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***Research paper***

# Fuzzy Sumudu Transforms of the Fuzzy Riemann-Liouville Fractional Derivatives About Order

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**Abstract**

This work involved fuzyy Sumudu transform (FST) for solving fuzzy fractional differential equations (FFDEs) involving Riemann-Liouvillefuzzy fractional derivative and we find with proof the formulas of fuzzy sumudu transforms for Riemann-Liouville fuzzy fractional derivative about order $0<\beta<1$. Addition, we use the resulting fuzzy Sumudu transform to solve (FFDEs) of order $0<\beta<1$ .

***Keywords:*** *Fuzzy sumudu transform, fuzzy Riemann-Liouville fractional derivatives, Mittag-leffeler. Mathematic applied/MSc 2015.*

## 1. Introduction

Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. the work in [19] One of the real commitments in this field , which examined the subject seriously. From that point onward, it was considered in [15], where the creators proposed a few applications. Necessary changes have for quite some time been utilized in unraveling direct customary differential conditions, and additionally straight fragmentary differential conditions. The fundamental changes were gone before by Fourier change. Afterward, a few new necessary changes have been proposed, to be specific, Laplace, Mellin, and Hankel changes [16, 23, 24]. In any event F .Jarad and K.Tas ponder use of Sumudu changes to Riemann-Liouville partial differential conditions. In this paper, we include new outcomes the fluffy Sumudu change for fluffy partial differential conditions (FFDEs) comprise of Riemann-Liouville fluffy fragmentary subordinate about request $0<\beta<1$

### Basic concepts

This section consists of basic concepts which are needed in this paper.

### Some Definitions and Theories

**Definition 2.1 [2]**

A fuzzy number $u$ in parametric form is a pair $(\underline{u},\overline{u})$ of functions $\underline{u}(r),\overline{u}(r)$ , $0\le r\le 1$ , which satisfy the following requirements:

$\underline{u}(r)$ is a bounded non-decreasing left continuous function in $(0,1]$ ,and right continuous at $0$ .

$\overline{u}(r)$ is a bounded non-increasing left continuous function in $(0,1]$ ,and right continuous at $0$ .

- $\underline{u}(r)\le\overline{u}(r)$, $0\le r\le 1$

**Theorem 2.2**([26]). Let and it is represented by $[\underline{f}_\alpha(x),\overline{f}_\alpha(x)]$. For any fixed $\alpha\in(0,1]$ assume $\underline{f}_\alpha(x)$ and $\overline{f}_\alpha(x)$ are Riemann-integrable on [a, b] for every $b\ge a$, and assume there are two positive $\underline{M}_\alpha$ and $\overline{M}_\alpha$ such that $\int_a^b|\underline{f}_\alpha(x)|\,dx\le\underline{M}_\alpha$ and $\int_a^b|\overline{f}(x)|\,dx\le\overline{M}_\alpha$ for every $b\ge a$ .Then, $\tilde{f}(x)$ is improper fuzzy Riemann-integrable on $[a,\infty)$ and the improper fuzzy Riemann-integrable is a fuzzy number. Furthermore, we have

$$\int_a^\infty f(x)\,dx=\left[\int_a^\infty\underline{f}(x)\,dx,\int_a^\infty\overline{f}(x)\,dx\right]$$

H-difference of fuzzy numbers is defined as follows

**Definition 2.3** [20] Let $x,y\in E$ . If there exists $z\in E$ such that $x+y=z$, then $z$ is called the Hukuhara – difference of $x$ and $y$, and it is denoted by $x\ominus y$ . The sign "$\ominus$" always stands for H–difference and also note that $x\ominus y\neq x+(-1)y$ .

**Definition 2.4** [20] A two – parameter function of the Mittag-Leffler type is defined by the series expansion:

$$E_{\alpha,\beta}(z)=\sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+\beta)},(\alpha>0,\beta>0)$$

**Definition 2.5**[20] Let $f(x):(a,b)\to E$ and $x_0\in(a,b)$, then $f(x)$ is differentiable at $x_0$, in the first form, if for $h>0$ sufficiently near 0, there exist the H-differences $f(x_0+h)\ominus f(x_0), f(x_0)\ominus f(x_0-h)$ and the limits

$$f'(x_0)=\lim_{h\to 0^+}\frac{f(x_0+h)\ominus f(x_0)}{h}=\lim_{h\to 0^+}\frac{f(x_0)\ominus f(x_0-h)}{h}$$

or

$f(x)$ is differentiable at $x_0$, in the second form, if for $h>0$ sufficiently near 0, there exist the H-differences $f(x_0)\ominus f(x_0+h), f(x_0-h)\ominus f(x_0)$ and the limits

$$f'(x_0)=\lim_{h\to 0^+}\frac{f(x_0)\ominus f(x_0+h)}{-h}=\lim_{h\to 0^+}\frac{f(x_0-h)\ominus f(x_0)}{-h}$$

In this paper, we denote the space of all continuous fuzzy functions on $[a,b]\subseteq\mathbb{R}$ and the space of all Lebesgue integrable fuzzy functions on the bounded interval [a, b] by $C^F[0,b]$ and $L^F[0,b]$ respectively

## 2. Riemann-Liouville Fuzzy Fractional Derivative

In this subsection, we provide some definitions and theorems on Riemann-Liouvillefuzzy fractional derivative.

**Definition 2.6** ([21]). Let $f(x)\in C^F[0,b]\cap L^F[0,b]$ be a fuzzy function. The fuzzy Riemann-Liouville integral of the fuzzy function $f$ is defined as follows

$$(I^\beta f)(x)=\frac{1}{\Gamma(\beta)}\int_0^x\frac{f(t)}{(x-t)^{1-\beta}}dt,\quad x,\beta\in\mathbb{R}_+$$

**Theorem 2.7**[2]. Let $f(x)\in C^F[0,b]\cap L^F[0,b]$ be a fuzzy function. The fuzzy Riemann-Liouville integral of the fuzzy function $f$ is as follows

$$\left[(I^\beta f)(x)\right]_\alpha=\left[I^\beta\underline{f}_\alpha(x),I^\beta\overline{f}_\alpha(x)\right],\quad 0\le\alpha\le 1,$$

$$(I^\beta\underline{f}_\alpha)(x)=\frac{1}{\Gamma(\beta)}\int_0^x\frac{\underline{f}_\alpha(t)}{(x-t)^{1-\beta}}dt,\quad x,\beta\in\mathbb{R}_+,$$

$$(I^\beta\overline{f}_\alpha)(x)=\frac{1}{\Gamma(\beta)}\int_0^x\frac{\overline{f}_\alpha(t)}{(x-t)^{1-\beta}}dt,\quad x,\beta\in\mathbb{R}_+,$$

**Definition2.8** [16] Let $f\in C^F[a,b]\cap L^F[a,b]$ and $x_0$ in $(a,b)$ and $\phi(x)=\frac{1}{\Gamma(1-\beta)}\int_a^x\frac{f(t)dt}{(x-t)^\beta}$ .We say that $f(x)$ is Riemann–Liouville H- differentiable about order $0<\beta<1$ at $x_0$, if there exists an element $\left({}^{RL}D^\beta_{a+}f\right)(x_0)\in E$, such that for all $h>0$, sufficiently small

$$\left({}^{RL}D^\beta_{a+}f\right)(x_0)=\lim_{h\to 0+}\frac{\phi(x_0+h)\ominus\phi(x_0)}{h}=\lim_{h\to 0+}\frac{\phi(x_0)\ominus\phi(x_0-h)}{h}$$

$$\left({}^{RL}D^\beta_{a+}f\right)(x_0)=\lim_{h\to 0+}\frac{\phi(x_0)\ominus\phi(x_0+h)}{-h}=\lim_{h\to 0+}\frac{\phi(x_0-h)\ominus\phi(x_0)}{-h}$$

For the sake of simplicity, we say that the fuzzy – valued function $f$ is ${}^{RL}[(i)-\beta]$ - differentiable if it is differentiable as in the Definition **2.8** case (i), and is ${}^{RL}[(ii)-\beta]$- differentiable if it is differentiable as in the Definition **2.8**case (ii) .

**Theorem 2.9** [16] Let $f\in C^F[a,b]\cap L^F[a,b]$, $x_0$ in $(a,b)$, $0<\beta<1$. Then:

Let $f(x)$ be a ${}^{RL}[(i)-\beta]$ - differentiable fuzzy – valued function then:

$$\left({}^{RL}D^\beta_{a+}f\right)(x_0)=\left[\left({}^{RL}D^\beta_{a+}\underline{f}\right)(x_0;r),\left({}^{RL}D^\beta_{a+}\overline{f}\right)(x_0;r)\right],$$
$0\le r\le 1$

- Let $f(x)$ be a ${}^{RL}[(ii)-\beta]$- differentiable fuzzy – valued function then:

$$0\le r\le 1,\quad \left({}^{RL}D^\beta_{a+}f\right)(x_0)=\left[\left({}^{RL}D^\beta_{a+}\overline{f}\right)(x_0;r),\left({}^{RL}D^\beta_{a+}\underline{f}\right)(x_0;r)\right]$$

where

$$\left({}^{RL}D^\beta_{a+}\underline{f}\right)(x_0;r)=\left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x\frac{\underline{f}(t;r)dt}{(x-t)^\beta}\right]_{x=x_0}$$

And

$$\left({}^{RL}D^\beta_{a+}\overline{f}\right)(x_0;r)=\left[\frac{1}{\Gamma(1-\beta)}\frac{d}{dx}\int_a^x\frac{\overline{f}(t;r)dt}{(x-t)^\beta}\right]_{x=x_0}$$

### Fuzzy Sumudu Transform

**Definition 3.1** [1,4] Let be a continuous fuzzy function. Suppose is improper fuzzy Riemann-integrable on $[0,\infty)$, then is called fuzzy Sumudu transform and is denoted by

$$G(u)=S\left[\tilde{f}(x)\right](u)=\int_0^\infty\tilde{f}(ux)e^{-x}dx,u\in[-\tau_1,\tau_2]$$

where the variable u is used to factor the variable x in the argument of the fuzzy function and. $\tau_1,\tau_2>0$.

The FST can also be written into the following parametric form

$$S\left[\tilde{f}(x)\right](u)=\left[s\left[\underline{f}_\alpha(x)\right](u),s\left[\overline{f}_\alpha(x)\right](u)\right]$$

Duality Properties of the Fuzzy Laplace and Fuzzy Sumudu Transform FLT has a close relationship with FST. It is necessary for us to be able to link between the two transforms in order to prove theorems and properties of the FST. The definition for FLT is given as follows.

**Definition 3.2**[1] Let $f(x)$ be a continuous fuzzy-valued function. Suppose that is improper fuzzy Riemann-integrable on $[0,\infty)$ then

is called the fuzzy Laplace transform and is denoted by:

**Theorem3.3** [1] Let $f(x)$ be a continuous fuzzy-valued function. If $F$ is the fuzzy Laplace transform of $f(x)$ and $G$ is the fuzzy Sumudu transform of $f(x)$, then:

$$G(u)=\frac{F\left(1/u\right)}{u}$$

Next, we give the definition for the classical Sumudu transform when dealing with Riemann-Liouville fractional derivative

### Fuzzy Sumudu transform for Riemann-Liouville fuzzy fractional derivative

In this part, we recall the theorem of FST and later we propose a new result on the property of FST for Riemann-Liouville fuzzy fractional derivative

**Theorem 4.1**[12] Let $f$ satisfy the relation

$$|f(t)| < \begin{cases} Me^{\frac{-t}{\tau_1}} \text{if } t \le 0 \\ Me^{\frac{-t}{\tau_2}} \text{if } t > 0 \end{cases}$$ and $f \in AC^n([0,b))$. Then

$$S\{D_{0+}^{\alpha} f(t)\}(u) = u^{-\alpha}F(u) - \sum_{k=0}^{n-1} u^k D_{0+}^{\alpha-k-1} f(t)\Big|_{t=0}.$$

Since in this paper, we only consider $0 < \beta < 1$, Theorem 4.1 can be simplified as follows

$$S\left[{}^{RL}D^{\beta} f(x)\right](u) = u^{-\beta}F(u) - {}^{RL}D^{\beta-1} f(0), \ 0 < \beta < 1.$$

**Theorem 4.2** :Suppose that $f(x) \in C^F[0,\infty) \cap L^F[0,\infty)$ and $0 < \beta < 1$ then :

1. If $({}^{RL}D^{\beta} f)(x)$ is ${}^{RL}[i-\beta]$ -differentiable fuzzy-valued function, then

$$S\left[\left({}^{RL}D_1^{\beta} f\right)(x)\right](u) = \frac{F(u)}{u^{\beta}} \ominus {}^{RL}D^{\beta-1} f(0)$$

2. If $({}^{RL}D^{\beta} f)(x)$ is ${}^{RL}[ii-\beta]$ - differentiable fuzzy-valued function, then

$$S\left[\left({}^{RL}D_2^{\beta} f\right)(x)\right](u) = -{}^{RL}D^{\beta-1} f(0) \ominus \frac{(-F(u))}{u^{\beta}}$$

**Now we will proof cases**

**Prove1** as follows: Since ${}^{RL}D^{\beta} f(x)$ is ${}^{RL}[i-\beta]$ -differentiable fuzzy-valued function, then we get:

$$\left({}^{RL}D_1^{\beta} f\right)(x) = \left[\left({}^{RL}D^{\beta}\underline{f}\right)(x;r), \left({}^{RL}D^{\beta}\overline{f}\right)(x;r)\right].$$

Therefore, we get:

$$\boxed{\left(\underline{{}^{RL}D^{\beta} f}\right)(x;r) = \left({}^{RL}D^{\beta}\underline{f}\right)(x;r)},$$

$$\left(\overline{{}^{RL}D^{\beta} f}\right)(x;r) = \left({}^{RL}D^{\beta}\overline{f}\right)(x;r) \quad (1)$$

Then from (1), we get:

$$S\left[\left({}^{RL}D_1^{\beta} f\right)(x)\right](u) = S\left[\left(\underline{{}^{RL}D^{\beta} f}\right)(x;r), \left(\overline{{}^{RL}D^{\beta} f}\right)(x;r)\right](u)$$

$$= s\left[\left({}^{RL}D^{\beta}\underline{f}\right)(x;r)\right](u), s\left[\left({}^{RL}D^{\beta}\overline{f}\right)(x;r)\right](u) \quad (2)$$

$$= \left[u^{-\beta}F(u) - {}^{RL}D^{\beta-1}\underline{f}(x;r), u^{-\beta}F(u) - {}^{RL}D^{\beta-1}\overline{f}(x;r)\right]$$

$$= \left[\frac{s\left[\underline{f}(x,r)\right](u)}{u^{\beta}} - {}^{RL}D^{\beta-1}\underline{f}(0;r), \frac{s\left[\overline{f}(x,r)\right](u)}{u^{\beta}} - {}^{RL}D^{\beta-1}\overline{f}(0;r)\right] \quad (3)$$

Then we get:

$$S\left[\left({}^{RL}D_1^{\beta} f\right)(x)\right](u) = \frac{F(u)}{u^{\beta}} \ominus {}^{RL}D^{\beta-1} f(0)$$

**prove 2** as follows: Since ${}^{RL}D^{\beta} f(x)$ is ${}^{RL}[ii-\beta]$ - differentiable fuzzy-valued function, then we get:

$$\left({}^{RL}D_2^{\beta} f\right)(x) = \left[\left({}^{RL}D^{\beta}\overline{f}\right)(x;r), \left({}^{RL}D^{\beta}\underline{f}\right)(x;r)\right].$$

Therefore, we get:

$$\left(\underline{{}^{RL}D^{\beta} f}\right)(x;r) = \left({}^{RL}D^{\beta}\overline{f}\right)(x;r),$$

$$\left(\overline{{}^{RL}D^{\beta} f}\right)(x;r) = \left({}^{RL}D^{\beta}\underline{f}\right)(x;r) \quad (4)$$

Then from (4), we get:

$$S\left[\left({}^{RL}D_2^{\beta} f\right)(x)\right](u) = S\left[\left(\underline{{}^{RL}D^{\beta} f}\right)(x;r), \left(\overline{{}^{RL}D^{\beta} f}\right)(x;r)\right](u)$$

$$= s\left[\left({}^{RL}D^{\beta}\overline{f}\right)(x;r)\right](u), s\left[\left({}^{RL}D^{\beta}\underline{f}\right)(x;r)\right](u) \quad (5)$$

$$= \left[u^{-\beta}F(u) - {}^{RL}D^{\beta-1}\overline{f}(x;r), u^{-\beta}F(u) - {}^{RL}D^{\beta-1}\underline{f}(x;r)\right]$$

$$= \left[\frac{s\left[\overline{f}(x,r)\right](u)}{u^{\beta}} - {}^{RL}D^{\beta-1}\overline{f}(0;r), \frac{s\left[\underline{f}(x,r)\right](u)}{u^{\beta}} - {}^{RL}D^{\beta-1}\underline{f}(0;r)\right] \quad (6)$$

Then we get :

$$S\left[\left({}^{RL}D_2^{\beta} f\right)(x)\right](u) = -{}^{RL}D^{\beta-1} f(0) \ominus \frac{(-F(u))}{u^{\beta}}$$

**Application**

In thissection we shall solve a FFIVP of order $0 < \beta < 1$ by using theorem 4.2 as in the following

**Example 5.1** Consider the following FFIVP:

$$\left({}^{RL}D^{\beta} y\right)(x) = y(x), \ 0 < \beta < 1$$
(7)

$${}^{RL}D^{\beta-1} y(0) = {}^{RL}y_0^{(\beta-1)}$$

$$S\left[\left({}^{RL}D^{\beta} y\right)(x)\right](u) = S\left[y(x)\right](u). \quad (8)$$

Now, by using theorem (4.2) we have $2^1 = 2$ cases as follows:

**Case 1** Let us consider $\left({}^{RL}D^{\beta} y\right)(x)$ be ${}^{RL}[i-\beta]$ -

$$\frac{F(u)}{u^{\beta}} \ominus {}^{RL}D^{\beta-1} f(0) = S\left[y(x)\right](u).$$

Then, we get :

$$\frac{s\left[\underline{y}(x,r)\right](u)}{u^{\beta}} - {}^{RL}D^{\beta-1}\underline{y}(0,r) = s\left[\underline{y}(x,r)\right](u),$$

$$\frac{s\left[\overline{y}(x,r)\right](u)}{u^{\beta}} - {}^{RL}D^{\beta-1}\overline{y}(0,r) = s\left[\overline{y}(x,r)\right](u)$$

Therefore we have :

$$\left(1-u^{\beta}\right)s\left[\underline{y}\left(x;r\right)\right]={}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)u^{\beta},\quad \left(1-u^{\beta}\right)s\left[\underline{y}\left(x;r\right)\right]={}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)u^{\beta}$$

$$\underline{y}\left(x;r\right)={}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)s^{-1}\left[\frac{u^{\beta}}{1-u^{\beta}}\right],$$

$$\overline{y}\left(x;r\right)={}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)s^{-1}\left[\frac{u^{\beta}}{1-u^{\beta}}\right]$$

$$S\left[x^{\gamma-1}E_{\beta,\gamma}\left(\mp x^{\beta}\right)\left(u\right)\right]=\frac{u^{\gamma-1}}{1\pm u^{\beta}}$$ ) as follows :

$$\underline{y}\left(x;r\right)={}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)x^{\beta-2}E_{\beta,\beta-1}\left(x^{\beta}\right)$$

$$\overline{y}\left(x;r\right)={}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)x^{\beta-2}E_{\beta,\beta-1}\left(x^{\beta}\right)$$

where $E_{\alpha,\beta}(z)$ denotes the Mittag-Leffler function defined in definition (2.4).

**Case 2** Let us consider $\left({}^{RL}D^{\beta}y\right)(x)$ be ${}^{RL}\left[ii-\beta\right]$ -

$$-{}^{RL}D^{\beta-1}f\left(0\right)\ominus\frac{\left(-F\left(u\right)\right)}{u^{\beta}}=S\left[y\left(x\right)\right]\left(u\right).$$

Then, we get :

$$\frac{s\left[\overline{y}\left(x,r\right)\right]\left(u\right)}{u^{\beta}}-{}^{RL}D^{\beta-1}\overline{y}\left(0,r\right)=s\left[\underline{y}\left(x,r\right)\right]\left(u\right),$$

$$\frac{s\left[\underline{y}\left(x,r\right)\right]\left(u\right)}{u^{\beta}}-{}^{RL}D^{\beta-1}\underline{y}\left(0,r\right)=s\left[\overline{y}\left(x,r\right)\right]\left(u\right)$$

Therefore we have :

$$\left(1-u^{\beta}\right)s\left[\underline{y}\left(x;r\right)\right]={}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)u^{\beta},\quad \left(1-u^{\beta}\right)s\left[\underline{y}\left(x;r\right)\right]={}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)u^{\beta}$$

Consequently:

$$\underline{y}\left(x;r\right)={}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)s^{-1}\left[\frac{u^{2\beta}}{1-u^{2\beta}}\right]+{}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)s^{-1}\left[\frac{u^{\beta}}{1-u^{2\beta}}\right]$$

$$\overline{y}\left(x;r\right)={}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)s^{-1}\left[\frac{u^{2\beta}}{1-u^{2\beta}}\right]+{}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)s^{-1}\left[\frac{u^{\beta}}{1-u^{2\beta}}\right]$$

Finally:

$$S\left[x^{\gamma-1}E_{\beta,\gamma}\left(\mp x^{\beta}\right)\left(u\right)\right]=\frac{u^{\gamma-1}}{1\pm u^{\beta}}$$ ) as follows :

$$\underline{y}\left(x;r\right)={}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)x^{2\beta}E_{2\beta,2\beta+1}\left(x^{2\beta}\right)+{}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)x^{\beta}E_{2\beta,\beta+1}\left(x^{2\beta}\right)$$

$$\overline{y}\left(x;r\right)={}^{RL}\underline{y}_0^{(\beta-1)}\left(r\right)x^{2\beta}E_{2\beta,2\beta+1}\left(x^{2\beta}\right)+{}^{RL}\overline{y}_0^{(\beta-1)}\left(r\right)x^{\beta}E_{2\beta,\beta+1}\left(x^{2\beta}\right)$$

Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17]. Partial math is the speculation of common analytics. This incorporates the capacities subsidiary of self-assertive request. Numerous specialists in numerous fields are investigate and consider The subject, for example, building, arithmetic et cetera [3, 6, 7, 8, 10, 17].

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