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Research paper

The Development of Transformation Elements between the Fracture Mechanics Dependences and the Equations of the Reinforced Concrete Theory

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Abstract

It has been developed a transformational element that relates the dependencies of the fracture mechanics to the calculation of reinforced concrete structures by the second group of limiting states. It is described the features of cutting a two-cantilever element including a crack for constructing an effective instrument of calculation for reinforced concrete with allowance for physical nonlinearity, cracking processes, bond of reinforcement with concrete and the effect of discontinuity. The results of development of two-cantilever elements of fracture mechanics for various force effects are presented: bending, eccentric compression, central extension, and also in the zone of inclined cracks. It is obtained a new solution to the problem of the stressed-strained state of the reinforced concrete element in the zone immediately adjacent to the crack.

Keywords: *transformation element, fracture mechanics, reinforced concrete structures, two-cantilever element, calculating apparatus, crack resistance, bond, discontinuity effect.*

1. Introduction

The main characteristic feature of bearing reinforced concrete structures is the presence of cracks in resisting its force and deformation influences.
Involvement of a finer calculating apparatus associated with the fracture mechanics (FM) has significant advantages. Now days FM has long been included in various fields of technology, aircraft engineering, shipbuilding, etc. in the study of various processes of deformation and damage to the model environment. It is possible to more precisely study the stress-strain state in zones adjacent to the cracks with the help of FM tools.
Unfortunately, attempts to apply the basic provisions of the FM for the calculation of reinforced concrete structures have not yet been adequately reflected in the theory of reinforced concrete. With regard to such a differential parameter as the width of the opening of cracks w_k – the differences between experimental and theoretical values can reach more than 200%. To date, there is practically no development that establishes the dependence of traditional parameters of reinforced concrete calculation (the resistance of stretched concrete between cracks, the distance between cracks, the width of opening cracks) with new elements of FM.
It should be noted that the difficulties that arise here are the main cause (along with the apparent need for the use of complex numbers), according to which the detailed tools of the FM have not yet been properly applied in the theory of reinforced concrete. So, it is a problem that is investigated by the FM, actively developing in recent years [1–4].
The results achieved in this area allow already today to extend the accumulated information to the calculation of reinforced concrete [1, 2, 5]. It should be borne in mind that for concrete as an elasto-plastic material, laws of linear FM are not applicable.
The specificity of the material manifests itself not only in the basic dependence of the FM, which relates the stress intensity factor to the amount of released energy per unit of the newly created crack surface ζ_I, but also in such concepts as the prefracture zone at the crack mouth, the critical stress intensity factor K_{cr} and the corresponding value ζ_{cr}, the limiting value of ζ_{cu}, corresponding to continuation of the crack, etc. Much depends on the successful separation of the two-cantilever element, including the crack [3, 5–8].
The number of studies have established that the parameter characterizing the crack initiation is the magnitude of the critical crack opening, designated by different authors in different ways: η_{max} – in the Sih's model (Evanston, USA) [10], ω_{max} – in the Hillerborg-Moder-Peterson's model – (Lund, Sweden) [11], δ – in the Leonov-Panasyuk's model, etc. The determination of this quantity by the majority of authors is carried out identically – on the basis of tests for axial extension (Sih's experiments [10], Hillerborg [11], Bažant [1, 12], etc.). The peculiarity of these tests is that the test unit allows you to set the amount of displacements, and, consequently, to obtain complete diagrams σ–ω with a descending branch of deformation. And phenomenologically, it is believed that such diagrams can be used in the predestruction zone. Indeed, even an approximate analysis of these zones shows that along with tensile stresses there are also compressive forces that are successfully modeled in the noted tests for axial tension.

2. Problem definition

In the case of a solid body whose strain-strain state is analyzed by the methods of the elasticity and plasticity theory, an elementary cube is distinguished. It describes the connection between stresses and deformations at a point. Further, in the transition to a cross-section, the established link integrates across the cross-section. As a result, the problem is reduced to differential equations, the exact solution of which, as a rule, is very difficult. In the material resistance, a hypothesis of flat deformation for the entire cross section is adopted, which greatly simplifies the solution of the problem. For a non-permeable body with a crack (where body integrity is disturbed), when establishing a connection between stresses and displacements, the methods developed in the elasticity and plasticity theory and the resistance of materials are not applicable. Nevertheless, the use of the basic cross-sectional method for cracking material brings its positive results. This also applies to the approximate reception of the determination of the intensity coefficient of stresses; it can also be used for the allocation of a special two-cantilever element, which has found application in the FM.

3. Development of two-cantilever elements

The selection of two-cantilever element, including a crack, with its core reinforced concrete element has its own specificity [6, 7, 13]:

– it is allocated to the entire height of the crack, and not for some of its elementary section;

– the forces in the sections passing at a distance t from the crack must be related to the required parameters of the stress-strain state of the reinforced concrete element;

– we should not forget about the virtual movements of the dedicated consoles when the neutral axis of the reinforced concrete element is rotated, that is, cantilever support is not absolutely rigid.

Thus, this important and difficult problem must be linked not only with the task of determining the stress-strain state of the cross-section of the reinforced concrete element, but also with the task of distributing the bond between the reinforcement and concrete, since the appearance of a crack in the solid body can be considered as a deformation effect that affects the bonding characteristics of reinforcement and concrete in the zones adjacent to the crack. With the aid of a two-cantilever element, the connection of its stress-strain state with the value of ζ_{cu} in the predestruction zone seems to be the most successful. In this case, the compliance of the crack's shores is determined by using conventional methods of structural mechanics, through which the value of ζ_{cu} can be expressed. Thus, a two-cantilever element is used as a link between the dependencies of the solid deformed body mechanics and the FM.

The above considerations [3, 5–8, 13] were used to isolate the two-cantilever elements shown in figures 1–3.

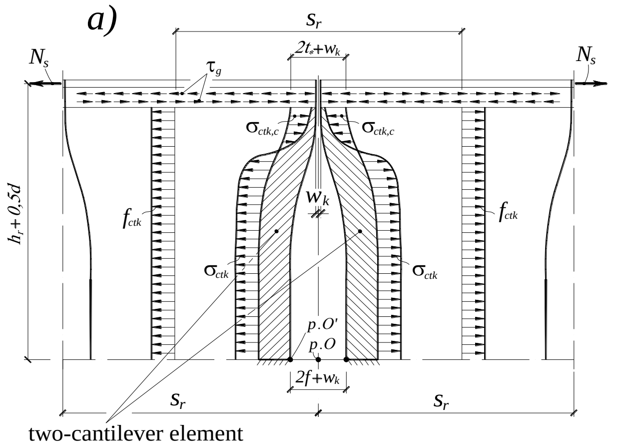
Here, the parameter t (characterizing the size of the compressed concrete zone in the vicinity adjacent to the crack), in accordance with the Saint-Venant's principle and with studies of the near-reinforcement zone, involving semi-analytical and numerical methods, is in the first approximation equal to one and a half diameter of reinforcement. In the following, the value of parameter t is refined by solving the coupling problem. The tensile stresses in the emitting sections are distributed by the law of a square parabola from the neutral axis to the point where the sign of these stresses changes.

At the same time, their maximum value is limited by the value of f_{ctk}; therefore, in a significant area, the actual distribution of tensile stresses is close to a rectangle, regardless of the law of their distribution in the elastic stage. Compressive stresses in the same sections in areas adjacent to the reinforcement are distributed by triangle.

To uncover the static uncertainty of the carved two-cantilever element, it is performed the transformation of the functional FM, which binds the specific surface of the formation of the crack (ζ_{cu}) to the increase of the potential energy δV and the additional work in the propagation of the crack of the body δW, which is expressed through the flexibility function, in relation to the reinforced concrete:

$$\zeta_{cu} = \lim_{\delta A \to 0}\left(\frac{\delta W - \delta V}{\delta A}\right) = \frac{1}{3}\left(P^2 \frac{\partial C}{\partial A} - CP\frac{\partial P}{\partial A}\right), \qquad (1)$$

where δA – increase in the area of the formed surface of the crack.



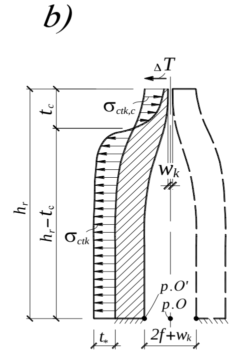
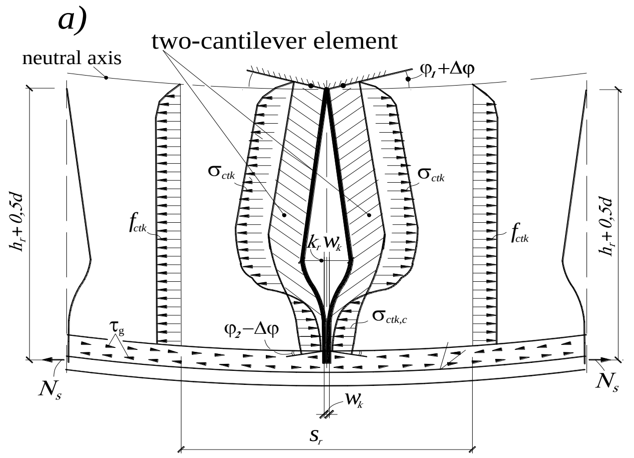


Fig. 1: To the implementation of the fracture mechanics dependencies in reinforced concrete structures for ***central tension***:
a – characteristic stress diagrams in the stretched concrete and cutting a special two-cantilever element in the vicinity of the crack; *b* – to calculate the flexibility of the console;
s_r = distance between cracks; w_k = the width of the crack opening at the level of the working reinforcement axis; h_r = length of crack; d or (Ø) = diameter of working reinforcement;
N_s = forces arising in working reinforcement; τ_g = conditional tangential bond stresses between concrete and reinforcement; f = distance, which is sought as a parameter equal to $w_k k_r$, where k_r is the deplanation coefficient shores of the crack; t_* = area near the crack vicinity (for practical calculations is equal 1.5Ø of the working reinforcement); ΔT = resultant conditional tangential stresses in the local zone adjacent to the crack; $\sigma_{ctk,c}$ = compression stress in concrete of a stretched zone; f_{ctk} = the characteristic value of the strength of concrete on the axial tension of the resistance of the stretched concrete; σ_{ctk} = stresses of the stretched concrete



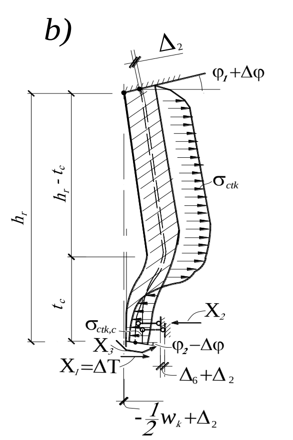


Fig. 2: To the implementation of the fracture mechanics dependencies in reinforced concrete structures for ***non-eccentric compression***: *a* – characteristic stress diagrams in the stretched concrete and cutting a special two-cantilever element in the vicinity of the crack; *b* – calculation scheme for flexibility of the console with unknown efforts and displacements; (φ_2+Δφ) & (φ_2–Δφ) = corresponding deformation angles of the pinch turning at the intersection of the working reinforcement rod; Δ_3 & Δ_6 = displacements caused by crack opening at the level of the reinforcement axis and in the place of maximum opening and equal to half (due to symmetry) of these values; Δ_2 = displacements, caused by the biases of the longitudinal geometric axis of the cantilever, which is caused by deformations of shortening from the longitudinal compressive force applied on the neutral axis, in the cross-section of the reinforced concrete element passing along the crack; $X_1=\Delta T$ = shear force in close proximity to the vicinity of the crack, in the area of t_*; X_2 = resultant compressed in cross-section, passing at a distance t_* from the crack, in the zone adjacent to the working reinforcement; X_3 = moment component in the valve.

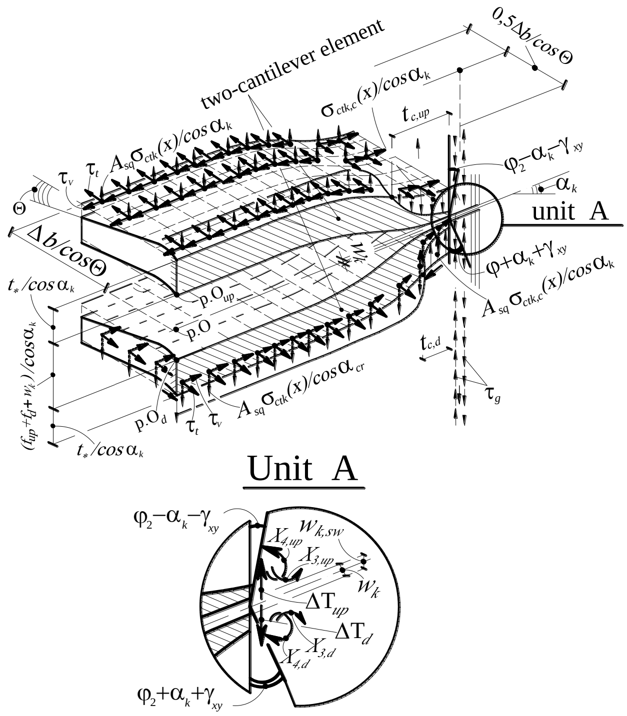


Fig.3: Universal two-cantilever element to realize the dependence

of FM in reinforced concrete in the zone of spatial cracks

Note: the symbols given in figure 3 are similar to the notation in figures 1 and 2; the index "***up***" refers over crack; the index "***d***" stands under crack.

The two-cantilever element fluidity is associated with movements of the entire reinforced concrete structure, and therefore with its rigidity, including in the presence of cracks. In view of the clear physical content proposed by the transformation two-cantilever elements and the engineered visibility of the functional of the fracture mechanics, they can be included directly in the developed physical models of resistance, the more so that the two-cantilever element used here applies to any cases of a stress-strain state, Figures 1–3.

In general, the length of the crack h_r is determined, using the following condition of fracture mechanics [13, 14]:

$$\frac{d\zeta_{cu}}{dh_r}=0\,, \qquad (2)$$

where

$$\zeta_{cu}=\frac{1}{3b}\cdot\left(\Delta T^2\cdot\frac{\partial C_I}{\partial h_r}+P_1^2\cdot\frac{\partial C_{II}}{\partial h_r}+P_2^2\cdot\frac{\partial C_{III}}{\partial h_r}\right)+$$
$$+b^2 f_{ctk}^2\cdot\frac{\partial C_q}{\partial h_r}+M_{con}^2\cdot\frac{\partial C_0}{\partial h_r}-C_I\Delta T\cdot\frac{\partial \Delta T}{\partial h_r}-$$
$$-C_{II}P_1\cdot\frac{\partial P_1}{\partial h_r}-C_{III}P_2\cdot\frac{\partial P_2}{\partial h_r}-C_0 M_{con}\cdot\frac{\partial M_{con}}{\partial h_r}\,. \qquad (3)$$

In equation (3) C_I – C_{III} are compliances of the corresponding consoles of the calculation schemes (Figures 1, *b* & 2, *b*).

After differentiation (2) taking into account the corresponding coefficients $\eta_1-\eta_{15}$, we will have:

$$0.5\cdot\eta_{14}\cdot h_r^2+\left(\Delta T\eta_8-S-\eta_5-\eta_8-\eta_{14}-\Delta T\eta_1-2\Delta T\cdot\eta_7\right)\cdot h_r-$$
$$-S-\eta_5-\eta_8-\Delta T\eta_1-2\cdot\Delta T\eta_7+\Delta T\eta_8-$$
$$-\xi_\tau\cdot\varepsilon_{q_1el}\cdot b\cdot t\cdot\eta_2-\Delta T\eta_2=0\,. \qquad (4)$$

Thus, we have an analytic dependence (4) for determining the length of the crack h_r.

It is necessary to emphasize the fact that in the selection of two-cantilever element (figures 1–3), which includes a crack, there are two possible options: the first one considers a long two-cantilever element, which completely includes a crack for its entire length, and the second – a short two-cantilever element, allocated between the tiers longitudinal working reinforcement or between transverse rebar (clamps, bends). The value of h_r is also known here and equals half the distance between the working reinforcement. In this case, the solution is considerably simplified.

With regard to determining the length of the inclined or spatial crack projection (in the case of a complex stress-strain state, for example, under the action torsion with bending, figure 3), it can be found by involving the function of many variables, using Lagrange multipliers λ_i, equation (6).

The projection of inclined crack whose width of disclosure has a maximum value is determined from the extremum condition of the many variables functions [15]

$$F_i=f\left(q_{sw},x_B,\sigma_s,x,\sigma_c,\sigma_{s,I},\sigma_{c,1},C_2,\lambda_1,\lambda_2,\lambda_3,\lambda_4,\lambda_5,\lambda_6,\lambda_7\right), \qquad (5)$$

where *i* is the index that determines the location of the desired cracks opening width or the minimum cracking load or load that corresponds to the minimum rigidity, etc.).

The conditions for the equality of zero partial derivatives arising from this condition are:

$$\left.\begin{array}{l}\dfrac{\partial f}{\partial x_1}+\lambda_1\dfrac{\partial\theta_1}{\partial x_1}+\lambda_2\dfrac{\partial\theta_2}{\partial x_1}+\ldots+\lambda_m\dfrac{\partial\theta_m}{\partial x_1}=0\\ \dfrac{\partial f}{\partial x_2}+\lambda_1\dfrac{\partial\theta_1}{\partial x_2}+\lambda_2\dfrac{\partial\theta_2}{\partial x_2}+\ldots+\lambda_m\dfrac{\partial\theta_m}{\partial x_2}=0\\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\ \dfrac{\partial f}{\partial x_n}+\lambda_1\dfrac{\partial\theta_1}{\partial x_n}+\lambda_2\dfrac{\partial\theta_2}{\partial x_n}+\ldots+\lambda_m\dfrac{\partial\theta_m}{\partial x_n}=0\end{array}\right\}. \qquad (6)$$

In this case, it is possible to determine the functional distance between different cracks s_r along the axis of longitudinal reinforcement, having the stresses in longitudinal reinforcement in a section with a dangerous inclined crack (in terms of the maximum width of the opening at the level of the axis of longitudinal reinforcement) [15].

Then we may determine the distance between cracks s_r along the axes of the transverse reinforcement, knowing the stresses in the cross-sectional reinforcement in a section with a dangerous inclined crack (by the criterion of the maximum opening width at the level of the axis of the transverse reinforcement).

The increase of deformations in the reinforcement with increasing loading causes a decrease in the distance between the cracks. In this case, the appearance of a new level of cracking corresponds to the load level, which adheres to the following inequality [15]:

$$s_{r,i} \le \beta \cdot s_{r,i\text{-}1}\,, \tag{7}$$

where parameter β is taken from the ratio between stresses in transverse reinforcement according to Figure 4.

Thus, the process of formation of cracks continues until destruction of the reinforced concrete construction (RCC). There is not one, as it is accepted in a number of known techniques, and several levels of cracking:

$$\left.\begin{array}{c} s_r > s_{r,1} - RCC\; has\; no\; cracks; \\ s_{r,1} > s_r > s_{r,2} - first\ level\ of\ cracks\ formation; \\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ \ldots\ldots\ldots\ ; \\ s_{r,n} > s_r > s_{r,n+1} - n - level\ of\ cracks\ formation \end{array}\right\}. \tag{8}$$

The degree of crack's implementation (crossing these cracks transverse reinforcement or it will only cross the dangerous inclined crack) is determined from the consideration of the stress-strain state along the transverse rebar based on the calculation scheme of the next level shown in Figure 4. It "closes" to a multi-threaded process performed analytically or automated with the PC "Lira-CAD" [14].

The volumes of concrete (formed by the thickness of the structure and the dimensions, compared with the step of the transverse reinforcement or with the double thickness of the protective layer of longitudinal reinforcement, respectively) are cut from the reinforced concrete structure. They include transverse or longitudinal reinforcement and are considered as level models. They are used to determine the deformations of the stretched concrete $\varepsilon_{stk}(y)$ along the axis of the transverse rebar of the *i*-th level of the of various cracks formation and the distance between them in reinforced concrete structures.

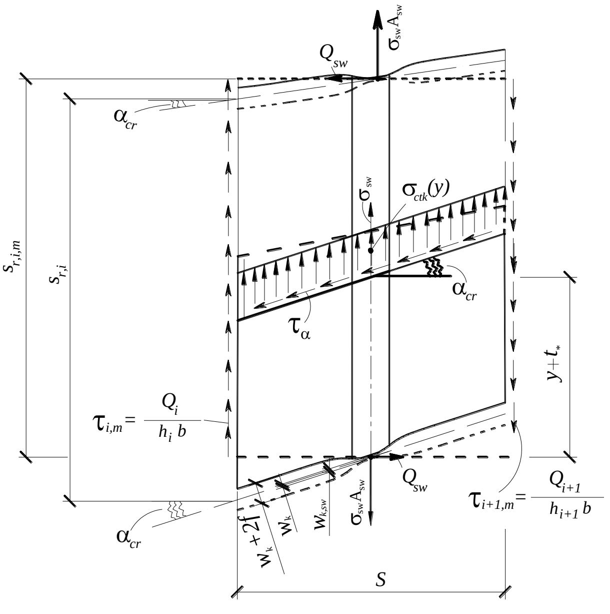


Fig. 4: Level model for determining the deformation of stretched concrete $\varepsilon_{ctk}(y)$ along the axis of the transverse of *i*-th level

of the various cracks formation and the distance between them in RCC

α_{cr} = angle of the crack inclination of in relation to the horizontal axes; Q_{sw} = transverse force in rebar; $w_{k,sw}$ = width of opening of cracks at the level of axis of transverse rebar; A_{sw} = the area of the transverse rebar; stresses arising in the transverse rebar from the force Q_{sw}; $s_{r,i}$ = the distance between *i*-th cracks; $s_{r,i,m}$ = the averaged distance between the projections of the cracks on the horizontal; $\tau_{i,m}$ & $\tau_{i+1,m}$ = corresponding tangential stresses acting along the cut volume of concrete; *S* = step of transverse rebar

In some cases, a significant simplification can be achieved by using long two-cantilever element [6, 7, 13, 14]. For example, for non-centric compressed reinforced concrete structures in a compressed zone, the development of a crack stops (Figure 2). Here the length of crack h_r is a constant value, equal (d–x) (where *d* is a working height of the cross-section; *x* is a height of the compressed zone). In this particular case, the derivative using of the fracture mechanics functional (condition (2) is not required, because it is a known value (Figure 2, *b* & 5). In addition, the flexibility of the crack shores is determined using ordinary methods of construction mechanics through which the value of ζ_{cu} can be expressed.

With regard to the dedicated two-cantilever element (Figure 2, *b*), which is under the influence of five forces (ΔT, P_1, P_2, q, M_{con}), expression (7) takes the form:

$$\zeta_{cu} = \frac{1}{3}\sum_{i=1}^{5}\left(P_i^2 \frac{\partial C_i}{\partial A} - C_i P_i \frac{\partial P_i}{\partial A} \right). \tag{9}$$

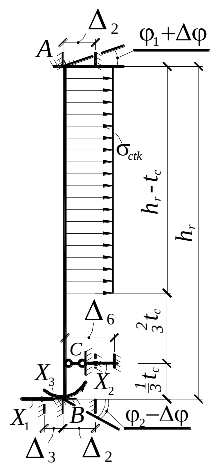


Fig. 5: Calculation scheme of the construction mechanics to determine the consistency of the console of non-centric compressed reinforced concrete structures

The malleability of the two-cantilever element is determined from the design scheme and equals:

$$C_I = \frac{2 \cdot \Delta_I}{\Delta T}\,; \tag{10}$$

$$C_{II} = \frac{2 \cdot \Delta_{II}}{-P_1}\,; \tag{11}$$

$$C_{III} = \frac{2 \cdot \Delta_{III}}{P_2}\,; \tag{12}$$

$$C_0 = \frac{2 \cdot \varphi_2}{M_{con}}\,. \tag{13}$$

After differentiation and algebraic transformations from equation (9) we obtain:

$$\Delta T = \frac{h_r(\eta_{15} - \eta_5 - \eta_8) - G_\tau \varepsilon_{q,el} b t \eta_2 + 0{,}5 h_r^2 \eta_{14}}{\eta_2 + \eta_r(\eta_1 + 2\eta_7 - \eta_8)}\,. \tag{14}$$

where η_1, η_2, η_5, η_7, η_8, η_{14}, η_{15} are the functions of such parameters f_{ctk}, E_c, *m*, *t*, *b*, I_{con}. In addition, one of the parameters depends on the turning angle of the neutral axis φ_1 of reinforced concrete construction (Figure 2, b & 5) and parameter η_{15} depends on the concrete constant: ζ_{cu}.

Dependence (14) allows us to find the component of tangential stress in a zone that is directly adjacent to a crack. Experimental and numerical studies show that it is in this zone that there is a sharp perturbation of tangential stresses, which is accompanied by a change in their sign. In this case, signs of normal stresses in concrete are also changing (which is also confirmed by experiments [2, 3, 14, 16–19]) due to the effect of breaking the solidity. In those cases where h_r is known (Figures 1–3 & 5), the parameters $X_1=\Delta T$, $X_2=P_{ctk,c}$, … , X_n can be determined using conventional methods of construction mechanics.

There are deformation influences $\Delta_2 \ldots \Delta_6$ due to the crack opening and relative displacements in Figure 5; Δ_2 – due to displacements of the longitudinal geometric axis of the console caused by deformations of the shortening of the longitudinal compressive force applied on the neutral axis in the cross-section of the reinforced concrete element, which passes through the crack; Δ_3 and Δ_6 – due to the opening of the crack at the level of the axis of the reinforcement and in the place of maximum disclosure, they are equal to half (due to the symmetry) of these values.

The paper also considers the peculiarities of angular displacements φ_1, φ_2, $\Delta\varphi$ (Figure 6), which are connected the width of crack opening and deformability of the reinforced concrete constructions buildings and facilities.

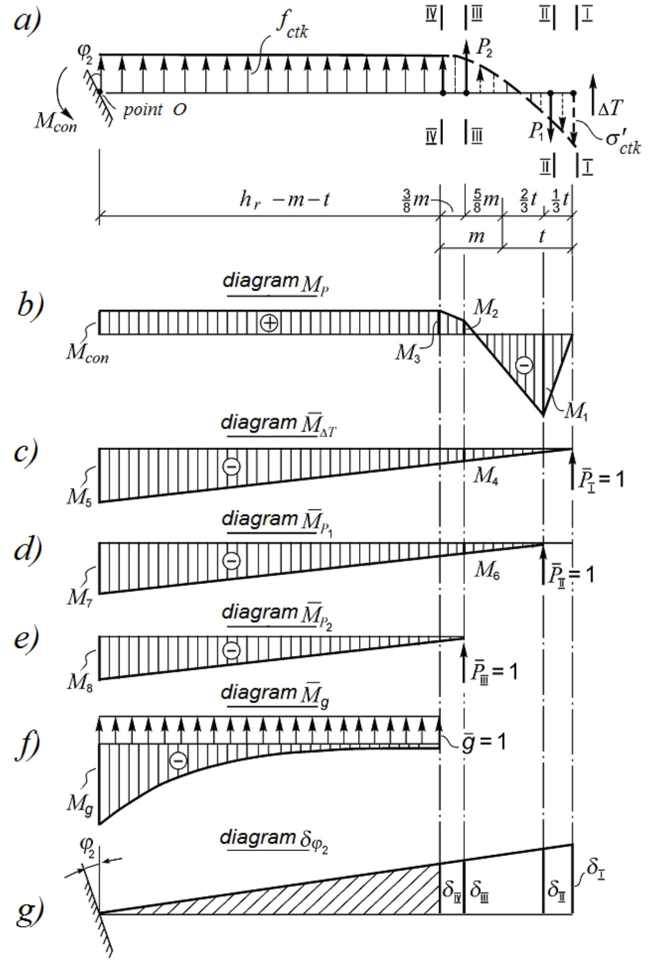


Fig. 6: The calculated cantilever scheme (*a*), the load diagram of the moments (b), the unit diagrams of the moments (*c*)–(*f*) and the displacement diagram from the rotation of the seal by the angle φ_2 (*g*)

Displacements in cross-sections I–I, II–II, III–III are determined by construction mechanics methods with the use of Figures 2, 5 and 6:

$$\Delta_I = \delta_I + \frac{1}{E_c I_{con}}\left[\frac{M_4 + M_5}{2}\left(h_r - t - \frac{5}{8}m\right)\left(-M_{con}\right)\right]; \quad (15)$$

$$\Delta_{II} = \delta_{II} + \frac{1}{E_c I_{con}}\left[\frac{M_6 + M_7}{2}\left(h_r - t - \frac{5}{8}m\right)\left(-M_{con}\right)\right]; \quad (16)$$

$$\Delta_{III} = \delta_{III} + \frac{1}{E_c I_{con}}\left[\frac{M_8}{2}\left(h_r - t - \frac{5}{8}m\right)\left(-M_{con}\right)\right], \quad (17)$$

where $M_4 = t + \frac{5}{8}m;\quad M_5 = h_r;\quad M_6 = \frac{2}{3}t + \frac{5}{8}m;$

$$M_7 = h_r - \frac{1}{3}t;\quad M_8 = h_r - t - \frac{5}{8}m.$$

In equations (16)–(18), the terms containing the areas of individual diagrams $\overline{M}_{\Delta T}$, $\overline{M}_{P1}$ and in sections III–I and III–II (due to their smallness) are excluded. The displacements associated with the rotation of the pinch by the angle φ_2 are determined from simple geometric relations:

$$\delta_I = \varphi_2 h_r; \quad (18)$$

$$\delta_{II} = \varphi_2\left(h_r - \frac{1}{3}t\right); \quad (19)$$

$$\delta_{III} = \varphi_2\left(h_r - t - \frac{5}{8}m\right). \quad (20)$$

The angle φ_2 is determined by the angle of rotation of the neutral axis of the reinforced concrete element φ_1 from the equation:

$$\varphi_2 = \varphi_1 + \Delta\varphi, \quad (21)$$

where $\Delta\varphi$ is the difference between the angles of the console rotation between points *O* and *B* (Figure 6, *a*).
The angle φ_1 is determined by the curvature of the reinforced concrete element. In this case, it is necessary to subtract from this part of the rotation angle of the neutral axis, which is caused by the deformation of the concrete element before the appearance of cracks. Taking the additional assumption that the appeared crack immediately extends over the length of h_r with respect to the two-cantilever element, we write:

$$\varphi_1 = \frac{t}{d}\left(\varepsilon_s \cdot \psi_s + \varepsilon_c\right) - \frac{M_{cr} \cdot t}{0{,}85 E_c I_{red}}, \quad (23)$$

where M_{cr} is the moment of first crack formation; ψ_s is coefficient of stretched concrete resistance; *d* is the working height of cross-section; ε_s are deformations in working reinforcement; ε_c are deformations in compressed zone of reinforced concrete cross-section; E_c is the modulus of elasticity of concrete; I_{red} is reduced inertia moment of reinforced concrete cross-section.
To determination the angle difference $\Delta\varphi$, we turn to Figure 6:

$$\Delta\varphi \approx -\frac{1}{E_c I_{red}} \cdot \frac{M_0 + M_{cr}}{2} \cdot m. \quad (24)$$

Thus, all the parameters included in formulas (15)–(17) and, consequently, the displacements Δ_I, Δ_{II}, Δ_{III} and the rotation angle φ_2 are defined.
The dependencies (10)–(13) are determined corresponding to the compliance of the reinforced concrete element. According to the calculation scheme (Figure 6), we find next equations:

$$P_1 = 0{,}5\sigma'_{ctk} \cdot b \cdot t; \quad (25)$$

$$P_2 = \frac{2}{3} f_{ctk} \cdot b \cdot m. \quad (26)$$

The flexibility corresponding to the distributed load can be expressed in the form (Figure 6):

$$C_q = \frac{2A_{\Delta q}}{q}, \quad \text{where} \quad (27)$$

$$A_{\Delta q} = A_{\delta\varphi} + \frac{1}{E_c I_{con}}\frac{1}{3}M_g\left(h_r - t - m\right)\left(-M_{con}\right), \qquad q = b \cdot f_{ctk},$$

$$M_g = \frac{1}{2}\left(h_r - t - m\right)^2.$$

In relation to the calculation scheme (Figure 5), as well as for short two-cantilever elements [14], on the basis of the theorem on the reciprocity of works and taking into account that $\delta_{21}=\delta_{12}$; $\delta_{31}=\delta_{13}$; $\delta_{31}=\delta_{23}$; $A_{p1}=A_{1p}=1\times\Delta_{1p}$; $A_{p2}=A_{2p}=1\times\Delta_{2p}$; $A_{p3}=A_{3p}=1\times\Delta_{3p}$; $A_{p4}=A_{4p}=1\times\Delta_{4p}$, the equations acquire the usual form of canonical equations of the forces method:

$$\begin{cases} X_{*,1}\delta_{11}+X_{*,2}\delta_{12}+X_{*,3}\delta_{13}+\Delta_{1p}+\Delta_3+h_r(\varphi_1+\Delta\varphi)=0 \\ X_{*,1}\delta_{21}+X_{*,2}\delta_{22}+X_{*,3}\delta_{23}+\Delta_{2p}-\Delta_6+(h_r-t_c)(\varphi_1+\Delta\varphi)=0 \\ X_{*,1}\delta_{31}+X_{*,2}\delta_{32}+X_{*,3}\delta_{33}+\Delta_{3p}+\varphi_2+\varphi_1=0 \end{cases} \quad (28)$$

From system (28) we obtain:

$$X_{*,1}=\Delta T=X_{*,2}(h_r-t_c)^2\frac{1}{h_r^2}-$$

$$-\frac{2}{h_r}\left(X_{*,2}\cdot A_{*,1}+A_{*,2}\right)-C_*\le 0,5\cdot 2\pi r t_c f_{ck}\,; \quad (29)$$

$$X_{*,2}=\frac{B_{*,2}}{B_{*,1}}\le 0.5\cdot f_{ck}\cdot b\cdot t_c\,; \quad (30)$$

$$X_{*,3}=X_{*,2}\cdot A_{*,1}+A_{*,2}\le M_{*,s}\,, \quad (31)$$

where

$$A_{*,1}=3\cdot(h_r-t_c)^2\cdot\left[2h_r+\left(\frac{1}{3}h_r+\frac{1}{6}t_c\right)\frac{2}{h_r^2}\right], \quad (32)$$

$$B_{*,1}=1-\left(h_r+\frac{1}{2}t_c\right)(h_r-t_c)\frac{1}{h_r^2}+$$

$$+A_{*,1}\frac{2}{h_r}\left(h_r+\frac{1}{2}t_c\right)\frac{1}{(h_r-t_c)}+A_{*,1}\frac{3}{2}\frac{1}{(h_r-t_c)}, \quad (33)$$

$$C_*=\frac{\chi\cdot f_{ctk}\cdot b}{6}(h_r-t_c)^3\frac{1}{h_r^2}-$$

$$-\chi_c P_{ctk,c}\cdot(h_r-\frac{1}{3}t_c)^2\frac{1}{h_r{}^2}+(\varphi_2+\varphi_1)E_c(\lambda)I_{con}\frac{2}{h_r^2}\,, \quad (34)$$

$$A_*=\frac{\chi f_{ctk}b}{12}(h_r-t_b)^3(3h_r+t_c)\frac{1}{h_r^2}-(h_r-\frac{1}{3}t_c)^2\cdot(\frac{1}{3}\chi_c P_{bt,c})\times$$

$$\times(h_r+\frac{1}{6}t_c)\cdot\frac{2}{h_r^2}++\left(\Delta_3+h_r(\varphi_1+\Delta\varphi)\right)\cdot E_c(\lambda)I_{con}\frac{2}{h_r^2}\,; \quad (35)$$

$$A_{*,2}=3\cdot A_*-2\cdot C_*\cdot h_r\,; \quad (36)$$

$$B_{*,2}=-A_{*,2}\frac{3}{2}\frac{1}{(h_r-t_c)}-B_*-A_{*,2}\frac{2}{h_r}\left(h_r+\frac{1}{2}t_c\right)\frac{1}{(h_r-t_c)}-$$

$$-C_*\left(h_r-\frac{1}{2}t_c\right)\frac{1}{(h_r-t_c)}\,. \quad (37)$$

The parameters required for this are determined from the following dependencies:

$$\varphi_1=\frac{0.5f}{h_1}\,,\ h_1=h_r-t_1\,,\ f=\Delta_6\,,\ \varphi_2=\frac{0.5f}{t_1}\,,\ t_1=t_c=2\cdot\varnothing\,.$$

In this case, the condition $\Delta\varphi<\varphi_1$, $\Delta\varphi<\varphi_2$, else – $\varphi_1=\Delta\varphi$ & $\varphi_2=\Delta\varphi$.

$$B_*=-\frac{3}{8}\chi f_{ctk}b\cdot(h_r-t_c)+\frac{\chi_c P_{ctk,c}h_r}{(h_r-t_c)}+$$

$$+\frac{3}{(h_r-t_c)^3}\left(\Delta_6+(h_r-t_c)(\varphi_1+\Delta\varphi)\right)E_c(\lambda)I_{con}\,, \quad (38)$$

where $\chi_c=\left(2\pi(r_1+r_2)\right)/b$ in the first step of the iterative process (r_1 & r_2 are radius of working reinforcment), $P_{ctk,c}=0.5\cdot f_{ck}\cdot b\cdot t_c$.

Similar equations can be written for a universal two-cantilever element, Figure 3. Already, the use of the developed two-cantilever elements, in relation to reinforced concrete, can bring its positive results in assessing the resistance of stretched concrete, the distance between cracks and the width of their opening and rigidity of reinforced concrete constructions of buildings and structures.

4. Conclusions

1. It has developed hypotheses of fracture mechanics in order to take into account of the effect of continuity violation of reinforced concrete structures with different power influences with the simplification of its energy functional. Received further development of the fracture mechanics functional in relation to the calculation of reinforced concrete structures. The peculiarity of construction of a two-cantilever element in zones adjacent to cracks is considered.
2. A new analytical dependence has been obtained that relates the tangential effort that arises in the immediate proximity of the crack (ΔT) with the length of its development h_r due to the specific energy of the formation of new surfaces of the crack ζ_{cu}. This dependence allows us to find tangential stresses in a zone that is directly adjacent to a crack. It is here, as experimental and numerical studies show, that there is a sharp perturbation of tangential stresses, which is accompanied by a jump-like increase and a sign change. This changes the sign and normal stresses in the concrete (from the stresses of tension they turn into compression stresses), which is confirmed by numerous experiments.
3. There is a concentration of deformations in areas of concrete adjacent to cracks that overcomes the needs of the system (consisting of concrete blocks and reinforcement with a given static pattern) in deformations. After the formation of cracks, the solidity of the concrete is broken and its deformation no longer complies with the laws of the solid body – in the cracks there is an additional deformation effect, which must be taken into account in the calculation of the RCC.
4. The connection of the components of the stress-strain state in the zone of perturbation with the specific energy of the new surfaces crack formation of ζ_{cu}, which is released in the zone of predestruction. As a result, a new solution was obtained for the stress-strain state of the reinforced concrete element in the area adjacent to the crack.
5. The series of two-cantilever elements of fracture mechanics was developed: in the presence of various cracks for reinforced concrete structures for different types of resistance (bending, central tensile (compression), non-eccentric compression); including a new universal two-cantilever element in the zone of spatial cracks for use in the complex resistance of reinforced concrete structures. The constructed elements are a connecting link and act as transformational elements between the dependences of fracture mechanics and the equations of the reinforced concrete theory.
The solution of the problem posed in the work allows, even remaining within the framework of the traditional model of reinforced concrete resistance, to significantly clarify its differential parameters, measured in experiments using a microscope, and to explain a lot of observed phenomena observed in experiments that are observed and occur when supporting the reinforced concrete with strength and deformation influences.

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