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Research paper

Oscillation of Second-Order Quasilinear Generalized Difference Equations

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Abstract

Authors present sufficient conditions for the oscillation solutions of the generalized perturbed quasilinear difference equation
$\Delta_\ell\Big(a((k-1)\ell+j)|\Delta_\ell v((k-1)\ell+j)|^{\gamma-1}\Delta_\ell v((k-1)\ell+j)\Big)+F(k\ell+j, v(k\ell+j))=G(k\ell+j, v(k\ell+j),\Delta_\ell v(k\ell+j))$
where $0<\gamma<1$, $k\in[0,\infty)$. Examples are illustrates the importance of our results are also included.

Keywords: *Generalized difference equation; Oscillation; Quasilinear.*

1. Introduction

Difference equations represent a captivating mathematical field, has rich field of the applications in such diverse disciplines as population dynamics, operations research, ecology, economics, biology etc. For the background of difference equations and its application in diverse fields with examples, see [1,13,20,27], based on assumption $\Delta u(k)=u(k+1)-u(k),\ k\in[0,\infty)$.
Though some authors [1],[19] have recommended the definition of Δ as

$$\Delta_\ell\, u(k)=u(k+\ell)-u(k),\quad \ell\in(0,\infty), \tag{iE}$$

no notable progress have been taken on this line. But in [14] the authors took up the definition of Δ as given in (E), and given many important results and applications. They labelled the operator Δ defined by (E) as Δ_ℓ and its inverse by Δ_ℓ^{-1} , many interesting results in number theory were obtained. Qualitative properties like rotator, expanding, shrinking, spiral and web like were established by extending theory of Δ_ℓ to complex function, for the solutions of difference equations involving Δ_ℓ in [2-12,14-18,21-26].
In the sequel, in this paper we consider the generalized perturbed quasilinear difference equation

$$\Delta_\ell\Big(a((k-1)\ell+j)|\Delta_\ell v((k-1)\ell+j)|^{\gamma-1}\Delta_\ell v((k-1)\ell+j)\Big)+F(k\ell+j, v(k\ell+j))=G(k\ell+j, v(k\ell+j),\Delta_\ell v(k\ell+j)),\quad k\in[0,\infty) \tag{1}$$

where $0<\gamma<1$, $a(k\ell+j)$ is an eventually positive real valued function, and Δ_ℓ is the generalized forward difference operator defined as

$$\Delta_\ell v(k\ell+j)=v((k+1)\ell+j)-v(k\ell+j).$$

By a solution of (1), we mean a nontrivial real valued function $v(k\ell+j)$ satisfying (1) for $k\in[0,\infty)$. A solution $v(k\ell+j)$ is said to be oscillatory if it is neither eventually positive nor negative, and nonoscillatory otherwise.

2. Main Results

Throughout this paper we assume that there exist real valued functions $q(k\ell+j)$, $p(k\ell+j)$ and a function $f:R\to R$ such that
(i). $xf(x)>0$ for all $x\neq 0$;
(ii). $f(x)-f(y)=g(x,y)(x-y)$ for $x,y\neq 0$, where g is a nonnegative function; and
(iii) $\dfrac{F(k\ell+j, x\ell+j)}{f(x\ell+j)}\geq q(k\ell+j)$,
$\dfrac{G(k\ell+j, x\ell+j, y\ell+j)}{f(x\ell+j)}\leq p(k\ell+j)$ for $x,y\neq 0$.
The conditions used in the main results are listed as follows:

$$\sum\frac{1}{a^{1/\gamma}((k-1)\ell+j)}=\infty, \tag{2}$$

$$\sum_{k=1}^{\infty}(q(k\ell+j)-p(k\ell+j))=\infty, \tag{3}$$

$$\sum_{k=1}^{\infty}(q(k\ell+j)-p(k\ell+j))<\infty, \tag{4}$$

$$\lim_{k\to\infty}\sum_{r=k_0}^{k}\left[\frac{1}{a(r\ell+j)}\sum_{s=r+1}^{\infty}(q(s\ell+j)-p(s\ell+j))\right]^{1/\gamma}=\infty, \tag{5}$$

$$\int_{\theta}^{\infty} \frac{du}{f(u)^{1/\gamma}} < \infty, \int_{-\theta}^{-\infty} \frac{du}{f(u)^{1/\gamma}} < \infty \text{ for all } \theta > 0 \qquad (6)$$

$$\liminf_{k\to\infty} \sum_{r=k_0}^{k} (q(r\ell+j) - p(r\ell+j)) \ge 0 \text{ for all large } k_0, \qquad (7)$$

$$\int_{0}^{\theta} \frac{du}{f(u)^{1/\gamma}} < \infty, \int_{0}^{-\theta} \frac{du}{f(u)^{1/\gamma}} < \infty \text{ foriall } \theta > 0, \qquad (8)$$

Theorem 2.1: *Suppose (2) and (3) hold. The all solutions of (1) are oscillatory.*

Proof. Let $v(k\ell+j)$ be a nonoscillatory solution of (1), say, $v(k\ell+j) > 0$ for $k \ge k_0 \ge 1$. We shall consider only this case because the proof for the case $v(k\ell+j) < 0$ for $k \ge k_0 \ge 1$ is similar. We begin with the identity

$$\Delta_\ell \left[\frac{a((k-1)\ell+j)|\Delta_\ell v((k-1)\ell+j)|^{\gamma-1} \Delta_\ell v((k-1)\ell+j)}{f(v(k\ell+j))} \right]$$
$$= \frac{G(k\ell+j, v(k\ell+j), \Delta f(v(k\ell+j)))}{f(v(k\ell+j))} - \frac{F(k\ell+j, v(k\ell+j))}{f(v(k\ell+j))}$$
$$- \frac{a(k\ell+j)\, g(v((k+1)\ell+j), v(k\ell+j))(\Delta_\ell v(k\ell+j))^2}{f(v(k\ell+j))}$$
$$\times \frac{|\Delta_\ell v(k\ell+j)|^{\gamma-1}}{f(v((k+1)\ell+j))} \qquad (9)$$

which in view of (1)-(3) provides

$$\Delta_\ell \left[\frac{a((k-1)\ell+j)|\Delta_\ell v((k-1)\ell+j)|^{\gamma-1} \Delta_\ell v((k-1)\ell+j)}{f(v(k\ell+j))} \right]$$
$$\le (p(k\ell+j) - q(k\ell+j)) \qquad (10)$$

$k \ge k_0$. Summing (10) from (k_0+1) to k gives

$$\frac{a(k\ell+j)|\Delta_\ell v(k\ell+j)|^{\gamma-1} \Delta_\ell v(k\ell+j)}{f(v((k+1)\ell+j))}$$
$$\le \frac{a(k_0\ell+j)|\Delta_\ell v(k_0\ell+j)|^{\gamma-1} \Delta_\ell v(k_0\ell+j)}{f(v((k_0+1)\ell+j))}$$
$$- \sum_{r=k_0+1}^{k} (q(r\ell+j) - p(r\ell+j)). \qquad (11)$$

By (3), the right side of (11) tends to $-\infty$ as $k\to\infty$. This implies that there exists an integer $k_1 \ge k_0$ such that $\Delta_\ell v(k\ell+j) < 0$ for $k \ge k_1$. Condition (3) also implies that there exists an integer $k_2 \ge k_1$ such that

$$\sum_{r=k+1}^{k} (q(r\ell+j) - p(r\ell+j)) \ge 0, \; k \ge k_2+1 \qquad (12)$$

Using (3) and summing (1) from (k_2+1) to k, and then using Abel's transformation [1], we get

$$a(k\ell+j)|\Delta_\ell v(k\ell+j)|^{\gamma-1} \Delta_\ell v(k\ell+j)$$
$$\le a(k_2\ell+j)|\Delta_\ell v(k_2\ell+j)|^{\gamma-1} \Delta_\ell v(k_2\ell+j)$$
$$- \sum_{s=k_2+1}^{k} f(v(r\ell+j))(q(r\ell+j) - p(r\ell+j))$$
$$= a(k_2\ell+j)|\Delta_\ell v(k_2\ell+j)|^{\gamma-1} \Delta_\ell v(k_2\ell+j)$$
$$- f(v((k+1)\ell+j)) \sum_{s=k_2+1}^{k} (q(r\ell+j) - p(r\ell+j))$$
$$+ \sum_{r=k_2+1}^{k} \Delta_\ell f(r\ell+j) \left[\sum_{s=k_2+1}^{r} (q(s\ell+j) - p(s\ell+j)) \right]$$
$$= a(k_2\ell+j)|\Delta_\ell v(k_2\ell+j)|^{\gamma-1} \Delta_\ell v(k_2\ell+j)$$
$$- f(v((k+1)\ell+j)) \sum_{r=k_2+1}^{k} (q(r\ell+j) - p(r\ell+jv))$$
$$+ \sum_{r=k_2+1}^{k} g(v((r+1)\ell+j), v(r\ell+j))\Delta_\ell v(r\ell+j)$$
$$\times \left[\sum_{s=k_2+1}^{r} (q(s\ell+j) - p(s\ell+j)) \right]$$
$$\le a(k_2\ell+j)|\Delta_\ell v(k_2\ell+j)|^{\gamma-1} \Delta_\ell v(k_2\ell+j), \; k \ge k_2+1 \quad (13)$$

where we have also used (12) in the last inequality. Since $\Delta_\ell v(k\ell+j) < 0$ for $k \ge k_1$, it follows from (13) that

$$a(k\ell+j)|\Delta_\ell v(k\ell+j)|^{\gamma-1} \ge a(k_2\ell+j)|\Delta_\ell v(k_2\ell+j)|^{\gamma-1}$$
$$\times \Delta_\ell v(k_2\ell+j, \; k \ge K_2+1$$
$$\Delta_\ell v(k\ell+j) \le -a(k_2\ell+j)^{1/\gamma} |\Delta_\ell v(k_2\ell+j)| \frac{1}{a(k\ell+j)^{1/\gamma}}, \qquad (14)$$
$$k \ge k_2+1.$$

Summing (14) from (k_2+1) to k provides

$$v((k+1)\ell+j) \le v((k_2+1)\ell+j)$$
$$- a(k_2\ell+j)^{1/\gamma} |\Delta_\ell v(k_2\ell+j)| \sum_{r=k_2+1}^{k} \frac{1}{a(r\ell+j)^{1/\gamma}}. \qquad (15)$$

By (2), the right side of (15) tends to $-\infty$ as $k\to\infty$. This contradicts the assumption that $v(k\ell+j)$ is eventually positive.

Example 2.2: *Consider the generalized difference equation*

$$\Delta_\ell (k\ell+j)|\Delta_\ell v((k-1)\ell+j)|^{\gamma-1} \Delta_\ell v((k-1)\ell+j)$$
$$+ v(k\ell+j)[b(k\ell+j, v(k\ell+j)) + 2^\gamma + 2^\gamma(2(k\ell+j)+\ell)]$$
$$= b(k\ell+j, v(k\ell+j))\, v(k\ell+j) \qquad (16)$$

where $\gamma \ge 1$ and $b(k\ell+j, v(k\ell+j))$ is any function of $k\ell+j$ and $v(k\ell+j)$. Clearly, (2) holds. By taking $f(v(k\ell+j)) = v(k\ell+j)$, we have

$$\frac{F(k\ell+j, v(k\ell+j))}{f(v(k\ell+j))} = b(k\ell+j, v(k\ell+j)) + 2^\gamma(2(k\ell+j)+\ell)$$
$$\equiv q(k\ell+j),$$
$$\frac{G(k\ell+j, v(k\ell+j), \Delta_\ell v(k\ell+j))}{f(v(k\ell+j))} = b(k\ell+j, v(k\ell+j))$$
$$\equiv p(k\ell+j)$$

and so (3) holds. Hence, by Theorem 1 all solutions of (16) are oscillatory. One such solution is given by $v(k\ell+j) = (-1)^k$.

Theorem 2.3: *Suppose (2) and (4)-(7) hold. Then all solutions of (1) are oscillatory.*

Proof. Suppose that $v(k\ell+j)$ is a nonoscillatory solution of (1), say, $v(k\ell+j) > 0$ for $k \ge k_0 \ge 1$.

Case 1. Suppose that $\Delta_\ell v(k\ell+j) \ge 0$ for $k \ge k_1 \ge k_0$. We sum (10) from (k_1+1) to k to get

$$0 \le \frac{a(k\ell+j)\left(\Delta_\ell v(k\ell+j)\right)^\gamma}{f(v((k+1)\ell+j))} \le \frac{a(k_1\ell+j)\left(\Delta_\ell v(k_1\ell+j)\right)^\gamma}{f(v((k_1+1)\ell+j))} - \sum_{r=k_1+1}^{k}(q(r\ell+j)-p(r\ell+j)). \tag{17}$$

In view of (3), it follow from (11) that

$$0 \le \frac{a(k_1\ell+j)\left(\Delta_\ell v(k_1\ell+j)\right)^\gamma}{f(v((k_1+1)\ell+j))} - \sum_{r=k_1+1}^{k}(q(r\ell+j)-p(r\ell+j))$$

and therefore for $k \ge k_1$

$$\sum_{r=k+1}^{\infty}(q(r\ell+j)-p(r\ell+j)) \le \frac{a(k\ell+j)\left(\Delta_\ell v(k\ell+j)\right)^\gamma}{f(v((k+1)\ell+j))}$$

$$\left[\frac{1}{a(k\ell+j)}\sum_{r=k+1}^{\infty}(q(r\ell+j)-p(r\ell+j))\right]^{1/\gamma} \le \frac{\Delta_\ell v(k\ell+j)}{f(v((k+1)\ell+j))^{1/\gamma}}. \tag{18}$$

Summing (18) from k_1 to k, we get

$$\sum_{r=k_1}^{k}\left[\frac{1}{a(r\ell+j)}\sum_{s=r+1}^{\infty}(q(s\ell+j)-p(s\ell+j))\right]^{1/\gamma} \le \sum_{r=k_1}^{k}\frac{\Delta_\ell v(r\ell+j)}{f(v((r+1)\ell+j))^{1/\gamma}} \le \int_{v(k_1)}^{v(k+1)}\frac{du}{f(u)^{1/\gamma}}. \tag{19}$$

By (5), the left side of (19) tends to ∞ as $k\to\infty$. However, the right side of (19) is finite by (6).

Case 2. Suppose that $\Delta_\ell v(k\ell+j)$ is oscillatory. Hence, there exists a real valued function $(k_n\ell+j)\to\infty$ such that $\Delta_\ell v(k_n\ell+j)<0$. We choose k so large that (7) holds. Then, summing (10) from (k_n+1) to k followed by taking limit supremum provides

$$\limsup_{k\to\infty}\frac{a(k\ell+j)|\Delta_\ell v(k\ell+j)|^{\gamma-1}\Delta_\ell v(k\ell+j)}{f(v((k+1)\ell+j))} \le \frac{a(k_n\ell+j)|\Delta_\ell v(k_n\ell+j)|^{\gamma-1}\Delta_\ell v(k_n\ell+j)}{f(v((k_n+1)\ell+j))} + \limsup_{k\to\infty}\left[-\sum_{r=k_n+1}^{k}(q(r\ell+j)-p(r\ell+j))\right]<0 \tag{20}$$

where we have used (7) in the last inequality. It follows from (20) that $\lim_{k\to\infty}\Delta_\ell v(k\ell+j)<0$. This contradicts the assumption that $\Delta_\ell v(k\ell+j)$ oscillates.

Case 3. Suppose that $\Delta_\ell v(k\ell+j)<0$ for $k\ge k_1\ge k_0$. We note that condition (7) implies the existence of an integer $k_2\ge k_1$ such that (12) holds. The rest of the proof is similar to that of Theorem 1.

Corollary 2.4: *Suppose (2), (4), (5) and (7) hold. Then, al bounded solutions of (1) are oscillatory.*

Proof. The condition (6) is used only in Case 1 of the proof of Theorem 3iSuppose $v(k\ell+j)$ is a bounded nonoscillatory solution of (1). In Case 1 we have $v(k\ell+j)>0$ and $\Delta_\ell v(k\ell+j)\ge 0$ for $k\ge k_1$. Hence, in view of (2), we have $f(v(k\ell+j))\ge f(v(k\ell+j))$ for $k\ge k_1$. It follows from (19) that

$$\sum_{r=k_1}^{k}\left[\frac{1}{a(r\ell+j)}\sum_{s=r+1}^{\infty}(q(s\ell+j)-p(s\ell+j))\right]^{1/\gamma} \le \sum_{r=k_1}^{k}\frac{\Delta_\ell v(r\ell+j)}{f(v((r+1)\ell+j))^{1/\gamma}} \le \frac{1}{f(v(k_1\ell+j))^{1/\gamma}}\sum_{r=k_1}^{k}\Delta_\ell v(r\ell+j) = \frac{(v((k+1)\ell+j)-v(r_1\ell+j))}{f(v(k_1\ell+j))^{1/\gamma}}. \tag{21}$$

By (5), the left side of (21) tends to ∞ as $k\to\infty$ whereas the right side is finite.

Example 2.5: *Consider the generalized difference equation*

$$\Delta_\ell\left(\frac{1}{k^2\ell+j}|\Delta_\ell v((k-1)\ell+j)|^{\gamma-1}\Delta_\ell v((k-1)\ell+j)\right) + v(k\ell+j)\left[\begin{matrix} b(k\ell+j, v(k\ell+j)) \\ +2^\gamma\dfrac{(2k^2\ell+j)+((2k+1)\ell+j)+1}{(k^2\ell+j)((k+1)^2\ell+j)}\end{matrix}\right] = b(k\ell+j, v(k\ell+j))\,v(k\ell+j),\ k\ge 1 \tag{22}$$

where $\gamma>0$ and $b(k\ell+j, v(k\ell+j))$ is any function of k and $v(k\ell+j)$. Clearly, (2) holds. Taking $f(v(k\ell+j))=v(k\ell+j)$, gives

$$\frac{F(k\ell+j, v(k\ell+j))}{f(v(k\ell+j))} = b(k\ell+j, v(k\ell+j)) + 2^\gamma\frac{(2k^2\ell+j)+((2k+1)\ell+j)+1}{(k^2\ell+j)((k+1)^2\ell+j)} \equiv q(k\ell+j)$$

and

$$\frac{G(k\ell+j, v(k\ell+j), \Delta_\ell v(k\ell+j))}{f(v(k\ell+j))} = b(k\ell+j, v(k\ell+j)) \equiv p(k\ell+j)$$

and hence (7) is satisfied. Next, we find that

$$\sum_{k=1}^{\infty}(q(k\ell+j)-p(k\ell+j)) = 2^\gamma\frac{(2k^2\ell+j)+((2k+1)\ell+j)+1}{(k^2\ell+j)((k+1)^2\ell+j)} = 2^\gamma\sum\left[\frac{1}{(k^2\ell+j)}+\frac{1}{((k+1)^2\ell+j)}\right]<\infty$$

and so (4) holds. To see that (5) satisfied, we note that

$$\sum_{r=k_0}^{\infty}\left[\frac{1}{a(r\ell+j)}\sum_{s=r+1}^{\infty}(q(s\ell+j)-p(s\ell+j))\right]^{1/\gamma}$$

$$= 2\sum_{r=k_0}^{\infty}\left[(r^2\ell+j)\sum_{s=r+1}^{\infty}\frac{((2s^2)\ell+j)+((2s+1)\ell+j)+1}{(s^2\ell+j)((s+1)^2\ell+j)}\right]^{1/\gamma}$$

$$= 2\sum_{r=k_0}^{\infty}\left[(r^2\ell+j)\left(\sum_{s=r+1}^{\infty}\frac{1}{(s^2\ell+j)}+\sum_{s=r+1}^{\infty}\frac{1}{((s+1)^2\ell+j)}\right)\right]^{1/\gamma}$$

$$\geq 2\sum_{r=k_0}^{\infty}\left[(r^2\ell+j)\sum_{s=r+1}^{2r}\frac{1}{(s^2\ell+j)}\right]^{1/\gamma}$$

$$\geq 2\sum_{r=k_0}^{\infty}\left[(r^2\ell+j)\sum_{s=r+1}^{2r}\frac{1}{((2r)\ell+j)^2}\right]^{1/\gamma}$$

$$= 2\sum_{r=k_0}^{\infty}\left(\frac{(r\ell+j)}{4}\right)^{1/\gamma}=\infty .$$

Hence, the conclusion of Corollary 4 follows and all bounded solutions of (22) are oscillatory. One such solution is given by $v(k\ell+j)=(-1)^k$.

Remark 2.6: *In equation (22) if we let* $0<\gamma<1$ *, then* $f(v(k\ell+j))=v(k\ell+j)$ *also satisfies (6). Hence, it follows from Theorem 2.2 that all solutions of (22) are oscillatory when* $0<\gamma<1$.

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