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***Research paper***

# Characterizations of $(\gamma\beta)^{\alpha}(I,K)$-Continuous Mappings in Ideal Topological Spaces

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**Abstract**

In this paper the notion of $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings is introduced and some of their properties are studied. Further the concept of contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings and($\gamma^{\alpha}$, β)-I-Continuous Mappings are introduced and properties are analyzed.

***Keywords:*** *$(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings; contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings; $(\gamma^{\alpha},\beta)$ - $I$ -Continuous Mappings (2010)AMS Mathematics Subject Classification: 54C05, 54C08.*

## 1. Introduction

The $\alpha$ -open sets, operation on topological spaces, $\tau_{\alpha-\gamma}$ , $\tau_{\alpha-\gamma-I}$ , $\tau_{\alpha-\gamma-I}$ interior and $\tau_{\alpha-\gamma-I}$ closure operators are introduced respectively by Njastad [1], Kasahara [3], Ogata [4], Kalaivani [5] and Kalaivani et al [6].

In this paper, the notion of $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings, properties are studied. Further the contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings is introduced and their properties are analyzed. Also the $(\gamma^{\alpha},\beta)-I$ -Continuous Mappings, their properties are studied.

## 2. Preliminaries

The concepts of ideal[2], its properties[7], $A^*(I)$ -local function [8], $X=X^*$ [9], $\tau\bigcap I=\phi$ [10], $\beta(I,\tau)$ [11], Kuratowski closure operator, Kuratowski* closure operator, $\gamma$ -semi-open set[12], ( $\alpha$ - $\gamma$ , $\beta$ )-continuous mappings[13] are defined, studied and discussed earlier.

Dontechev and Przemki[14], Reilly and Vamanamurthy[15], Tong[16] and Maximilian Ganster and Ivan Reilly [17] studied about α-continuity and decompositions of continuity.

Here $f_M$ denotes the mapping $f:(X,\tau,I)\to(Y,\sigma,K)$ and $TX_I,TY_K$ denotes the ideal topological spaces $(X,\tau,I)$ , $(Y,\sigma,K)$ .Then CM denotes continuous mapping, C denotes continuity, OS denotes open set, CS denotes closed set, inv denotes inverse image, M denotes mapping, IM denotes the identity mapping and $(\gamma\beta)^{\alpha}(I,K)$ - CM denotes the $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mapping.

## 3. $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings

**Definition 3.1.** A mapping $f_M$ is said to be a $(\gamma\beta)^{\alpha}(I,K)$ - CM if for each $\beta^{\alpha}K$ -OS,G of $TY_K$ , the inv $f_M^{-1}(G)$ is a $\gamma^{\alpha}I$ -OS in $TX_I$ .

**Theorem3.1.**Let $f_M$ be a mapping, then the following statements are equivalent:

(i) $f_M$ is a $(\gamma\beta)^{\alpha}(I,K)$ -CM.

(ii) For each element $a\in TX_I$ and $Q\in\sigma_{\beta^{\alpha}K}$ containing $f_M(a)$ , there exists $P\in\tau_{\gamma^{\alpha}I}$ containing $a$ such that $f_M(P)\subseteq Q$ .

(iii) The inv of each $\sigma_{\beta^{\alpha}K}$ -CS of $TY_K$ is a $\tau_{\gamma^{\alpha}I}$ -CS in $TX_I$ .

(iv) $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(D))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(D))$ for each $D\subseteq TY_K$ .

(v) $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(C))))\subseteq f_M(C)$ for each $C\in TX_I$ .

**Proof.** $(i)\Rightarrow(ii)$ Let $a\in TX_I$ and $Q$ be any $\beta^{\alpha}K$ - OS of $TY_K$ containing $f_M$ (a). Let $H=f_M^{-1}(Q)$, then by Definition 3.1 , $H$ is a $\tau_{\gamma^{\alpha}I}$ - OS containing $a$ and $f_M(H)\subseteq Q$ .

$(ii)\Rightarrow(iii)$ Let $C$ be a $\beta^{\alpha}K$ - CS of $TY_K$ . Set $S=TY_K-C$ then $S$ is a $\beta^{\alpha}K$ - OS in $TY_K$ .Let $b\in f_M^{-1}(S)$ , by $(ii)$ , there exists a $\gamma^{\alpha}I$ -

OS, $G$ of $TX_I$ containing $b$ such that $f_M(G)\subseteq S$ . We obtain $b\in G\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(G)))\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(f_M^{-1}(S))))$ and hence $b\in f_M^{-1}(S)\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(f_M^{-1}(S))))$ . This shows that $f_M^{-1}(S)$ ) is a $\tau_{\gamma^\alpha I}$ -OS in $TX_I$ . Hencewe obtain that,

$f_M^{-1}(C)=TX_I - f_M^{-1}(TY_K - C)=TX_I - f_M^{-1}(S)$

is a $\gamma^\alpha I$ -CS in $TX_I$ .

$(iii)\Rightarrow(iv)$ Let $M$ be any subset of $TY_K$ .Since $cl_{\sigma_\beta}(M)$ is a CS in $TY_K$ , by $(iii)$ , $f_M^{-1}(cl_{\sigma_\beta}(M))$ is a $\gamma^\alpha I$ -CS and $TX_I - f_M^{-1}(cl_{\sigma_\beta}(M))$ is a $\gamma^\alpha I$ -OS. Thus

$TX_I - f_M^{-1}(cl_{\sigma_\beta}(M))\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(TX_I - f_M^{-1}(cl_{\sigma_\beta}(M)))))=$

$TX_I - cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(cl_{\sigma_\beta}(M)))))$ .

Hence $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(M))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(M))$ .

$(iv)\Rightarrow(v)$ Let $L$ be any subset of $TX_I$ . By $(iv)$ , $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(L)))\subseteq cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(f_M(L)))))\subseteq f_M^{-1}(f_M(L))$ and hence $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(L))))\subseteq cl_{\sigma_\beta}(f_M(L))$ .

$(v)\Rightarrow(i)$ Let $H$ be any $\beta^\alpha K$ -OS of $TY_K$ . Then, by $(v)$ $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(TY_K - H)))))\subseteq cl_{\sigma_\beta}(f_M(f_M^{-1}(TY_K - H)))$ $\subseteq cl_{\sigma_\beta}(TY_K - H)=TY_K - H$ . Therefore,

$cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(TY_K - H))))\subseteq f_M^{-1}(TY_K - H)\subseteq TX_I - f_M^{-1}(H)$

. Hence we obtain that $f_M^{-1}(H)\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(f_M^{-1}(H))))$ . This implies that $f_M^{-1}(H)$ is a $\gamma^\alpha I$ -OS.Hence, $f_M$ is a $(\gamma\beta)^\alpha(I,K)$ -CM.

**Remark 3.1.** Every CM need not be a $(\gamma\beta)^\alpha(I,K)$ -CM: Illustrated by the following example.

**Example 3.1.**Let $X=\{d,e,f\},\tau=\{\phi,X,\{d\},\{e\},\{d,e\},\{d,f\}\}$ $\sigma=\{\phi,Y,\{d\},\{e\}\{d,e\},\{d,f\}\},I=\{\phi,\{d\},\{f\},\{d,f\}\}$ and $K=\{\phi,\{d\}\}$ .

The $\gamma$ operation on $\tau$ is given as follows: $\gamma(\text{M})=cl(\text{M})$ , for $\text{M}\in\tau$ .Then $\tau_{\gamma^\alpha I}=\{\phi,X,\{e\},\{d,f\}\}$ .

We define $\beta$ on $\sigma$ as follows $\text{H}^\beta=\begin{cases}\text{H} & if\ e\in\text{H}\\ cl(\text{H}) & if\ e\notin\text{H}\end{cases}$ .

Then $\sigma_{\beta^\alpha K}=\{\phi,Y,\{e\},\{d,e\},\{d,f\}\}$.

Hence the IM $f_M$ is a CM, but it is not a $(\gamma\beta)^\alpha(I,K)$ - CM, since $\{d,e\}\in\sigma_{\beta^\alpha K}$ and $f_M^{-1}(\{d,e\})=\{d,e\}\notin\tau_{\gamma^\alpha I}$ .

**Theorem 3.2.**Let $f_M$ be a M and $I,K$ are ideals of $TX_I$ , $TY_K$ . If $I = K = \{\phi\}$ or $I_n$ , then the concept of $\alpha$ - $(\gamma,\beta)$ - C and $(\gamma\beta)^\alpha(I,K)$ - C are equivalent.

**Proof.** Follows from the 3.1.Theorem's Proof.

**Theorem3.3.**Let $f_M$ be a M and $L\in\sigma$ .Then $f_M^{-1}(L^*_\gamma)=(f_M^{-1}(L))^*$ implies that $f_M^{-1}(cl^*_{\tau_\gamma}(L))\subseteq cl^*_{\tau_\gamma}(f_M^{-1}(L))$ .

**Proof.** Since $f_M^{-1}(cl^*_{\tau_\gamma}(L))=f_M^{-1}(L\cup L^*_\gamma)=f_M^{-1}(L)\cup f_M^{-1}(L^*_\gamma)\subseteq$

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**Remark 3.2.** The converse of the Theorem 3.3 is not true . It is demonstrated by the example given below.

**Example 3.2.** Let $X=Y=\{1,2,3\}$ ,

$\tau=\{\phi,X,\{1\},\{3\},\{1,2\},\{1,3\}\}$ , $\sigma=\{\phi,Y,\{1\},\{3\},\{1,2\},\{1,3\}\}$ , $I=\{\phi,\{1\},\{3\},\{1,3\}\}$ and $K=\{\phi,\{2\}\}$ .

$\gamma$ on $\tau$ is defined as follows : $F^\gamma=\begin{cases}\text{F}\ if\ \text{F}=\{1\}\\ F\cup\{3\}\ if\ \text{F}\neq\{1\}\end{cases}$ for $F\in\tau$ .Then $\tau_{\gamma^\alpha I}=\{\phi,X,\{1\},\{3\},\{1,3\}\}$ .

Also we define the operation $\beta$ on $\sigma$ : for every $\text{F}\in\sigma$ , $\text{F}^\beta=\begin{cases}\text{F}\ if\,\text{F}=\{1\}\\ \text{F}\cup\{3\}\ if\,\text{F}\neq\{1\}\end{cases}$

Then $\sigma_{\beta^\alpha K}=\{\phi,Y,\{1\},\{3\},\{1,2\},\{1,3\}\}$

We define a mapping $f_M$ as $f_M(\{1\})=\{1\}$; $f_M(\{2\})=\{2\}$; $f_M(\{3\})=\{3\}$ .

For the subset $\{1,2\}\in\sigma$ we have $f_M^{-1}(\{1,2\})^*=\{1,2\}$ and( $f_M^{-1}(\{1,2\}))^*=(\{1,2\})^*=\{2\}$ .

Thus $f_M^{-1}(P^*)\subseteq(f_M^{-1}(P))^*$ for $P=\{1,2\}\in\sigma$ . But $f_M^{-1}(cl^*_{\tau_\gamma}(P))\subseteq cl^*_{\tau_\gamma}(f_M^{-1}(P))$for each $P\in\sigma$ .

**Lemma 3.1.** Let $H,C\in TX_I$ .Then the following properties hold:

(i) $H\in\tau_{\gamma^\alpha I}$ if and only if there exists $U\in\tau$ such that $U\subseteq H\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(U))$

(ii) If $H\in\tau_{\gamma^\alpha I}$ and $H\subseteq C\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(H))$ , then $C\in\tau_{\gamma^\alpha I}$ .

**Proof .** (i) Let $H$ be a $\gamma^\alpha I$ -OS, then $H\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(H)))$ .Let $\text{A}=\text{int}_{\tau_\gamma}(H)$.Then, $\text{A}\subseteq H\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{A}))$ .

Conversely, let $\text{A}\subseteq H\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{A}))$ for some $\text{A}\in\tau$ . Since $\text{A}\subseteq H$ , $\text{A}\subseteq \text{int}_{\tau_\gamma}(H)$ and hence $cl^*_{\tau_\gamma}(\text{A})\subseteq cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(H))$ .

Hence we obtain that $H\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(H))))$ .Then $H$ is a $\gamma^\alpha I$ -OS.

(ii) Since $H$ is a $\gamma^\alpha I$ -OS, there exists an OS, k such that $\kappa\subseteq H\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\kappa))$ .We have $\kappa\subseteq H\subseteq C\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(H))$ implies that $\kappa\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\kappa))$ . Using the $(i)$ result, $C\in\tau_{\gamma^\alpha I}$ .

## 4. Contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings

**Definition 4.1.** A M $f_M$ is said to be a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM if the set $f_M^{-1}(\wp)$ is a $\gamma^{\alpha}I$ -CS in $\tau_{\gamma^{\alpha}I}$ for each $\beta^{\alpha}K$ -OS, $\wp$ of $\sigma_{\beta^{\alpha}K}$ .

**Definition 4.2.** A M $f_M$ is said to be a $(\gamma\beta)^{\alpha}(I,K)$ -CM if the set $f_M^{-1}$( S ) is a $\gamma^{\alpha}I$ -OS in $\tau_{\gamma^{\alpha}I}$ for each $\beta^{\alpha}K$ -OS, S of $\sigma_{\beta^{\alpha}K}$ .

**Remark 4.1.** The concept of contra $(\gamma\beta)^{\alpha}(I,K)$ C and $(\gamma\beta)^{\alpha}(I,K)$ C are independent as shown by the following two examples.

**Example 4.1.** Let $X=\{k,l,m\},\tau=\{\phi,X,\{k\},\{l\},\{k,l\},\{k,m\}\}$ and $I=\{\phi,\{k\}\}$ .

The $\gamma$ on $\tau$ is described as: $B^{\beta}=\begin{cases} B & if\ l\in B \\ cl(B) & if\ l\notin B \end{cases}$

Then $\tau_{\gamma^{\alpha}I}=\{\phi,X,\{k,l\},\{k,m\}\}$

Let $Y=\{k,l,m\},\sigma=\{\phi,Y,\{k\},\{l\},\{k,l\},\{k,m\}\}$ and $K=\{\phi,\{k\},\{m\},\{k,m\}\}$ . $\beta$ on $\sigma$ is given as follows: for $\square\in\sigma$, $\beta(\square)=cl(\square)$ .Then $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{l\},\{k,m\}\}$ .

The M $f_M$ is defined as $f_M(\{k\})=\{m\}; f_M(\{l\})=\{k\}; f_M(\{m\})=\{l\}$ . Then for every $P\in\sigma_{\beta^{\alpha}K}$ , $f_M^{-1}(P)$ is a $\gamma^{\alpha}I$ -CS in $TX_I$ . Hence $f_M$ is a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM. But $f_M^{-1}$ $(\{l\})=\{m\}$ is not a $\gamma^{\alpha}I$ -OS in $TX_I$. Hence $f_M$ is not a $(\gamma\beta)^{\alpha}(I,K)$ -CM.

**Example 4.2.** Let $X=\{p,q,r\}$ , $\tau=\{\phi,X,\{p\},\{r\},\{p,q\},\{p,r\}\}$ and $I=\{\phi,\{q\}\}$ .

Then $\gamma$ on $\tau$ is defined for $D\in\tau$ as follows:

$$D^{\gamma}=\begin{cases} D & if\ D=\{p\} \\ D\cup\{r\} & if\ D\neq\{p\} \end{cases}$$

Then $\tau_{\gamma^{\alpha}I=}\{\phi,X,\{p\},\{r\},\{p,q\},\{p,r\}\}$

Let $Y=\{p,q,r\}$ , $\sigma=\{\phi,Y,\{p\},\{q\},\{p,q\},\{p,r\}\}$ and $K=\{\phi,\{p\},\{r\},\{p,r\}\}$ .

We also define $\beta$ on $\sigma$ as follows: $\beta(\mathrm{E})=cl(\mathrm{E})$ for every $\mathrm{E}\in\sigma$ .Then $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{q\},\{p,q\},\{p,r\}\}$ .

We define $f_M$ as $f_M(\{p\})=\{q\}; f_M(\{q\})=\{r\}; f_M(\{r\})=\{p\}$ . Then for every $E\in\sigma_{\beta^{\alpha}K}$ , $f_M^{-1}(E)$ is a $\gamma^{\alpha}I$ -OS in $TX_I$ . Hence $f_M$ is a $(\gamma\beta)^{\alpha}(I,K)$ -CM. But $f_M^{-1}(\{q\})=\{p\}$; $f_M^{-1}(\{p,q\})=\{p,r\}$ are not $\gamma^{\alpha}I$ -CS in $TX_I$ .Hence $f_M$ is not a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM.

**Theorem 4.1.** A mapping $f_M$ is a contra $(\gamma\beta)^{\alpha}(I,K)$ - CM if and only if the inv of each $\beta^{\alpha}K$ -CS in $TY_K$ is a $\gamma^{\alpha}I$-OS in $TX_I$ .

**Proof.** Proof follows from the Definition 4.1.

**Theorem 4.2.** Let $f_M$ and $g:(Y,\sigma,K)\to(Z,\varsigma,W)$ be any two mappings.

(i) $g\circ f$ is a contra $(\gamma\beta)^{\alpha}(I,W)$ -CM , if $g$ is $(\gamma\beta)^{\alpha}(K,W)$ -CM and $f_M$ is contra $(\gamma\beta)^{\alpha}(I,K)$ -CM.

(ii) $g\circ f$ is a contra $(\gamma\beta)^{\alpha}(I,W)$ -CM , if $g$ is a contra $(\gamma\beta)^{\alpha}(K,W)$ -CM and $f_M$ is an $(\gamma\beta)^{\alpha}(I,K)$ - CM.

**Proof.** Proof follows from the Definition 4.1.

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**Definition 4.3.** A mapping $f_M$ is called a perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM if the inv of each $\beta^{\alpha}K$ -OS in $TY_K$ is a $\gamma^{\alpha}I$ - clopen set in $TX_I$

**Example 4.4.** Let $X=\{1,2,3\},\tau=\{\phi,X,\{1\},\{2\},\{1,2\},\{1,3\}\}$, and $I=\{\phi,\{1\}\}$ .

$\gamma$ on $\tau$ as follows: $B^{\gamma}=\begin{cases} B & if\ 2\in B \\ cl(B) & if\ 2\notin B \end{cases}$

Then $\tau_{\gamma^{\alpha}I=}\{\phi,X,\{2\},\{1,2\},\{1,3\}\}$

Let $Y=\{1,2,3\},\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\},\{1,3\}\}$ and $K=\{\phi,\{1\},\{3\},\{1,3\}\}$ $\beta$ on $\sigma$ as follows: $\beta(P)=cl(P)$ , for $P\in\sigma$ Then $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{2\},\{1,3\}\}$ .

We define the M $f_M$ as $f_M(\{1\})=\{3\}; f_M(\{2\})=\{2\}$; $f_M(\{3\})=\{1\}$ .

Then for $\mathrm{H}\in\sigma_{\beta^{\alpha}K}$ , $f_M^{-1}(H)$ is a $\gamma^{\alpha}I$ -clopen set in $TX_I$ . Hence $f_M$ is a perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM.

**Remark 4.3.** Every perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM is a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM and $(\gamma\beta)^{\alpha}(I,K)$ -CM.

**Remark 4.4.** A contra $(\gamma\beta)^{\alpha}(I,K)$ -CM may not be perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM. The example given below illustrates this remark.

Let $X=\{1,2,3\}$ , $\tau=\{\phi,X,\{1\},\{2\},\{1,2\},\{1,3\}\}$ and $I=\{\phi,\{1\}\}$ .

$\gamma$ on $\tau$ as follows: $B^{\gamma}=\begin{cases} B & if\ 2\in B \\ cl(B) & if\ 2\notin B \end{cases}$

Then $\tau_{\gamma^{\alpha}I=}\{\phi,X,\{2\},\{1,2\},\{1,3\}\}$ .

Let $Y=\{1,2,3\},\sigma=\{\phi,Y,\{1\},\{2\},\{1,2\},\{1,3\}\}$ and $K=\{\phi,\{1\},\{3\},\{1,3\}\}$ . $\beta$ on $\sigma$ as follows: $\beta(P)=cl(P)$ , for $P\in\sigma$ and we obtain that $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{2\},\{1,3\}\}$

We define M $f_M$ as $f_M(\{1\})=\{3\}$; $f_M(\{2\})=\{1\}; f_M(\{3\})=\{2\}$ . Then for every $\mathbf{Z}\in\sigma_{\beta^{\alpha}K}$ , $f_M^{-1}$ ( $\mathbf{Z}$ ) is a $\gamma^{\alpha}I$ -CS in $TX_I$ . Hence

$f_M$ is a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM. But $f_M^{-1}(\{2\})=\{3\}$ is not a $\gamma^{\alpha}I$ -OS in $TX_I$. Hence $f_M$ is not a $(\gamma\beta)^{\alpha}(I,K)$ -CM.

**Theorem 4.3.** For the M $f_M$, statements described below are equivalent:
(i) $f_M$ is a perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM
(ii) $f_M$ is contra $(\gamma\beta)^{\alpha}(I,K)$ -CM and $(\gamma\beta)^{\alpha}(I,K)$ -CM.

**Proof.** Follows from the Definitions 4.1, 4.2 and 4.3.

## 5. $(\gamma^{\alpha},\beta)$ - $I$ -Continuous Mappings

**Definition 5.1.** A subset $A$ of an ideal topological space is called a $I\beta$ -OS if $A\subseteq cl_{\tau_\gamma}(\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(A)))$

**Definition 5.2.** A M $f_M$ is defined as a $(\gamma^{\alpha},\beta)$ -$I$-CM if for every $I\beta$ -open set $V$ of $TY_K$, $f_M^{-1}(V)$ is a $\gamma^{\alpha}I$ -OS in $TX_I$.

**Theorem 5.1.** If a M $f_M$ is a $(\gamma^{\alpha},\beta)$ - $I$ -CM, then $f_M$ is an $(\gamma^{\alpha},\beta)$ -CM.
**Proof.** Immediate from the above illustrated Theorem 4.1.

**Theorem 5.2.** Let $f_M$ be a M. Statements given below are same:
(i) $f_M$ be a $(\gamma^{\alpha},\beta)$ - $I$ -CM.
(ii) For each $b$ and each $I\beta$ -OS, $H\subseteq TY_K$ containing $f_M(b)$, there exists $G\in\tau_{\alpha_\gamma}$ such that $b\in G$, $f_M(G)\subseteq H$.
(iii) The inv of each $I\beta$ -CS in $TY_K$ is a $\gamma^{\alpha}I$ -CS.
(iv) $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(B))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(B))$ for $B\subseteq TY_K$.
(v) $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}((A))))\subseteq cl_{\sigma_\beta}(f_M(A))$ for $A\subseteq TX_I$.

**Proof .** Proof is analogous to the proof of the 4.1.Theorem.

**Corollary 5.1.** Let $f_M$ be a $(\gamma^{\alpha},\beta)$ - $I$ - CM, then
(i) $f_M(cl^*_{\tau_\gamma}(\Upsilon))\subseteq cl_{\sigma_\beta}(f_M(\Upsilon))$ for each $\Upsilon\in\gamma$ - $PIO(X)$.
(ii) $cl^*_{\tau_\gamma}((f_M^{-1}(\Im))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\Im))$ for each $\Im\in\gamma$ - $PIO(Y)$.

**Proof.** Proof is analogous to the proof to the corollary 4.1.
s
**Definition 5.3.** A subset □ of an ideal TS is said to be a $\gamma I$ -open set with respect to the operation $\gamma$ on $\tau$ if
□ $\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(□))\cup cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(□))$.

**Definition 5.4.** A M $f_M$ is called a $(\gamma,\beta^{\alpha})$ - $I$ -open mapping if the image of each $\gamma I$ -OS in $TX_I$ is a $\beta^{\alpha}I$ -OS of $TY_I$.

**Theorem 5.3.** A M $f_M$ is said to be a $(\gamma,\beta^{\alpha})$ - $I$ -open mapping if and only if for each subset $\aleph\subseteq TY_I$ and each $\gamma I$ -CS, $F$ of $TX_I$ containing $f_M^{-1}(\aleph)$, there exists a $\beta^{\alpha}I$ -CS, $H\subseteq TY_I$ containing $W$ such that $f_M^{-1}(H)\subseteq F$.

**Proof.** Let $\wp=TY_I-f_M(TX_I-\Im)$. Since $f_M^{-1}(\aleph)\subseteq\Im$ ,s. Since $f_M$ is a $(\gamma,\beta^{\alpha})$ - $I$ - open mapping, then $\wp$ is a $\beta^{\alpha}I$ - CS and $f_M^{-1}(\wp)=TX_I-f_M^{-1}(f_M(TX_I-\Im))\subseteq TX_I-(TX_I-\Im)=\Im$.
Conversely, let $\ell$ be any $\gamma I$ -OS of $TX_I$ and $\aleph=TY_I-f_M(\ell)$.
Then, $f_M^{-1}(\aleph)=TX_I-f_M^{-1}(f_M(\ell))\subseteq TX_I-\ell$ and $TX_I-\ell$ is a $\gamma I$ -CS. There exists a $\beta^{\alpha}I$ - CS,$\mathfrak{H}$ of $TY_I$ containing $\aleph$ such that $f_M^{-1}(\wp)\subseteq TX_I-\ell$. Then, $f_M^{-1}(\wp)\cap\ell=\phi$ and $\wp\cap f_M(\ell)=\phi$.
Therefore, $TY_I-f_M(\ell)\supseteq\wp\supseteq\aleph=TY_I-f_M(\ell)$ and $f_M(\ell)$ is a $\beta^{\alpha}I$ - OS in $TY_I$. This implies that $f_M$ is a $(\gamma,\beta^{\alpha})$ - $I$ -open mapping.

**Corollary 5.1.** If $f_M$ is a $(\gamma,\beta^{\alpha})$ - $I$ - open mapping, then these properties hold:
(i) $f_M^{-1}(cl_{\sigma_\beta}(\text{int}_{\sigma_\beta}(cl_{\sigma_\beta}(\aleph))))\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$ for $\aleph\subseteq TY_I$.
(ii) $f_M^{-1}(cl^*_{\sigma_\beta}(\text{O}))\subseteq cl_{\tau_\gamma}(f_M^{-1}(\text{O}))$ for $\gamma$ -preopen set O of $TY_I$.

**Proof.** (i) Let $\aleph$ be any subset of $TY_I$, then $cl_{\tau_\gamma}(f_M^{-1}(\aleph))$ is a $\gamma I$ -CS in $TX_I$. By Theorem 4.3, there exists a $\beta^{\alpha}I$ -CS,□ $\subseteq TY_I$ containing $\aleph$ such that $f_M^{-1}(□)\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$. Since $TY_I-□$ is a $\beta^{\alpha}I$ - OS, $f_M^{-1}(TY_I-□)\subseteq f_M^{-1}(\text{int}_{\sigma_\beta}(cl^*_{\sigma_\beta}(\text{int}_{\sigma_\beta}(TY_I-□))))$
and we obtain that

$$f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\aleph))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(□))))$$
$$\subseteq f_M^{-1}(□)\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$$

Therefore $f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\aleph))))\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$
(ii) Let O be any $\gamma$ - preopen set of $TY_I$. By using (i) result,

$$f_M^{-1}(cl^*_{\sigma_\beta}(\text{O}))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{O}))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{int}_{\sigma_\beta}(cl_{\sigma_\beta}(\text{O}))))$$
$$\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\text{O}))))$$

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