma_{ij}\left(G_{1}+G_{2}\right)\right]$

3. Conclusion

The Signless Laplacian matrix and energy for an intuitionistic fuzzy graph are defined. Some results on Laplacian spectra of intuitionistic fuzzy graphs may reveal more analogous results of these kinds and will be discussed in the forthcoming papers. This is a text of acknowledgements. Do not forget people who have assisted you on your work. Do not exaggerate with thanks. If your work has been paid by a Grant, mention the Grant name and number here.

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