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Research paper

Iterative application method for solving system modified korteweg-de vries equations

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Abstract

In this paper we use a new iterative method (NIM) to solution the floating wave the new modified Korteweg-de Vries equations. We have shown that a more accurate NIM solution with modified Korteweg-de Vries equations is very efficient, convenient and completely accurate for nonlinear equations systems. The NIM solution is expected to find a wide-ranging application in engineering or physics. The results also show that the NIM solution is more reliable and easy to compute and compute speed than HPM and He's Method (NIM) is introduced to overcome the difficulty arising in calculating Adomian polynomials.

Keywords: *Modified Korteweg-De Vries Equations Combined; A New Iterative Method (NIM).*

1. Introduction

In 1895 Korteweg and Vries KdV An derived the model of the Russell phenomenon of Salton [1].Such as shallow water waves of small capacities but limited [2]. Solitons are local waves that are constantly changing their shape, speed characteristics and stable against mutual collision [3]. It was additionally used to portray various vital physical wonders, for example, magneto hydrodynamics in hot plasma, sound waves in joined precious stone and ionic sound waves [4].
Different techniques have been proposed to acquire exact answers for nonlinear partial differential conditions. These include the Backacklund conversion method [5,6], the Hirota's bilinear method [7], the reverse scattering method [8], the extended tanh method [9-11], the approximation of Adomian [12-14] [15-16], Variable Frequency Method [17,18], Lindstead-Boincher's Multiple Methods [19-20], Edomin Dissociation Method [14,17,18], Homotopy Perturbation method for solving nonlinear vectors and the revised revised Korteweg-de Vries equations in 2008 [21].
In the present paper, we will utilize the NIM strategy to develop voyaging wave arrangements the new nonlinear coupled adjusted Korteweg-de Vries (MKdV) system [22]

$$\frac{\partial r}{\partial t}=\frac{1}{2}\frac{\partial^3 r}{\partial x^3}-3r^2\frac{\partial r}{\partial x}+\frac{3}{2}\frac{\partial^3 z}{\partial x^3}+3r\frac{\partial z}{\partial x}+3z\frac{\partial r}{\partial\,\partial x}-3\lambda\frac{\partial r}{\partial x} \tag{1}$$

$$\frac{\partial z}{\partial t}=\frac{\partial^3 z}{\partial x^3}-3z\frac{\partial z}{\partial x}-3\frac{\partial z}{\partial x}\frac{\partial r}{\partial x}+3r^2\frac{\partial z}{\partial x}+3\lambda\vartheta\frac{\partial z}{\partial x} \tag{2}$$

NIM, arranged by Daftardar-Gejji and Jafari in 2006 [23] and was enhanced by Hemeda [24], was viably connected to an assortment of linear and nonlinear equations, for example, arithmetical equations, integrative equations, vital differential equations, ordinary and fractional differential equations. equations of whole number and number request, arrangement of equations also. NIM is easy to comprehend and simple to actualize utilizing PC bundles and accomplish preferable outcomes [16] over current ADM [25], homotopy clutter (HPM) [26] or VIM [27]. Furthermore, utilizing Allawee to solve nonlinear Burger Line conditions and Burger Coupled 2018 conditions [28]. The organization of this paper is as follows: In Section 2, we present a basic introduction to NIM. The new MKdV equation solutions associated with using the (NIM) in Section 3. It can be concluded that this method is a very powerful and effective technique in finding accurate solutions to wide categories of problems. NIM's Turbulence Method offers fast convergence solutions and comparisons between numerical results of the proposed NIM solutions with HPM solutions.

2. (NIM) [29]

To explain the thought of the NIM, we see the following comprehensive functional equation:

$$r=f+M(r), \tag{2}$$

Where M is a nonlinear operator from a Banach space B → B and f is a known function. We are looking for a solution r of (2) having the series form

$$r = \sum_{i=0}^{\infty} r_i, \quad (3)$$

The nonlinear operator M can be decomposed as:

$$M\left(\sum_{i=0}^{\infty} r_i\right) = M(r_0) + \sum_{i=1}^{\infty}\left[M\left(\sum_{j=0}^{i} r_j\right) - M\left(\sum_{j=0}^{i-1} r_j\right)\right] \quad (4)$$

Now using the above eq.s (3) and (4) in (2):

$$\sum_{i=0}^{\infty} r_i = f + M(r_0) + \sum_{i=1}^{\infty}\left[M\left(\sum_{j=0}^{i} r_j\right) - M\left(\sum_{j=0}^{i-1} r_j\right)\right] \quad (5)$$

We define the recurrence relation in the following way:

$$r_1 = M(r_0)$$

$$r_2 = M(r_0 + r_1) - M(r_0)$$
$$r_3 = M(r_0 + r_1 + r_2) - M(r_0 + r_1) \quad (6)$$

$$r_{n+1} = M(r_0 + r_1 + \cdots + r_n) = M(r_0 + r_1 + \cdots + r_{n-1});\ n = 1,2,3,\ldots.$$

Then

$$r_0 + r_1 + \cdots + r_{n+1} = M(r_0 + r_1 + \cdots + r_n);\ n = 1,2,3,\ldots. \quad (7)$$

And

$$\sum_{i=0}^{\infty} r_i = f + M\left(\sum_{j=0}^{\infty} r_j\right),$$

The m-term approximate solution of (2) is given by $r \approx r_0 + r_1 + r_2 + \cdots + r_{m-1}$. For understanding the Convergence of this method we refer reader to [30]

3. Applications problem

In this subsection, we discover the arrangements r(x, t) and z(x, t) fulfilling the new coupled MKdV equations (2) with the initial conditions [25]

$$r(x,0) = \frac{P_1}{2k} + k\tanh(k\varphi) \quad (8)$$

$$z(x,0) = \frac{\lambda}{2}\left(1 + \frac{k}{P_1}\right) + P1\tanh(k\varphi) \quad (9)$$

Where

$$\varphi = x + \frac{t}{4}\left(-4k^2 - 6\lambda + \frac{64k\lambda}{P_1} + \frac{3P_1^2}{k^2}\right)$$

Where k, P1 and λ are constants.
With reference to (MIN) proposed by He [31], we construct two z_1 and z_2 of the MKdV (2) equations that meet

$$M(r,z) = \frac{1}{2}\frac{\partial^3 r}{\partial x^3} - 3r^2\frac{\partial z}{\partial x} + \frac{3}{2}\frac{\partial^3 z}{\partial x^3} + 3r\frac{\partial z}{\partial x} + 3z\frac{\partial r}{\partial\,\partial x} - 3\vartheta\frac{\partial r}{\partial x}$$

And

$$D(r,z) = \frac{\partial^3 z}{\partial x^3} - 3z\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial x}\frac{\partial r}{\partial x} + 3r^2\frac{\partial z}{\partial x} + 3\vartheta\frac{\partial z}{\partial x}$$

To solve this system of equations (NIM) determines how to solve the equation (7), Burger equation is given by, $M(r,z){:}\ \emptyset \times [0,1] \to R$ and $D(r,v){:}\ \emptyset \times [0,1] \to R$
For equation (14) and (15) respectively which satisfy

$$r(x,t) = \int_0^t M(r,z)\,dt = \int_0^t \frac{1}{2}\frac{\partial^3 r}{\partial x^3} - 3r^2\frac{\partial r}{\partial x} + \frac{3}{2}\frac{\partial^3 z}{\partial x^3} + 3r\frac{\partial z}{\partial x} + 3z\frac{\partial r}{\partial\,\partial x} - 3\vartheta\frac{\partial r}{\partial x}\ dt$$

And

$$z(x,t) = \int_0^t D(r,z)\,dt$$

$$= \int_0^t \frac{\partial^3 z}{\partial x^3} - 3z\frac{\partial z}{\partial x} - 3\frac{\partial z}{\partial x}\frac{\partial r}{\partial x} + 3r^2\frac{\partial z}{\partial x} + 3\vartheta\frac{\partial z}{\partial x}\,dt$$

New With initial conditions in (1 and 2) we get:

$r_1(x,t)=\int_0^t M(r_0,z_0)\,dt=\int_0^t \frac{1}{2}\frac{\partial^3 r_0}{\partial x^3}-3r_0^2\frac{\partial r_0}{\partial x}+\frac{3}{2}\frac{\partial^3 z_0}{\partial x^3}+3r\frac{\partial z_0}{\partial x}+3z_0\frac{\partial r_0}{\partial\,\partial x}-3\lambda\frac{\partial r_0}{\partial x}\,dt$

From the integrative equation we get:

$r_1(x,t)=-\frac{1}{4}\frac{\left(4b_1k^4-3b_1^3+6\lambda b_1-6\lambda k^3\right)t}{cosh(kx)^4 b_1}$ (10)

To Fond for all r_2 Were

$r_2(x,t)=\int_0^t(M\,(r_1+r_0,z_1+z_0)-M(r_0,z_0))dt$

From the integrative equation we get:

$r_2(x,t)=\int_0^t((\frac{1}{2}\frac{\partial^3}{\partial x^3}(r_0+r_1)-3(r_0+r_1)^2\frac{\partial}{\partial x}(r_0+r_1)+\frac{3}{2}\frac{\partial^2}{\partial x^2}(z_0+z_1)+3z_0\frac{\partial}{\partial x}(r_0+r_1)-3\lambda\frac{\partial}{\partial x}(r_0+r_1))-(\frac{1}{2}\frac{\partial^3 r_0}{\partial x^3}-3r_0 2\frac{\partial r_0}{\partial x}+\frac{3}{2}\frac{\partial^3 z_0}{\partial x^3}+3r_0\frac{\partial z_0}{\partial x}+3z_0\frac{\partial r_0}{\partial\partial x}-3\lambda\frac{\partial r_0}{\partial x}))dt$

For

$r_2(x,t)=-\frac{1}{128}\frac{1}{kcosh(kx)^7b_1^3}(t^2(-864\,sin(kx)k^{13}t^2b_1^2\lambda-1296\,sinh(kx)\,k^8t^2b_1^5\,\lambda+1296\,sinh(kx)k^9t^2b_1^4\,\lambda+1296\,sinh(kx)k^{10}t^2b_1^3\lambda^2-2592\,sinh(kx)\,k^{11}t^2b_1^2\lambda^2+\cdots\ldots\ldots\ldots\ldots\ldots\ldots$ (11)

Such that

$r_0(x,t)+r_1(x,t)+\cdots+r_{n+1}(x,t)=M\big(r_0(x,t)+r_1(x,t)+\cdots+r_n(x,t)\big);$

$\sum_{i=0}^{\infty} r_i\,(x,t)=r_0(x,t)+r_1(x,t)+\cdots+r_{n+1}(x,t)$

$r(x,t)=\frac{1}{128}\frac{1}{cosh(kx)^7b_1^3}(-192t^4sinh(kx)k^{14}b_1^3+432t^4sinh(kx)k^{10}b_1^5-324t^4\,sinh(kx)k^6\,b_1^7\quad+648t^4\,sinh(kx)\,k^{11}\lambda^3+81t^4sinh(kx)k^2b_1^9-640t^3k^{11}b_1^3cosh(kx)+960t^3k^7b_1^5\,cosh(kx)-360t^3k^3b_1^7\,cosh(kx)-\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (12)

And

$z_1(x,t)=\int_0^t D(r_0,z_0)\,dt=\int_0^t\frac{\partial^3 z}{\partial x^3}-3z\frac{\partial z}{\partial x}-3\frac{\partial z}{\partial x}\frac{\partial r}{\partial x}+3r^2\frac{\partial z}{\partial x}+3\vartheta\frac{\partial z}{\partial x}dt$

From the integrative equation we get:

$z_1(x,t)=\frac{3}{4}\frac{1}{cosh(kx)^4k}(2\lambda k^2b_1\ \ cosh(kx)^2-2k^3\lambda\,cosh(kx)^2+4b_1k^4cosh(kx)^2-8b_1k^4+b_1^3cosh(kx)^2)t)$ (13)

$z_2(x,t)=\int_0^t(D\,(r_1+r_0,z_1+z_0)-M(r_0,z_0))dt$

From the integrative equation we get:

$z_2(x,t)=\int_0^t(\frac{\partial^3}{\partial x^3}(z_0+z_1)-3(z_0+z_1)\frac{\partial}{\partial x}(z_0+z_1)-3\frac{\partial}{\partial x}(z_0+z_1)\frac{\partial}{\partial x}(r_0+r_1)+3(r_0+r_1)^2\frac{\partial}{\partial x}(z_0+z_1)+3\lambda\frac{\partial}{\partial x}(z_0+z_1)dt-(\frac{\partial^3 r_0}{\partial x^3}-3z_0\frac{\partial z_0}{\partial x}-3\frac{\partial z_0}{\partial x}\frac{\partial r_0}{\partial x}+3{r_0}^2\frac{\partial z_0}{\partial x}+3\lambda\frac{\partial z_0}{\partial x}))dt$

For

$z_2(x,t)=-\frac{1}{128}\frac{1}{cosh(kx)^7b_1^2k^2}(t^2(288\,sinh(kx)\,k^{13}t^2b_1^2\lambda+432\,sinh(kx)\,k^8t^2b_1^5\lambda-432\,sin\,h(kx)\,k^9t^2b_1^4\lambda+432\,sinh(kx)\,k^{10}t^2b_1^3\lambda^2-864\,sinh(kx)\,k^{11}t^2b_1^2\lambda^2+432\,sinh(kx)\,k^{12}t^2b_1\lambda^2-576k^3tcosh(kx)^2b_1^6\,sinh(kx)\,\lambda+\cdots\ldots\ldots$ (14)

Such that

$z_0(x,t)+z_1(x,t)+\cdots+z_{n+1}(x,t)=D\big(z_0(x,t)+z_1(x,t)+\cdots+z_n(x,t)\big);$

$\sum_{i=0}^{\infty} z_i\,(x,t)=z_0(x,t)+z_1(x,t)+\cdots+z_{n+1}(x,t)$

$z(x,t)=\frac{1}{128}\frac{1}{cosh(kx)^7b_1^2k^2}(192t^4\,sinh(kx)\,k^{14}b_1^3-432t^4\,sinh(kx)\,k^{10}b_1^5+324t^4\,sinh(kx)k^6\,b_1^7\quad+648t^4\,sinh(kx)\,k^{11}\lambda^3-81t^4\,sinh(kx)\,k^2b_1^9+1152t^3k^{11}b_1^3cosh(kx)-1728t^3k^7b_1^5\,cosh(kx)+648t^3k^3b_1^7\,cosh(kx)-\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (15)

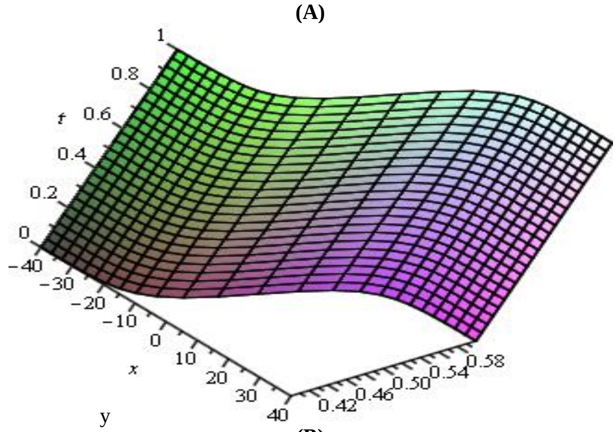
The numerical values and behavior of the solutions obtained by of the NIM method is shown for different values of time and HPM method

$t = 0.1, \varepsilon\ \lambda\ =\ 0.5, p1\ =\ 0.1, and\ k\ =\ 0.1 = 1, \beta = 0.6, \alpha = 0.4$

Table 1:

x	R_{HPM}	R_{NIM}	$EXAET$	$EROR$	Z_{HPM}	Z_{NIM}	$EXAET$	$EROR$
-50	-0.0999923	0.400009776	0.400001616	9.50×10^{-10}	0.500004	-0.0999902237	0.0999983838	6.90×10^{-10}
-40	-0.0999431	0.400072214	0.400001974	4.83×10^{-9}	0.500031	-0.0999277850	0.0999980260	4.61×10^{-9}
-30	-0.09958	0.400532373	0.400002411	3.34×10^{-8}	0.500227	-0.0994676271	0.0999975889	3.35×10^{-8}
-20	-0.0969362	0.403867982	0.400002944	2.09×10^{-7}	0.501616	-0.0961320283	0.0999970551	2.19×10^{-7}
-10	-0.0793302	0.425438484	0.400003596	3.12×10^{-7}	0.509353	-0.0745617738	0.0999964031	5.70×10^{-7}
0	-0.00755	0.503699931	0.400004393	1.62×10^{-6}	0.52	0.00370006845	0.0999956068	1.76×10^{-6}
10	0.0729886	0.577669373	0.400005365	6.90×10^{-7}	0.507446	0.0776694820	0.0999946342	5.81×10-7
20	0.0958693	0.596654836	0.400006553	2.15×10^{-7}	0.50121	0.0966548393	0.0999934462	2.12×10-7
30	0.099431	0.599540635	0.400008004	3.23×10^{-8}	0.500168	0.0995406357	0.0999919952	3.22×10-8
40	0.0999228	0.599937707	0.400009776	4.65×10^{-9}	0.500023	0.0999377080	0.0999902230	4.43×10^{-9}
50	0.0999895	0.599991567	0.400011941	7.90×10^{-10}	0.500003	0.0999915675	0.0999880585	5.30×10^{-10}

(A)



(B)

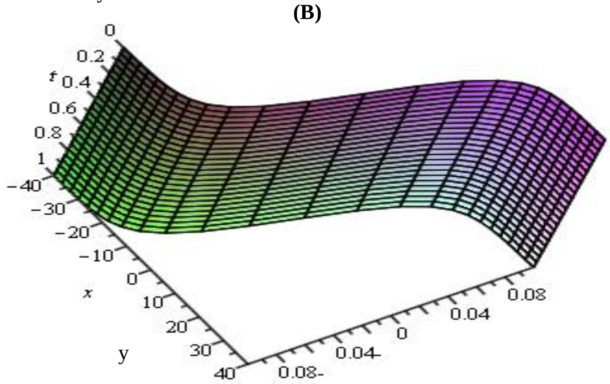


Fig. 1: If Λ = 0.5 And P1 = 0.1 And K = 0.1.

4. Conclusion

In the present paper a (NIM) is successfully applied to Solutions of the New Coupled MKdV Equation. The solution obtained by (NIM) method has produced less error than the methods listed in the tables for some text problems. The results are satisfactory and competent with some available solutions in the literature. Another advantages is that the method can be used without the complex calculations. to solve the system of differential equations reliably.

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