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Research paper

Geometric and Computer Modeling of Building Structures Forms

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Abstract

The current state of designing curvilinear objects of architecture and construction needs to take into account as many data and requirements as possible to ensure an appropriate model accuracy. In geometric modeling initial data, as a rule, are geometric characteristics and conditions, which are represented in numerical form (coordinates or values of parameters) with quite big arrays. In these conditions, methods of global continuous modeling with a single solution become ineffective. Because of this they require a usage of rather complicated mathematical algorithms and can not provide a necessary adequacy of models. Methods of discrete geometric modeling are free from these drawbacks.
The purpose of this article is expanding possibilities of the classical finite difference method and the static-geometric method by applying a geometric apparatus of superposition. In discrete modeling of geometric images this allows using hyperbolic functions as interpolators.
The result of this study is a computational template for continuous two-dimensional discrete interpolation. This allows to model geometric images of architectural and building constructions in the form of discrete frames of chain lines.

Keywords: *discrete modeling; geometric images; finite difference method; static-geometric method; geometric apparatus of superpositions; hyperbolic functions; chain line.*

1. Introduction

This Production development and improvement of technological processes create new scientific problems to construct adequate models of objects and phenomena for their effective analysis, calculation, optimization and forecasting. In the process of designing modern objects of construction, architecture, engineering, an important place belongs to a stage of geometric modeling. At this stage basic parameters of their geometric shape can be determined. At the same time, the quality of models depends on the ability to effectively manage their geometry, adjusting both models as a whole, and their individual parts, rapid analysis and comparative evaluation of the results.
Discrete geometric modeling [1,2,3] is the most promising direction of an applied geometry development in the modern period. It can be conventionally divided into studies on a discretization of continuous geometric images and a formation, using discrete source data.
Among the most common directions in the discrete surface modeling there is a finite element method. This method is based on a discrete representation of surfaces in the form of a set of individual elements that interact with each other in a finite number of nodal points.
The finite difference method favorably differs from the finite element method by simplicity, but loses in the universality and accuracy of results, which can be obtained, solving engineering problems.
On the basis of a static interpretation of the finite difference method, prof. S. Kovalev [4] created a static-geometric method of formatting discrete geometric images with certain properties. This method is the most obvious and understandable method of discrete modeling of continuous images, and usually takes into account static features of various objects.
Prof. Naydysh V. [5] has developed theoretical principles of the discrete geometric modeling, which is based on algorithms of condensation, using geometric relations, equalities, basic interpolation functions of an initial point array, to form a new set of discrete elements with given properties.
The last two methods actively develope, have practical application, and their effectiveness is confirmed by results.
Article [6] of prof. S. Pustyul′ha is devoted to a further development of the static-geometric method and an expansion of its form-forming properties. In this work it was proposed a usage of mathematical apparatus and geometric interpretation of numerical sequences together with the classical method of finite differences and static-geometric method to formate geometric images. It allows to have done a simple and effective transition to continuous analogues of formed discrete models and vice versa; to have solved a number of problems of discrete geometric modeling of balanced images of arbitrary number of measurements without solving cumbersome systems of linear equations. All this together allows to provide saving of computational resources.
Each of these methods has its advantages and disadvantages in solving specific practical problems. Therefore, their research, an enrichment by new effective algorithms, a studying of a possibility of their compilation, and on this basis, an expansion of the set of

output data, are relevant. Further development and improvement of the above-mentioned methods as a whole are also relevant. At the same time, from one side it is possible to enrich well-known methods of discrete geometric modeling by new algorithms and improve their modeling capabilities. From other side it is possible to expand a circle of practical tasks and optimize models, which were created for their realization.

In [7], prof. S. Kovalev defined a concept of "superposition" in applied geometry on the basis of a functional addition as a superposition of sets, between points of which a certain conformity is established.

The method of constructing a discrete mesh on the basis of superpositions of pre-calculated two or more meshes with the same topology allows to have determined coordinates of a free node of a new mesh, using coordinates of corresponding nodes of known meshes. Properties of such superpositions are insufficiently studied in terms of their invariants with respect to parameters of initial meshes.

This article proposes an application of the geometric apparatus of superposition in combination with the above-mentioned methods [8, 9, 10, 11, 12, 13, 14, 15, 16]. This allows significantly improving efficiency and expanding possibilities of a process of discrete modeling of geometric images. In particular, it allows investigating a possibility of usage of different elementary functional dependencies, not only parabolic functions, as interpolants.

2. Main body

An easy way to comply with the paper formatting requirements is to use this document as a template and simply type your text into it. In paper [8] it is noted that in nature and technology there is a lot of chain lines. A chain line is described by a hyperbolic cosine [17]:

$$y = a \cdot ch\frac{x}{a} \tag{1}$$

In architecture and construction, arches in the form of an inverted chain line have high stability due to the fact that internal compression forces are perfectly compensated and do not cause a deflection. Some properties of a discrete line model can be transferred to a surface model generated by the same laws, if this line is considered as a component of the surface of a frame. Other properties of a discrete surface model can be obtained as a result of generalization of corresponding properties of a line model [8].

Therefore, for a discrete determination of geometric images of building structures, a usage of hyperbolic functions of type (1) as interpolators is more appropriate.

Consider numerical sequence of two variables (2), which is depicted in Fig. 1.

$$z_{i,j} = n \cdot ch\frac{x_i}{n} + m \cdot ch\frac{y_j}{m} \tag{2}$$

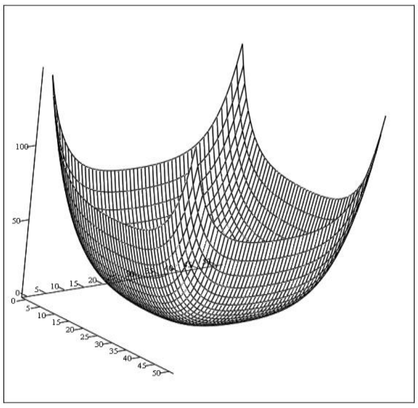

Fig. 1: Two-dimensional numeric sequence

$$z_{i,j} = n \cdot ch\frac{x_i}{n} + m \cdot ch\frac{y_j}{m} \quad \text{(n=1, m=1)}$$

Constituents of sequence (2) are two sequences of one variable (3) and (4):

$$z_i = n \cdot ch\frac{x_i}{n}, \tag{3}$$

$$z_j = m \cdot ch\frac{y_j}{m}. \tag{4}$$

On the basis of the formulas, which were defined in [4], they can be represented by recurrence dependences of superpositions in the form

$$z_i = k_1 z_{i-1} + k_2 z_{i+1} + k_3 z_{i+2} \tag{5}$$

$$z_j = k_4 z_{j-1} + k_5 z_{j+1} + k_6 z_{j+2}, \tag{6}$$

where, taking into account the results of research work [8]:

$k_1 = k_4 = 0{,}244728471054$; $k_2 = k_5 = 1$;

$k_3 = k_6 = (1 - k_1 - k_2) = (1 - k_4 - k_5) = -0{,}244728471054$

By adding (5) to (6) we obtain the recursive formula of sequence (2) in the form:

$2z_{i,j} = k_1 z_{i-1,j} + k_2 z_{i+1,j} + k_3 z_{i+2,j} + k_4 z_{i,j-1} + k_5 z_{i,j+1} + k_6 z_{i,j+2}$

or an identical formula:

$z_{i,j} = k_1 z_{i-1,j} + k_2 z_{i+1,j} + k_3 z_{i+2,j} + k_1 z_{i,j-1} + k_2 z_{i,j+1} + k_3 z_{i,j+2}$

where $k_1 = 0{,}122364235527$; $k_2 = 0{,}5$; $k_3 = -0{,}122364235527$,

or in the form of a computational pattern (Fig. 2) for continuous discrete interpolation by two-dimensional numerical sequences, whose components are hyperbolic functions of form (1).

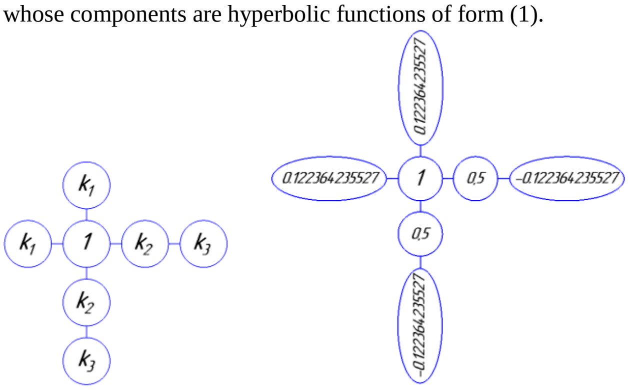

Fig. 2: Computational template for discrete interpolation of a numerical sequence of two variables

Example 1. Construct a discrete model of the curve surface on the mesh, reference contour and applicate of the central node (Fig. 3),

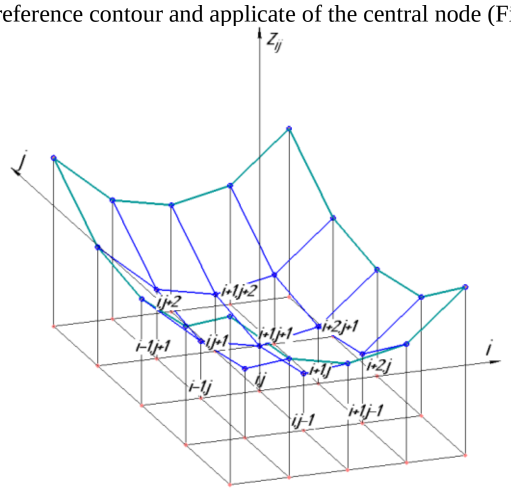

Fig. 3: Discrete surface frame

$$z_{i,j} = n \cdot ch\frac{x_i}{n} + m \cdot ch\frac{y_j}{m}$$

given in the mesh plan, according to the following initial data, belonging to surface (2):

$z_{i+2,j} = z_{i,j+2} = z_{20} = z_{02} = 4{,}762195691084$;

$z_{i+2,j+1} = z_{i+1,j+2} = z_{21} = z_{12} = 5{,}305276325899$;

$z_{i,j} = z_{00} = 2$,

Since the reference contour is symmetric with respect to the planes xOz, yOz and the diagonal plane, then

$z_{21} = z_{12} = z_{-12} = z_{-21} = z_{-2-1} = z_{-1-2} = z_{1-2} = z_{2-1}$;

$z_{10} = z_{01} = z_{-10} = z_{0-1}$;

$z_{11} = z_{-11} = z_{-1-1} = z_{1-1}$;

$z_{20} = z_{02} = z_{-20} = z_{0-2}$,

and therefore we formulate a system of equations for 1/8 part, determining applicates of only two nodes: $z_{i+1,i} = z_{10}$ and $z_{i+1,i+1} = z_{11}$.

$$\begin{cases} k_1 z_{i-1,j} + k_2 z_{i,j} + k_3 z_{i+2,j} + k_1 z_{i+1,j+2} + k_2 z_{i+1,j+1} + k_3 z_{i+1,j-1} = \\ \qquad = z_{i+1,j} \\ k_1 z_{i-1,j+1} + k_2 z_{i,j+1} + k_3 z_{i+2,j+1} + k_1 z_{i+1,j-1} + k_2 z_{i+1,j} + k_3 z_{i+1,j+2} = \\ \qquad = z_{i+1,j+1} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} k_1 z_{-10} + k_2 z_{00} + k_3 z_{20} + k_1 z_{12} + k_2 z_{11} + k_3 z_{1-1} = z_{10} \\ k_1 z_{-11} + k_2 z_{01} + k_3 z_{21} + k_1 z_{1-1} + k_2 z_{10} + k_3 z_{12} = z_{11} \end{cases} \tag{8}$$

Replacing in system of equations (8) the applicates of symmetric nodes and substituting values of the applicates of given nodes, we obtain two equations with two unknowns:

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 2 + 0{,}122364235527 \cdot 4{,}762195691084 - \\ -0{,}122364235527 \cdot 5{,}305276325899 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ \qquad = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}305276325899 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}305276325899 = \\ \qquad = z_{11} \end{cases}$$

Solutions of the system are given in Table 1.

Table 1: Values of the applicates of the points of a compartment of the discrete frame of the surface $z_{i,j} = n \cdot ch\frac{x_i}{n} + m \cdot ch\frac{y_j}{m}$

j	*i*				
	-2	-1	0	1	2
-2	*7,524391382168*	*5,305276325899*	*4,762195691084*	*5,305276325899*	*7,524391382168*
-1	*5,305276325899*	3,086161269636	2,543080634815	3,086161269636	*5,305276325899*
0	*4,762195691084*	2,543080634815	2	2,543080634815	*4,762195691084*
1	*5,305276325899*	3,086161269636	2,543080634815	3,086161269636	*5,305276325899*
2	*7,524391382168*	*5,305276325899*	*4,762195691084*	*5,305276325899*	*7,524391382168*

The obtained results completely coincide with the values of the applicates, calculated by formula (2).

Example 2. Construct discrete models of the curve surfaces on the mesh, reference contour and applicate of the central node (Fig. 4), according to the following initial data:

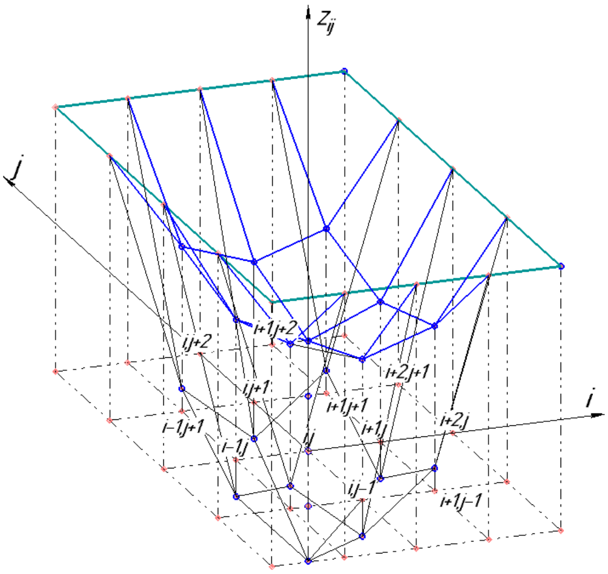

Fig. 4: Formation of discrete surface models by superpositions of point sets on the basis of two-dimensional hyperbolic interpolation.

$z_{i+2,j} = z_{i+2,j+1} = z_{i,j+2} = z_{i+1,j+2} = z_{20} = 4{,}8$;

$z_{i,j} = z_{00} = 2$; $z_{i,j} = z_{00} = 1$;

$z_{i,j} = z_{00} = -1$; $z_{i,j} = z_{00} = -2$.

Since the reference contour is symmetric with respect to the planes xOz, yOz and the diagonal plane, then

$z_{21} = z_{12} = z_{-12} = z_{-21} = z_{-2-1} = z_{-1-2} = z_{1-2} = z_{2-1}$;

$z_{10} = z_{01} = z_{-10} = z_{0-1}$; $z_{11} = z_{-11} = z_{-1-1} = z_{1-1}$;

$z_{20} = z_{02} = z_{-20} = z_{0-2}$,

and therefore we formulate a system of equations for 1/8 part, determining applicates of only two nodes:

$z_{i+1,j} = z_{10}$, $z_{i+1,j+1} = z_{11}$.

When substituting to this system the aforementioned initial conditions, the solving gives the following results:

Surface 1.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 2 + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot 4{,}8 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 = z_{11} \end{cases}$$

1. $z_{10} = 2.550513413074704$ $z_{11} = 2.992789319729712$.

Surface 2.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 1 + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot 4{,}8 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 = z_{11} \end{cases}$$

2. $z_{10} = 1.747125346315672$; $z_{11} = 2.347356933918896$.

Surface 3.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot -1 + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot 4{,}8 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 = z_{11} \end{cases}$$

3. $z_{10} = 0.14034921279761$; $z_{11} = 1.056492162297266$.

Surface 4.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot -2 + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot 4{,}8 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 4{,}8 = z_{11} \end{cases}$$

4. $z_{10} = -0.66303885396142$; $z_{11} = 0.41105977648645$.

Fig. (4) shows the discrete frames of surface 1 and surface 4, which were formed on one given reference contour with the applicates of the central node, respectively:

$z_{i,j} = z_{00} = 2$, $z_{i,j} = z_{00} = -2$.

Consider numerical sequence of two variables (9), which is depicted in Fig. 5.

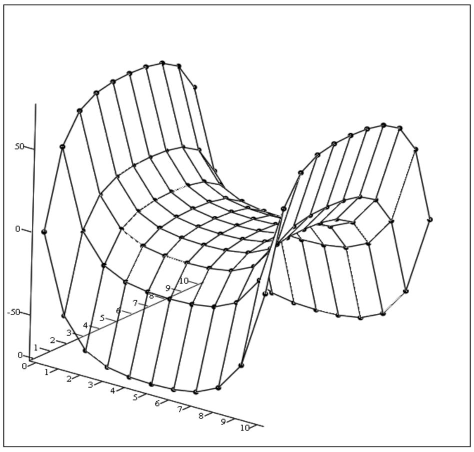

Fig. 5: Discrete frame of two-dimensional numerical sequence $z_{i,j} = n \cdot ch\frac{x_i}{n} - m \cdot ch\frac{y_j}{m}$ *(n=1, m=1)*

$$z_{i,j} = n \cdot ch\frac{x_i}{n} - m \cdot ch\frac{y_j}{m}. \tag{9}$$

Constituents of sequence (9) are two sequences of one variable (3) and (4), which can be represented by recurrence dependencies on the basis of superpositions in the form (5, 6), according to the formulas from [8].

For a continuous discrete interpolation with two-dimensional numerical sequence (9) we shall use the computational template, which is presented in Figure 2.

Example 3. Construct a discrete model of the curve surface on the mesh, reference contour and applicate of the central node (Fig. 5), given in the mesh plan according to the following initial data, belonging to the surface (9):

$z_{i+2,j} = z_{i,j+2} = z_{20} = -z_{02} = \pm 2{,}762195691084$;

$z_{i+2,j+1} = z_{i+1,j+2} = z_{21} = -z_{12} = \pm 2{,}219115056269$;

$z_{i,j} = z_{00} = 0$.

Since the reference contour is symmetric with respect to the planes xOz, yOz, then:

$z_{21} = -z_{12} = -z_{-12} = z_{-21} = z_{-2-1} =$
$= -z_{-1-2} = -z_{1-2} = z_{2-1}$;

$z_{10} = -z_{01} = z_{-10} = -z_{0-1}$;

$z_{11} = z_{-11} = z_{-1-1} = z_{1-1}$;

$z_{20} = -z_{02} = z_{-20} = -z_{0-2}$.

Replacing in system of equations (8) the applicates of symmetric nodes and substituting values of the applicates of given nodes, we obtain two equations with two variables:

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 0 + 0{,}122364235527 \cdot 2{,}762195691084 - \\ -0{,}122364235527 \cdot (-2{,}219115056269) + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot (-z_{10}) + 0{,}122364235527 \cdot 2{,}219115056269 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot (-2{,}219115056269) = \\ = z_{11} \end{cases}$$

$z_{10} = 0{,}54308063481377$; $z_{11} = 0$.

Solutions of the system are presented in Table 2.

Table 2: Values of the applicates of the points of a compartment of the discrete frame of the surface $z_{i,j} = n \cdot ch\frac{x_i}{n} - m \cdot ch\frac{y_j}{m}$

j	*i*				
	-2	-1	0	1	2
-2	*0*	*-2,219115056269*	*-2,762195691084*	*-2,219115056269*	*0*
-1	*2,219115056269*	0	-0,543080634815	0	*2,219115056269*
0	*2,762195691084*	0,543080634815	0	0,543080634815	*2,762195691084*
1	*2,219115056269*	0	-0,543080634815	0	*2,219115056269*
2	*0*	*-2,219115056269*	*-2,762195691084*	*-2,219115056269*	*0*

The obtained results completely coincide with the values of the applicates, calculated by formula (9).

Results of calculations of the applicates of unknown knots of discrete models of curve surfaces on the mesh, which is defined in the plan, the reference contour, which belongs to the surface (9) and the applicates of the central node $z_{i,j} = z_{00} = -2$; $z_{i,j} = z_{00} = -1$, $z_{i,j} = z_{00} = 1$; $z_{i,j} = z_{00} = 2$ are presented below.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot -2 + 0{,}122364235527 \cdot 2{,}762195691084 + \\ +0{,}122364235527 \cdot 2{,}219115056269 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot (-z_{10}) + 0{,}122364235527 \cdot 2{,}219115056269 + \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot (-2{,}219115056269) = \\ = z_{11} \end{cases}$$

$z_{10} = -0{,}3478957241491$; $z_{11} = 0$.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot -1 + 0{,}122364235527 \cdot 2{,}762195691084 + \\ +0{,}122364235527 \cdot 2{,}219115056269 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot (-z_{10}) + 0{,}122364235527 \cdot 2{,}219115056269 + \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot (-2{,}219115056269) = \\ = z_{11} \end{cases}$$

$z_{10} = 0{,}097592455332335$; $z_{11} = 0$.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 1 + 0{,}122364235527 \cdot 2{,}762195691084 + \\ +0{,}122364235527 \cdot 2{,}219115056269 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot (-z_{10}) + 0{,}122364235527 \cdot 2{,}219115056269 + \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot (-2{,}219115056269) = \\ = z_{11} \end{cases}$$

$z_{10} = 0{,}98856881429521$; $z_{11} = 0$.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 2 + 0{,}122364235527 \cdot 2{,}762195691084 + \\ +0{,}122364235527 \cdot 2{,}219115056269 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot (-z_{10}) + 0{,}122364235527 \cdot 2{,}219115056269 + \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot (-2{,}219115056269) = \\ = z_{11} \end{cases}$$

$z_{10} = 1{,}434056993776648$; $z_{11} = 0$.

Let's also consider numerical sequence of two variables (10), which is depicted in Figure 6.

$$z_{i,j} = n \cdot ch\frac{x_i}{n} \cdot m \cdot ch\frac{y_j}{m}. \tag{10}$$

Constituents of sequence (10) are two sequences of one variable (3) and (4), which can be represented by recurrence dependencies on the basis of superpositions in the form (5, 6), according to the formulas from [8].

For a continuous discrete interpolation with two-dimensional numerical sequence (10) we shall use the computational template, which is presented in Figure 2.

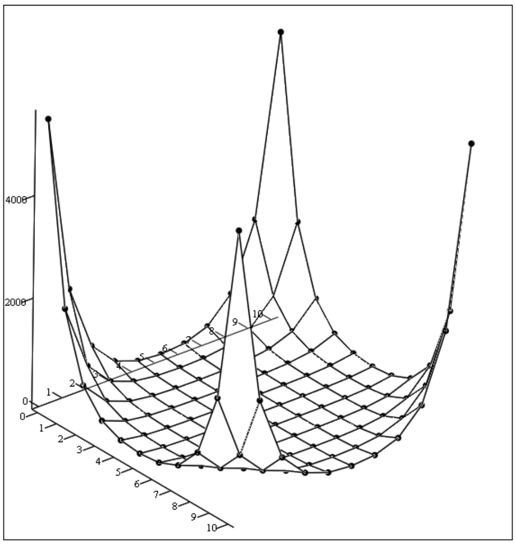

Fig. 6: Discrete frame of two-dimensional numerical sequence $z_{i,j} = n \cdot ch\frac{x_i}{n} \cdot m \cdot ch\frac{y_j}{m}$ *(n=1, m=1)*

Example 4. We construct a discrete model of the curve surface on the mesh, which is given in the mesh plan, the reference contour, and the applicate of the central node (Fig. 6), according to the following initial data, which belong to the surface (10):

$z_{i+2,j} = z_{i,j+2} = z_{20} = z_{02} = 3{,}762195691084$;

$z_{i+2,j+1} = z_{i+1,j+2} = z_{21} = z_{12} = 5{,}805371315296$;

$z_{i,j} = z_{00} = 1$.

Since the reference contour is symmetric with respect to the planes xOz, yOz , then:

$z_{21} = z_{12} = z_{-12} = z_{-21} = z_{-2-1} =$
$= z_{-1-2} = z_{1-2} = z_{2-1}$;
$z_{10} = z_{01} = z_{-10} = z_{0-1}$;
$z_{11} = z_{-11} = z_{-1-1} = z_{1-1}$;
$z_{20} = z_{02} = z_{-20} = z_{0-2}$,

and therefore we formulate a system of equations for 1/8 part, determining applicates of only two nodes:

$z_{i+1,j} = z_{10}$, $z_{i+1,j+1} = z_{11}$.

Replacing in system of equations (8) the applicates of symmetric nodes and substituting values of the applicates of given nodes, we obtain two equations with two unknowns:

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 1 + 0{,}122364235527 \cdot 3{,}762195691084 - \\ -0{,}122364235527 \cdot 5{,}805371315296 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 = \\ = z_{11} \end{cases}.$$

$z_{10} = 1{,}543080634813891$; $z_{11} = 2{,}381097845538435$.

Solutions of the system are presented in Table 3.

Table 3: Values of the applicates of the points of a compartment of the discrete frame of the surface $z_{i,j} = n \cdot ch\frac{x_i}{n} \cdot m \cdot ch\frac{y_j}{m}$

j	i				
	-2	-1	0	1	2
-2	*14,154116418011*	*5,805371315296*	*3,762195691084*	*5,805371315296*	*14,154116418011*
-1	*5,805371315296*	2,381097845541	1,543080634815	2,381097845541	*5,805371315296*
0	*3,762195691084*	1,543080634815	1	1,543080634815	*3,762195691084*
1	*5,805371315296*	2,381097845541	1,543080634815	2,381097845541	*5,805371315296*
2	*14,154116418011*	*5,805371315296*	*3,762195691084*	*5,805371315296*	*14,154116418011*

The obtained results completely coincide with the values of the applicates, calculated by formula (10).

Results of calculations of the applicates of unknown knots of discrete models of curve surfaces on the mesh, which is defined in the plan, the reference contour, which belongs to the surface (10) and the applicates of the central node $z_{i,j} = z_{00} = -2$; $z_{i,j} = z_{00} = -1$; $z_{i,j} = z_{00} = 0$; $z_{i,j} = z_{00} = 2$ are presented below.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot -2 + 0{,}122364235527 \cdot 3{,}762195691084 - \\ -0{,}122364235527 \cdot 5{,}805371315296 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 = \\ = z_{11} \end{cases}$$

$z_{10} = -0{,}8670835654632$; $z_{11} = 0{,}44480068810599$.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot -1 + 0{,}122364235527 \cdot 3{,}762195691084 - \\ -0{,}122364235527 \cdot 5{,}805371315296 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 = \\ = z_{11} \end{cases}$$

$z_{10} = -0{,}063695498704171$; $z_{11} = 1{,}090233073916805$.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 0 + 0{,}122364235527 \cdot 3{,}762195691084 - \\ -0{,}122364235527 \cdot 5{,}805371315296 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 = \\ = z_{11} \end{cases}$$

$z_{10} = 0{,}73969256805486$; $z_{11} = 1{,}73566545972762$.

$$\begin{cases} -0{,}122364235527 \cdot z_{10} + 0{,}5 \cdot 2 + 0{,}122364235527 \cdot 3{,}762195691084 - \\ -0{,}122364235527 \cdot 5{,}805371315296 + 0{,}5 \cdot z_{11} + 0{,}122364235527 \cdot z_{11} = \\ = z_{10} \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 - \\ -0{,}122364235527 \cdot z_{11} + 0{,}5 \cdot z_{10} + 0{,}122364235527 \cdot 5{,}805371315296 = \\ = z_{11} \end{cases}$$

$z_{10} = 2{,}346468701572922$; $z_{11} = 3{,}026530231349251$.

3. Conclusions

On the basis of the geometric apparatus of superpositions, a computational pattern for continuous discrete interpolation by two-dimensional numerical sequences, components of which are hyperbolic functions, are obtained. This method extends possibilities of discrete geometric modeling.

The developed method allows formatting two-dimensional geometric images in the form of discrete frames of hyperbolic curves through given nodal points, which is impossible to do in most cases, using usual interpolation methods.

The results of this work can be a basis for further research of the discrete formation of geometric images by two-dimensional numerical sequences of not only parabolic, hyperbolic, but also other functional dependencies.

References

[1] Samostyan V.R. Vplyv heometrychnykh vymoh na protsesy dyskretnoho modelyuvannya kryvoliniynykh ob″yektiv budivnytstva: dys. …kand. tekhn. nauk: 05.01.01 / V.R. Samostyan. – K., 2011. – 182 s.

[2] Guoliang Xu, Oing Pan, Chandrajit L. Bajaj. Discrete surface modeling using partial differential equations. Computer Aided Geometric Design. Volume 23, Issue 2, February 2006, pp. 125-145, https://doi.org/10.1016/j.cagd.2005.05.004

[3] Lienhardt P. (1997) Aspects in topology-based geometric modeling Possible tools for discrete geometry?. In: Ahronovitz E., Fiorio C. (eds) Discrete Geometry for Computer Imagery. DGCI 1997. Lecture Notes in Computer Science, vol 1347. Springer, Berlin, Heidelberg pp 33-48. https://doi.org/10.1007/BFb0024828

[4] Kovalev S.N. Formirovaniye diskretnykh modeley poverkhnostey prostranstvennykh arkhitekturnykh konstruktsiy: dis. … doktora tekhn. nauk: 05.01.01 / S.N. Kovalev – M.: MAI, 1986. – 348 s.

[5] Naydysh, V.M. Teoretycheskye osnovy dyskretnoho heometrycheskoho modelyrovanyya. / V.M. Naydysh // Prykladna heometri-

ya ta inzhenerna hrafika. – K.: KNUBA, 1995. – Vyp. 58. – S. 26 – 29.

[6] Pustyulʹha, S.I. Dyskretne vyznachennya heometrychnykh ob″yektiv chyslovymy poslidovnostyamy: dys. … doktora tekhn. nauk: 05.01.01 / S.I. Pustyulʹha. – K., 2006. – 322 s.

[7] Kovalev S.N. O superpozytsyyakh / S.N. Kovalev // Prykladna heometriya ta inzhenerna hrafika: zb. nauk. pratsʹ. – K.: KNUBA, 2010. − Vyp. 84. – S. 38 – 42. . ISSN 0131-579X

[8] Vorontsov O. Discrete modeling of building structures geometric images. / O. Vorontsov, L. Tulupova, O. Vorontsova // International Journal of Engineering & Technology. Vol. 7 No. 3.2 (2018). P. 727 – 731. ISSN: 2227-524X

[9] Vorontsov O. Recurrence formulae of a catenary in creation of geometric images. / O. Vorontsov., L. Tulupova // Oxford Journal of Scientific research No. 1. (9), January-June, 2015, Volume IV. P. 134 – 140. . ISSN 0305-4882.

[10] Vorontsov O.V. Vyznachennya dyskretnoho analohu polinoma n-ho stepenya superpozytsiyamy tochok chyslovoyi poslidovnosti n-ho poryadku / O.V. Vorontsov // Prykladna heometriya ta inzhenerna hrafika: zb. nauk. pratsʹ – K.: KNUBA, 2012. – Vyp. 90. – S. 63 – 67. . ISSN 0131-579X

[11] Vorontsov O.V. Dyskretna interpolyatsiya superpozytsiyamy tochok chyslovykh poslidovnostey drobovo-liniynykh funktsiy / O.V. Vorontsov, N.O. Makhinʹko // Prykladna heometriya ta inzhenerna hrafika: pratsi TDATA. – Melitopolʹ: TDATA, 2013. Vyp. 4. – T. 57. – S. 62 – 67.

[12] Vorontsov O.V. Vlastyvosti superpozytsiy tochkovykh mnozhyn / O.V. Vorontsov // Prykladna heometriya ta inzhenerna hrafika: zb. nauk. pratsʹ – K.: KNUBA, 2010. – Vyp. 86. − S. 345 − 349. . ISSN 0131-579X.

[13] Vorontsov O.V. Opredeleniye diskretnykh analogov klassov elementarnykh funktsiy superpozitsiyami odnomernykh tochechnykh mnozhestv [Elektronnyy resurs] / O.V. Vorontsov, L.A. Tulupova // Universsum. Ser.: Tekhnicheskiye nauki: elektron. nauchn. zhurn. − 2014. − № 3(4). − ISSN 2311-5122.

[14] Vorontsov O.V. Dyskretna interpolyatsiya superpozytsiyamy odnovymirnykh tochkovykh mnozhyn transtsendentnykh funktsionalʹnykh zalezhnostey na prykladi hiperbolichnykh funktsiy. / O.V. Vorontsov // Visnyk Khersonsʹkoho natsionalʹnoho tekhnichnoho universytethu / Vyp. 3(54). – Kherson: KHNTU, 2015. – S. 551-554. ISSN 2078-4481

[15] Vorontsov O.V. Dyskretna interpolyatsiya heometrychnykh obraziv superpozytsiyamy dvovymirnykh tochkovykh mnozhyn funktsionalʹnykh zalezhnostey / O.V. Vorontsov, L.O. Tulupova, I.V. Vorontsova // Visnyk Khersonsʹkoho natsionalʹnoho tekhnichnoho universytethu / Vyp. 3(62). T.2. – Kherson: KHNTU, 2017. – S. 66-70. ISSN 2078-4481

[16] Vorontsov O.V. Diskretnoye modelirovaniye krivykh poverkhnostey superpozitsiyami dvumernykh tochechnykh mnozhestv / O.V. Vorontsov, L.A. Tulupova // Sbornik statey po materialam XL mezhdunarodnoy nauchno-prakticheskoy konferentsii «Tekhnicheskiye nauki – ot teorii k praktike». – Novosibirsk, 2014. – №11 (36). – S. 7 – 16. ISSN 2308-5991.

[17] Savelov A.A. Ploskiye krivyye. Sistematika, svoystva, primeneniya. (Spravochnoye rukovodstvo). Pod redaktsiyey A.P. Nordena. Gosudarstvennoye izdatel'stvo fiziko-matematicheskoy literatury. Moskva 1960 g. – 293 s.

[18] Gutak, A. D. (2015). Experimental investigation and industrial application of ranque-hilsch vortex tube. International Journal of Refrigeration, 49, 93-98. https://doi.org/10.1016/j.ijrefrig.2014.09.021

[19] Cherniha, R., & Serov, M. (2006). Symmetries, ansätze and exact solutions of nonlinear second-order evolution equations with convection terms, II. European Journal of Applied Mathematics, 17(5), 597-605. https://doi.org/10.1017/S0956792506006681