International Journal of Engineering &Technology, 7 (4) (2018) 6856-6861

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i4.27144
Research paper

Analysis of semi buried cylindrical water tank by finite element method and classical approach

Niyonyungu Ferdinand [1] *, Niyonsaba Callixte [2]

[1] *School of Civil engineering, Lanzhou Jiao tong University, Lanzhou, China*
[2] *Department of structural and construction Engineering, University of Dar es Salaam, Tanzania*
**Corresponding author E-mail: niol20@yahoo.fr*

Abstract

Classical approach is mostly where the access to finite element software is expensive, but it is limited to some structures with non-uniformity and it is time consuming. This paper explains the results from classical and finite element analysis of water tank by considering fixed base and free base loading condition. Thereafter, there are the comparison of the results (hoop stresses) obtained from finite element method with those obtained from classical approach. The comparison shows that the good results of finite element analysis are obtained when the tank is meshed into more elements.

Keywords: *Finite Element Method; Classical Method; Cylindrical Water Tank.*

1. Introduction

Both classical method and finite element method (FEM) are used to analyze cylindrical tanks. But classical method can be challenging for tanks with irregular shapes or with holes and fluid-structure-soil interaction on it.

The main objective of this paper is to compare the use of finite element method and classical method in analyzing semi-buried cylindrical water tanks. As the specific objectives, this project is concerned with the assessment of the convergence of the approximate values from the finite element method to the reference value (or exact value) from the classical approach as long as the tank is discretized into small elements. The accuracy of finite element method with respect to classical method is assessed so that the engineers can reduce some complexity and errors when analyzing semi-buried cylindrical water tanks. The scope of this project is based on analysis of semi-buried cylindrical water tanks by using classical method and FEM (using LISA), and then showing the convergence and accuracy of FEM values to the reference value from classical method.

2. Theory and classical approach

2.1. Loading conditions

The wall of the cylindrical tank is primarily designed to resist ring tensions (also called hoop tension) due to the horizontal pressures of the contained liquid [1].

If the wall is free at the top and free-to-slide at the bottom then, when the tank is full, the ring tension at depth z is given by $n = \gamma zr$ [2]

Where $\gamma$the unit is weight of the liquid and r is the internal radius of the tank. In this condition, when the tank is full, no vertical bending or radial shear exists.

The Underground water tank has three basic components; i.e., top slab, sidewalls and base slab.

- The top slab will be designed as normal simply supported slab based on the self-weight and superimposed loads
- The design of sidewalls and the base slab will be based on assuming (i) Tank full of water but no soil outside, (ii) No water inside tank but soil pressure from outside and (iii) Tank full of water and soil outside.
- For the case of partially buried cylindrical water tank; the worst loading is due to combined effect of earth and water during operation of the tank.

2.2. Deformation and stresses of cylindrical tank

Generally, for thin walled cylinder, three types of stresses are developed in pressure: circumferential or ring stresses (σ_θ) longitudinal or axial stresses (σ_L) and radial stresses (σ_r).

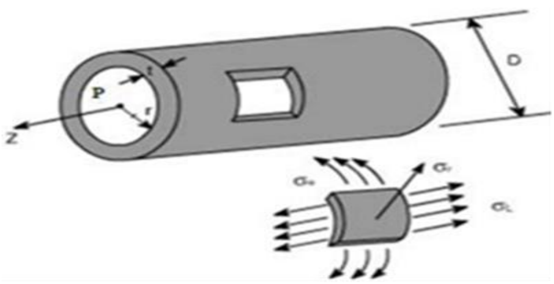


Fig. 1: Thin-Walled Cylinder under Pressure.

2.2.1. Ring stresses

It is a type of mechanical stress of a cylindrically shaped part as a result of internal or external pressure. It can be defined as the average force exerted circumferentially (perpendicular both to the axis and to the radius) on every particle in the cylinder wall.

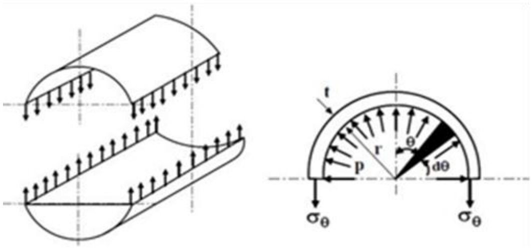


Fig. 2: Hoop Stress.

So long as the wall thickness is small compared to the diameter then the force trying to split the cylinder due to the pressure is

$$F = 2\int_0^{\frac{\pi}{2}} prL\cos\theta\, d\theta = 2prL = pD_iL$$
(1)

The cross-section area which sustains this force is given by

$$A = 2tL$$
(2)

Therefore, the ring stress is defined by

$$\sigma_\theta = \frac{F}{A} = \frac{PD_iL}{2tL} = \frac{pD_i}{2t}$$
(3)

2.2.2. Axial stress

In this case we consider the forces trying to split the cylinder along the length. The force due to pressure is

$$F = \int_0^r 2p\pi r dr = 2p\pi\frac{r^2}{2} = p\frac{\pi D_i^2}{4}$$
(4)

The cross-section area which sustains the force in this case is given by: $A = \pi D_i t$
This area has been approximated to a rectangle whose dimensions are the length of the circumference (πD_i) and the thickness.
Consequently, the axial stress is defined by:

$$\sigma_L = \frac{F}{A} = \frac{P\frac{\pi D_i^2}{4}}{\pi D_i t} = \frac{pD_i}{4t}$$
(5)

2.2.3. Radial stresses

The radial stresses are normal to the curved plane of the isolated element. In thin-walled cylinder theory, they are normally not considered, because they are too small compared to the other two stresses.

3. Finite element method

3.1. Introduction

Mathematicians and researchers continue to put the finite element method on sound theoretical ground whereas the engineers continue to find interesting application of it in different engineering branches. Hence, the FEM Knowledge makes a good engineer better while just user without intensive knowledge of FEM may produce more dangerous results.

3.2. Finite element equations

In the problems of linear elasticity of the mechanics of solids and structures the most common formulation employed consists in expressing the equilibrium differential equation in terms of displacement as the only independent field variable. The corresponding displacement formulation in the finite element method is based on the variation equation given by the minimum Total Potential Energy (TPE) [3].
By considering three-dimensional finite element having the vector of nodal displacement{q},

$$\{q\} = \{u_1 v_1 w_1 u_2 v_2 w_2\}$$
(6)

Displacement at any point of finite element {u} can be determined with the use of nodal displacement {q} and shape functionN_i.

$$\{q\} = [N]\{q\}$$
(7)

$$[N] = \begin{bmatrix} N_1 & 0 & 0 & N_2 \ldots \\ 0 & N_1 & 0 & 0 \ldots \\ 0 & 0 & N_1 & 0 \ldots \end{bmatrix}$$
(8)

Strains are determined from nodal displacement as follows,

$$\{u\} = [N]\{q\}$$
(9)

$$[B] = [D][N] = [B_1 \quad B_2 \ldots]$$
(10)

$$[B_i] = \begin{bmatrix} \partial N_i/\partial x & 0 & 0 \\ 0 & \partial N_i/\partial y & 0 \\ 0 & 0 & \partial N_i/\partial z \\ \partial N_i/\partial y & \partial N_i/\partial x & 0 \\ 0 & \partial N_i/\partial z & \partial N_i/\partial y \\ \partial N_i/\partial z & 0 & \partial N_i/\partial x \end{bmatrix}$$
(11)

By using the above equations, the total potential energy is expressed through nodal displacement as:

$$\prod = \int_V \frac{1}{2}[B]\{q\} - \{\varepsilon^t\}[E]([B]\{q\} - \{\varepsilon^t\})dV - \int_V([N]\{q\})^T\{P^V\}dV - \int_S([N]\{q\})^T\{P^S\}dS$$
(12)

The optimum nodal displacement is obtained by differentiating the equation [12] as$\left\{\frac{\partial \Pi.}{\partial q}\right\}$=0,
This gives

$$\int_V[B]^T[E][B]dV\{q\} - \int_V[B]^T[E][\varepsilon^t]dV - \int_V[N]^T\{P^V\}dV - \int_S[N]^T\{P^S\}dS = 0$$
(13)

And it can be presented in the form of:

$$[k]\{q\} = \{f\}, \{f\} = \{p\}$$
(14)

$$[k] = \int_V[B]^T[E][B]dV$$
(15)

$$\{P\} = \int_V[N]^T\{P^V\}dV + \int_S[N]^T\{P^S\}dS$$
(16)

Here $[k]$ is the element stiffness matrix; $\{f\}$ is the load vector and $\{P\}$ is the vector of actual forces
The total potential energy is the sum of elements potential energies:

$$\prod = \sum\prod_i = \sum\frac{1}{2}\{q_i\}^T[k_i]\{q_i\} - \sum\{q_i\}^T\{f_i\}$$
(17)

To obtain the finite element stiffness equation, the variation of TPE functional is decomposed into contributions from individual elements.

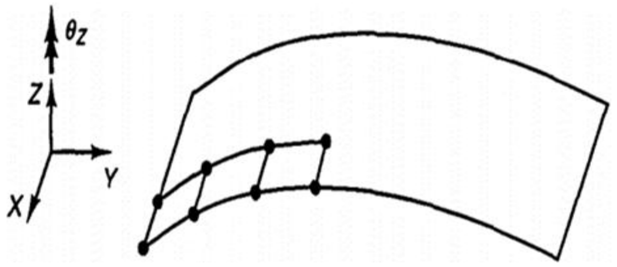


Fig. 3: Analysis of Slightly Curved Shell.

Thus,

$$\prod(u) = \sum_1^m \left\{\frac{1}{2}\int_{Be}(Du)^T H(Du)dV - \int_{Be}(Du)^T H\varepsilon dV - \int_{Be}\underline{u}^T \underline{P}dV - \int_{\partial Bt}\underline{u}^T \underline{\overline{t}}dS\right\}$$
(18)

The finite element approximation for displacement is given by,

$$\underline{u} = \sum \underline{N_i}\, \underline{q_i}$$
(19)

$N = N_m$, where N_m is the shape function for membrane element and q is the free parameter of displacement at the node to be determined.

$\underline{u} = \underline{N} * \underline{q}$ Then, $\underline{D} * \underline{u} = \underline{D} * \underline{N} * \underline{q} = DNq = \underline{B}q$

$$\underline{B} = \underline{D} * \underline{N}$$
(20)

Inserting [29] into [27], and taking $\prod$ for an element $\prod^e$ we get,

$$\prod^e(u) \cong \frac{1}{2}\int_{Be} q^T\,(B^T HB)qdV - \int_{Be} q^T\,(BH\overline{\varepsilon})dV - \int_{Be} q^T\,(N^T P)dV - \int_{\partial Bet} q^T \underline{N}\,\underline{\overline{t}}dS\text{N}$$

$$\prod^e(q) \cong \frac{1}{2}\int_{Be} q^T K_e q - q^T G_e - q^T F_e - q^T \overline{F}$$
(21)

Where,

$\underline{K_e} = \int_B \underline{B}\,\underline{H}\,\underline{B}dV$ Stiffness matrix of the element

$\underline{G_e} = \int_B \underline{B}\,\underline{H}\,\underline{\overline{\varepsilon}}dV$ Vector of equivalent nodal solution

$F_e = \int_B \underline{N}^T\, PdV$ Equivalent applied nodal loads (volume).

$\underline{\overline{F}}_e = \int_B \underline{N}^T\,\underline{\overline{t}}dV$ Equivalent loads on nodal boundary

Stationary condition of$\prod^e$ respect to q for such sub domain is

$$\forall q, \partial q, \delta\prod(q) = \underline{K_e}q - \underline{G_e} - F_e - \underline{\overline{F}}_e = \underline{K_e}\underline{q} - f = 0$$
(22)

Therefore, the consistent element nodal force vector is

$f = \underline{K_e}\underline{q}$, where

$$f = \underline{G_e} + F_e + \underline{\overline{F}}_e$$
(23)

For the stiffness matrix of the element, we have

$$\underline{\overline{K}}_e = \int_B \underline{B}^T\,\underline{H}\,\underline{B}dV$$
(24)

But $B = B_m$where

$$B_m = D_m * N_m$$
(25)

$$\underline{K_e} = \int_B B_m^t\, H_m B_m dV$$
(26)

Where,

$\underline{K_e}$-Stiffness matrix of shell element

And thus, the stiffness matrix of the shell element is the combination of stiffness matrix for plate in bending and the stiffness matrix for membrane. The resulting matrix has dimension of 20x20 and can be directly employed in the analysis of variety of shell structure like cylindrical tank. The governing equations for finite element method are as follow:

1) Equilibrium:$D^T\sigma + b = 0$
2) Compatibility strain-displacement:$e = Du$
3) Hook's law:$\sigma = Ee$

Where σ is the stress, e is the strain and u the displacement.
$\boldsymbol{D}$ Is the strain-displacement matrix, $\boldsymbol{E}$ is the elastic modulus.

4. Case study analysis

4.1. Description of the model

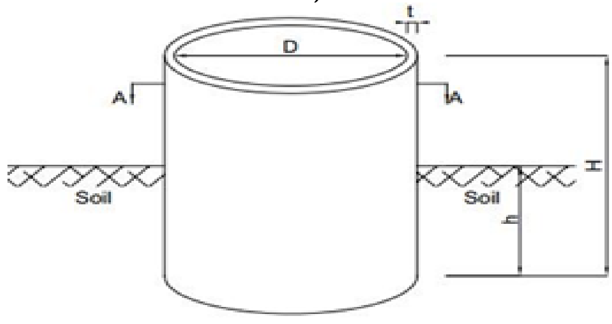
A semi buried reinforced concrete cylindrical purification tank with free at the top is used as model.
The model has the following characteristics:

- Total Height of vertical wall, H=6m
- Underground Height of vertical wall, h=3m
- Mean Diameter of tank, D=15m
- Thickness of vertical wall, t=30cm

The storage liquid (water) and construction materials have the following Properties:

- Density of water,$\gamma_w = 9.81\,kN/m^3$
- Density of the soil, $\gamma_s = 18\,kN/m^3$
- Density of concrete, $\gamma_c = 24\,kN/m^3$
- Friction angle of the soil,$\emptyset = 35°$
- Angle of internal friction, ,$\varphi = 30°$
- Coefficient of wall friction,$v = 0.45$
- Young modulus of elasticity of concrete,$E = 25E6Kpa$
- Characteristic strength of concrete,$f_{cu} = 25\,N/m^2$
- Poison ratio of concrete,$\mu = 0.2$
- Coefficient of thermal expansion of concrete $\alpha = 10E - 6/°F$

A)



B)

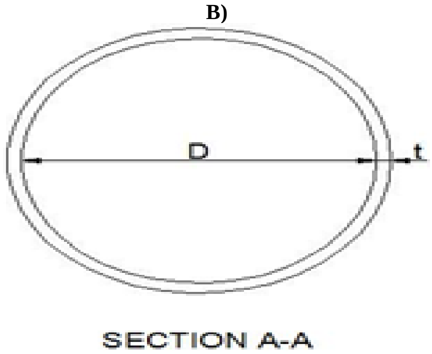


Fig. 4: A) Cylindrical Water Tank; B) Section A-A.

4.2. Analysis of semi buried cylindrical water tank by classical approach

4.2.1. Tank with sliding base

For the tank with sliding base, water pressure is fully resisted by ring action without developing any bending moment or shear. The wall of the cylindrical tank is primarily analyzed to resist hoop stresses due to the horizontal pressures of the contained liquid [1]. The hoop stresses at depth z is given by

$\sigma_h = \gamma zr/t, K_a = \frac{1-\sin\emptyset}{1+\sin\emptyset}$

Table 1: Variation of Hoop Stresses along the Side Walls for Tank with Sliding Base

Z from the top of the tank [m]	Stresses due to water pressure $\sigma_{hw} = \gamma_w Zr/_t [1]$	Stresses due to earth pressure, $\sigma_{he} = -(\gamma_s zK_a r/t)[2]$	Combination $\sigma_h = [1] + [2]$
0.0H	0	-	0.00
0.2H	294300	-	294300.00
0.4H	588600	-	588600.00
0.6H	882900	-73167.31	809732.69
0.8H	1177200	-219501.94	957698.06
H	1471500	-365836.57	1105663.43

4.2.2. Tank with fixed base

For the tank with fixed base, the water pressure will be resisted by hoop action in the horizontal direction and cantilever action in the vertical direction [1,7]. Due to fixity at base of wall, the upper part of the wall will have hoop stress and lower part will act as cantilever. For this paper, IS code method is used for analysis of circular water tank with rigid base.

Coefficients to determine values of Hoop stresses are calculated based on Table 9 of IS 3370 part IV.

$\frac{H^2}{Dt} = \frac{6^2}{(15-0.3)*0.3} = 8.16 \approx 8$

Table 2: Variation of Hoop Stresses along the Side Walls for Tank with Fixed Base

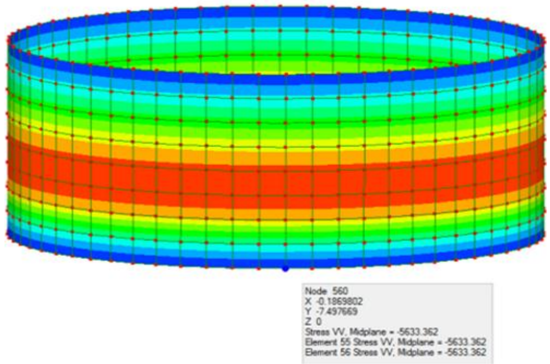
Z (From the top of the tank) [m]	Hoop stresses due to water pressure	Hoop stresses due to earth pressure	Stresses due to water pressure $\sigma_{hw} = coef * \frac{\gamma_w Hr}{t}$ [1]	Stresses due to earth pressure $\sigma_{he} = -(coef * \frac{\gamma_w Hr}{t})$ [2]	Combination $\sigma_h = (1) + (2)$
0.0H	-0.011		-16186.5		-16186.5
0.2H	0.218		320787		320787.0
0.4H	0.443		651874.5		651874.5
0.6H	0.575	0.218	846112.5	-79752.37	766360.1
0.8H	0.381	0.575	560641.5	-210356.03	350285.5
1.0H	0	0	0	0.00	0.0

Table 3 shows that the maximum hoop pressure is at 0.6H from the top. At the top, there is slight hoop compression in the wall. This is due to inward radial deflection at the top.

4.3. Analysis of semi buried cylindrical water tank by finite element method

The model is modeled and analyzed by using a finite element software LISA. The square finite elements are used, and results are computed for different number of elements [4-5].

A)



B)

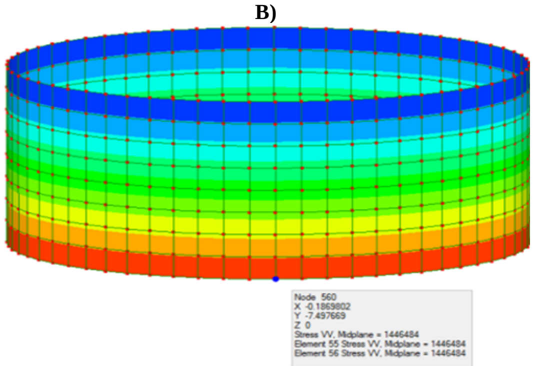
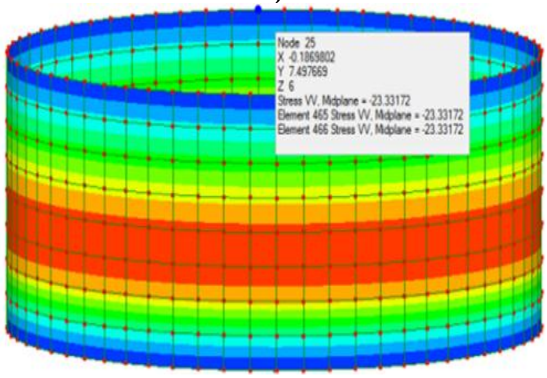


Fig. 5: Three-Dimensional View of the Tank Model (504 Elements) Showing the Results (Stress) at the Bottom Edge: A) when the Base Is Fixed, B) when the Base is Sliding.

A)



B)

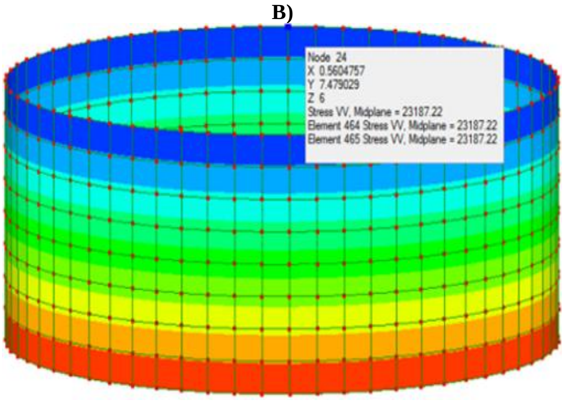


Fig. 6: Three-Dimensional View of the Tank Model (504 Elements) Showing the Results (Stress Vv) at the Top Edge: (A) when the Base is Fixed, (B) When the Base is Sliding.

Interpretation of results
The hoop stress is the resultant obtained from horizontal and vertical stress. The error is calculated by the formula:

$$Error = \left(\frac{(Exact\ value - Approximate\ value)}{Exact\ Value} * 100\right)\%$$

The exact values are the values obtained by analyzing cylindrical water tank with use of classical approach and approximate values are the values obtained by analyzing cylindrical water tank with use of finite element software LISA.

4.4. Comparison of the results from classical method and finite element method

The comparison is done by comparing hoop stresses results at the bottom edge and top edge from finite element and classical method. Also two cases of the base (fixed and sliding) [6] are considered.
The hoop stresses [8] from classical approach are considered as the benchmark and compared with that from finite element approach from different nodes.
The results are presented graphically as follows

4.4.1. Full tank with sliding base

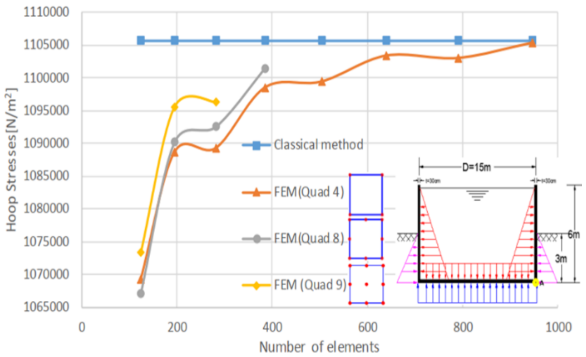


Fig. 7: "Hoop Stress Due to Both Water Pressure and Earth Pressure" Variation with Number of Meshing Elements at Point A for Tank with Sliding Base.

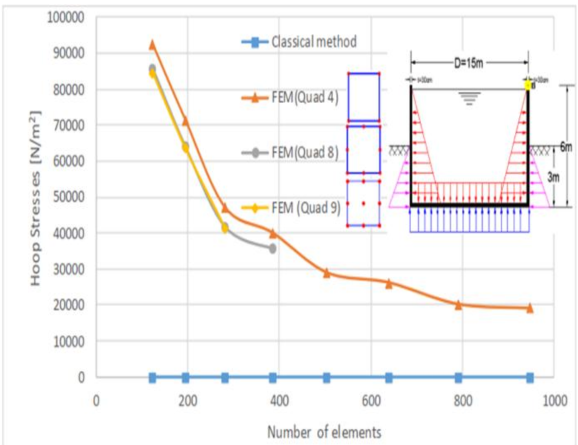


Fig. 8: "Hoop Stress Due to Both Water Pressure and Earth Pressure" Variation with Number of Meshing Elements at Point B for Tank with Sliding Base.

4.4.2. Tank with fixed base

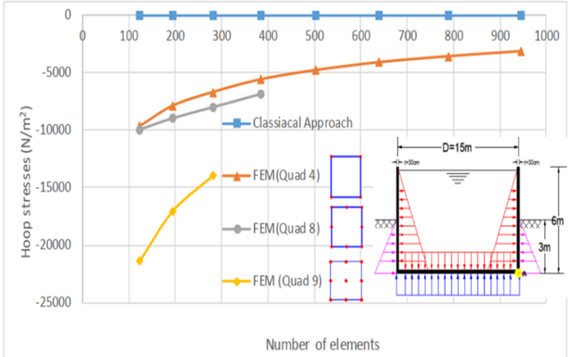


Fig. 9: "Hoop Stress Due to Both Water Pressure and Earth Pressure" Variation with Number of Meshing Elements at Point A for Tank with Fixed Base.

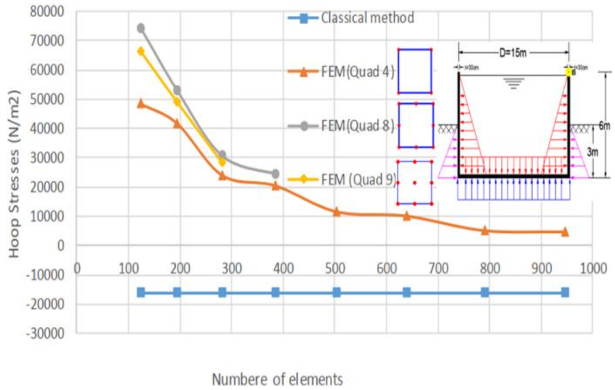


Fig. 10: "Hoop Stress Due to Both Water Pressure and Earth Pressure" Variation with Number of Meshing Elements at Point B for Tank with Fixed Base.

5. Discussion of the results

As the number of elements increases, the results from finite element Method approach the exact solution (classical approach results). Since in analysis of semi buried cylindrical tanks, engineers the mesh must be increased to infinite number of elements.
Even though the results show that as elements increase FEM results approach the exact solution, there are some errors that were mostly due to the fact that the whole finite element is assumed to be loaded with the same load. This is different from the real situation [9, 10] where the pressure distribution of water and soil in the side of the walls is triangular with the maximum of

$p = wh$ and $p = \gamma_s h\left(\frac{1-\sin\emptyset}{1+\sin\emptyset}\right)$respectively.

6. Conclusion and recommendations

Based on the results obtained, it can be concluded that if the interpolation functions satisfy certain mathematical requirements, a finite element solution for a problem converges to the exact solution of the problem. That is, as the number of elements is increased, the finite element solutions changes incrementally. The incremental changes decrease with the mesh refinement process and approach the exact solution asymptotically.
As recommendation; by analyzing the above graphs, it is clear that the quad 4 elements is more accurate than other elements like quad 8 and quad 9. That is using quad 4 in analyzing cylindrical water tank is recommended. Also, the good results of finite element analysis are obtained when the tank is meshed into more elements. It is recommended to mesh the cylindrical tank as much as possible so that the hoop stresses can significantly approach the exact solution.

References

[1] Reynolds, C., Steedman, J., Threlfall, A. (2008). Reinforced Concrete Designer's Handbook. London: CRC Press, https://doi.org/10.4324/9780203087756.

[2] ACI Committee 350.3-06, 2006, "Seismic design of liquid-containing concrete structures (ACI 350.3-06) and commentary (ACI 350.3R-06)", American Concrete Institute, Farmington Hills, MI, U.S.A.

[3] Zienkiewicz, O.C & Brotton, D.M & Morgan, L. 1976. A Finite Element Primer for Structural Engineers, the Structural Engineer Vol 54, No 10, October, Pages 389 - 397

[4] K. Ho-Le, “Finite element mesh generation methods: a review and classification,” Computer-Aided Design, vol. 20, no. 1, pp. 27–38, Jan. 1988. https://doi.org/10.1016/0010-4485(88)90138-8.

[5] E. Ventsel, T. Krauthammer, and E. Carrera, “Thin Plates and Shells: Theory, Analysis, and Applications,” Applied Mechanics Reviews, vol. 55, no. 4, p. B72, 2002. https://doi.org/10.1115/1.1483356.

[6] Atalay MB, Stine MD, Farnworth SK. Stress evaluation of the primary tank of a double-shell underground storage tank facility. Office of Scientific and Technical Information (OSTI); 1994 Dec 1; Available from: https://doi.org/10.2172/80957.

[7] T. A. Chapel and D. Laymon, “Site Characterization and Modeling for an Underground Water Storage Tank, Black Hawk, Colorado,” GeoFlorida 2010, Feb. 2010. https://doi.org/10.1061/41095(365)91.

[8] Hsu PW. Maximum Hoop Stresses in a Pressurized Buried Pipe under an Impact Load. Pipelines 2001 [Internet]. American Society of Civil Engineers; 2001 Jul; Available from: https://doi.org/10.1061/40574(2001)47.

[9] N. el Mezaini, “Effects of Soil-Structure Interaction on the Analysis of Cylindrical Tanks,” Practice Periodical on Structural Design and Construction, vol. 11, no. 1, pp. 50–57, Feb. 2006. https://doi.org/10.1061/(ASCE)1084-0680(2006)11:1(50).

[10] Circular Storage Tanks and Silos, Second Edition. CRC Press; 2003 Sep 2.