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Research paper

Solution of Non-Linear Ito System of Equations by Homotopy Analysis Method (HAM)

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Abstract

Nonlinear Ito system of equations have wide application in applied physics. Many authors have found solution of this complex problem by using Adomain Decomposition Method (ADM), Reduced Differential Transform Method (RDTM) etc. All of these methods have a drawback as their convergence is quite slow and it requires a very good approximation to converge these schemes in considerable iterations. To overcome this difficulty, Liao has proposedHomotopy Analysis Method (HAM) that is quite effective due to the presence of convergence control parameter$\hbar$. It has been shown that for $\hbar = -0.7$, the scheme converges after very few iterations. Analytical solution obtained by HAM has been compared with the exact solution and both are found in good agreement. Computations are performed using the software package MATHEMATICA.This work verifies the validity and the potential of the HAM for the study of nonlinear systems of partial differential equations.

Keywords: *Ito coupled system, Adomian decomposition method, homotopy analysis method, analytical solutions, symbolic computation, Mathematica.*

1. Introduction

Nonlinear PDEs are very important to study due to their wide application to describe important physics of a phenomenon involved in various scientific fields like Fluid Mechanics, Solid State Physics, Mathematical Biology etc. To find analytical solution of these nonlinear PDEs is a challenge to researchers from many decades. Availability of Mathematica, Matlab, Maple etc. has made it realistic to perform very tedious algebra involved in analytical solutions. A broad class of analytical solutions methods and numerical solutions methods was used to handle these problems [1].

Gardner [2] had proposed inverse scattering technique (IST) to get solitary wave solutions.Perturbation techniques [3, 4] were also proposed by researchers to find the approximate solution of nonlinear problems. This technique is based on perturbation quantities to couple the nonlinear system of equations. These types of perturbations are not present in many problems of science and engineering. To overcome this difficulty, some non-perturbed techniques [5, 6] have been developed. Adomian's decomposition method [7] has also been used by many researchers to find approximate analytical solution of nonlinear PDEs. All these techniques have not found useful when we are dealing with highly nonlinear system due to absence of convergence rate of series solution.

In 1992, Liao [8] has proposed homotopy analysis method (HAM) that can also control rate of convergence of series solution. In 2003, Liao has published his first book [9] on the method.Many researchers have applied this method successfully to find solution of system of nonlinear PDEs appearing in science & engineering such as KdV-type equations [10–11], nonlinear heat transfer [12] etc. Several other known nonlinear equations such as Laplace equation with Dirichlet and Neumann boundary conditions [13], the generalized Hirota-Satsuma coupled KdV equation [14] and the Benjamin-Bona-Mahony-Burgers (BBMB) equations [15] have also been solved by HAM [16].

In the present work, we have found analytical solution of highly nonlinear Ito system of equations. To show the efficiency of the method, the initial conditions are chosen for which the exact solution is known. It's important to mention however that HAM can be implemented with any initial condition.We consider anIto system [17]:

$$u_t - v_x = 0, \tag{1}$$

$$v_t + 2v_{xxx} + 6uv_x + 6vu_x - 6(wp)_x = 0, \tag{2}$$

$$w_t - w_{xxx} - 3uw_x = 0, \tag{3}$$

$$p_t - p_{xxx} - 3up_x = 0, \tag{4}$$

with the following initial conditions:

$$u(x,0)=\frac{2}{3}d^2-\frac{1}{3}c-2d^2(\tanh(dx))^2, \qquad v(x,0)=e+2cd^2(\tanh(dx))^2,$$
$$w(x,0)=a+b\tanh(dx), \; p(x,0)=\frac{d^2}{b^2}(2e+4cd^2+c^2)(a-b\tanh(dx)). \tag{5}$$

The exactsolution of the system (1)-(4) with initial conditions (5) is known and given by:

$$u(x,t)=\frac{2}{3}d^2-\frac{1}{3}c-2d^2\big(\tanh\big(d(x-ct)\big)\big)^2,$$
$$v(x,t)=e+2cd^2(\tanh(d(x-ct)))^2,$$
$$w(x,t)=a+b\tanh\big(d(x-ct)\big),$$
$$p(x,t)=\frac{d^2}{b^2}(2e+4cd^2+c^2)(a-b\tanh(d(x-ct))). \tag{6}$$

This paper is arranged in the following manner: In Section 2, we present some basic definitions of the Homotopy Analysis method. In Section 3, the implementation of HAMon the system of 1-D Ito system is given; in Section 4, convergence analysis for this problem is presented; in the end the conclusion ispresented in Section 5.

2. Basic Concepts of Ham

Let us consider the following differential equation

$$N[u(x,t)]=0, \tag{7}$$

where N is a nonlinear operator, $u(x,t)$is an unknown function, and x and t denote space and time variables, respectively. By means of generalizing the traditional homotopy method, Liao [5] constructs the so called zero-order deformation equation:

$$(1-q)L[\varphi(x,t;q)-u_0(x,t)]=q\hbar HN[\varphi(x,t;q)], \tag{8}$$

where$q\in[0,1]$is the embedding parameter, $\hbar$a non-zero auxiliary parameter, Ha non-zero auxiliary function,Lan auxiliary linear operator, $u_0(x,t)$an initial guess of$u(x,t)$and$\varphi(x,t;q)$is an unknown function. It is important that one has great freedom to choose auxiliary things in HAM.Obviously, when$q=0$and$q=1$, it holds that

$$\varphi(x,t;0)=u_0(x,t), \qquad \varphi(x,t;1)=u(x,t). \tag{9}$$

Thus, as$q$increases from 0 to 1, the solution$\varphi(x,t;q)$varies from the initial guess$u_0(x,t)$to the solution$u(x,t)$.Expanding$\varphi(x,t;q)$in Taylor seriesabout$q=0$, we have:

$$\varphi(x,t;q)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t)q^m, \tag{10}$$

where$u_m(x,t)=\frac{1}{m!}\frac{\partial^m\varphi(x,t;q)}{\partial q^m}\Big|_{q=0}$. (11)

If the auxiliary linear operator, the initial guess, the nonzero auxiliary function$H$, and the nonzero auxiliary parameter$\hbar$are properly chosen, the above series converges at$q=1$, and then we have:

$$\varphi(x,t;1)=u_0(x,t)+\sum_{m=1}^{\infty}u_m(x,t), \tag{12}$$

which must be one of the solutions of the original non-linear differential equation.
Differentiating the zero-order deformation equation (6)m-times with respect to$q$, putting $q=0$and finally dividing by$m!$ throughout, we obtain the m-th order deformation equation:

$$L[u_m(x,t)-\chi_m u_{m-1}(x,t)]=\hbar HR_m(u_{m-1},x,t), \tag{13}$$

where $R_m(u_{m-1},x,t)=\frac{1}{(m-1)!}\frac{\partial^{m-1}N[\varphi(x,t;q)]}{\partial q^{m-1}}\Big|_{q=0}$ and $\chi_m=\begin{cases}0, & m\le 1\\ 1, & m>1\end{cases}$. (14)

3. Application

We consider (1)-(4) subject to the initial conditions$u(x,0)$, $v(x,0)$, $w(x,0)$ and $p(x,0)$given by (5).
For application of the HAM, we define the linear operator$L$ as

$$L[\varphi(x,t;q)]=\frac{\partial\varphi(x,t;q)}{\partial t}, \tag{15}$$

with the property$L(c)=0$, where c is a constant. From (1)-(4), we define the system of nonlinear operators as

$$N_1[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)]=\frac{\partial\varphi_1(x,t;q)}{\partial \mathrm{t}}-\frac{\partial\varphi_2(x,t;q)}{\partial \mathrm{x}}, \tag{16}$$

$$N_2[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)] = \frac{\partial\varphi_2(x,t;q)}{\partial t} + 2\frac{\partial^3\varphi_2(x,t;q)}{\partial x^3} + 6\varphi_1(x,t;q)\frac{\partial\varphi_2(x,t;q)}{\partial x} + 6\varphi_2(x,t;q)\frac{\partial\varphi_1(x,t;q)}{\partial x} - 6\frac{\partial(\varphi_3(x,t;q)\varphi_4(x,t;q))}{\partial x}, \quad (17)$$

$$N_3[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)] = \frac{\partial\varphi_3(x,t;q)}{\partial t} - \frac{\partial^3\varphi_3(x,t;q)}{\partial x^3} - 3\varphi_1(x,t;q)\frac{\partial\varphi_3(x,t;q)}{\partial x}, \quad (18)$$

$$N_4[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)] = \frac{\partial\varphi_4(x,t;q)}{\partial t} - \frac{\partial^3\varphi_4(x,t;q)}{\partial x^3} - 3\varphi_1(x,t;q)\frac{\partial\varphi_4(x,t;q)}{\partial x}. \quad (19)$$

Using the above definition, we construct the zero-order deformation equations:

$$(1-q)L[\varphi_1(x,t;q) - u_0(x,t)] = q\hbar_1 N_1[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)],$$
$$(1-q)L[\varphi_2(x,t;q) - v_0(x,t)] = q\hbar_2 N_2[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)],$$
$$(1-q)L[\varphi_3(x,t;q) - w_0(x,t)] = q\hbar_3 N_3[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)],$$
$$(1-q)L[\varphi_4(x,t;q) - p_0(x,t)] = q\hbar_4 N_4[\varphi_1(x,t;q),\varphi_2(x,t;q),\varphi_3(x,t;q),\varphi_4(x,t;q)]. \quad (20)$$

Obviously, when$q=0$ and$q=1$,

$$\varphi_1(x,t;0) = u_0(x,t), \quad \varphi_1(x,t;1) = u(x,t), \quad (21)$$

$$\varphi_2(x,t;0) = v_0(x,t), \quad \varphi_2(x,t;1) = v(x,t). \quad (22)$$

$$\varphi_3(x,t;0) = w_0(x,t), \quad \varphi_3(x,t;1) = w(x,t), \quad (23)$$

$$\varphi_4(x,t;0) = p_0(x,t), \quad \varphi_4(x,t;1) = p(x,t). \quad (24)$$

Differentiating the zero-order deformation equations (20)m times with respect to$q$, and finally dividing by$m!$, we obtain the m-th order deformation equations:

$$L[u_m(x,t) - \chi_m u_{m-1}(x,t)] = \hbar_1 H_1 R_{1,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}), \quad (25)$$

$$L[v_m(x,t) - \chi_m v_{m-1}(x,t)] = \hbar_2 H_2 R_{2,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}), \quad (26)$$

$$L[w_m(x,t) - \chi_m w_{m-1}(x,t)] = \hbar_3 H_3 R_{3,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}), \quad (27)$$

$$L[p_m(x,t) - \chi_m p_{m-1}(x,t)] = \hbar_4 H_4 R_{4,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}), \quad (28)$$

subject to the initial conditions$u_m(x,0)=0,\ v_m(x,0)=0, w_m(x,0)=0,\ p_m(x,0)=0$,
where

$$R_{1,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}) = \frac{\partial u_{m-1}(x,t)}{\partial t} - \frac{\partial v_{m-1}(x,t)}{\partial x}, \quad (29)$$

$$R_{2,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}) = \frac{\partial v_{m-1}(x,t)}{\partial t} + \frac{\partial^3 v_{m-1}(x,t)}{\partial x^3} + 6\sum_{j=0}^{m-1} u_j(x,t)\frac{\partial v_{m-1-j}(x,t)}{\partial x} + 6\sum_{j=0}^{m-1} v_j(x,t)\frac{\partial u_{m-1-j}(x,t)}{\partial x} - 6\frac{\partial}{\partial x}\left(\sum_{j=0}^{m-1} w_j(x,t)p_{m-1-j}(x,t)\right), \quad (30)$$

$$R_{3,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1}) = \frac{\partial w_{m-1}(x,t)}{\partial t} - \frac{\partial^3 w_{m-1}(x,t)}{\partial x^3} - 3\sum_{j=0}^{m-1} u_j(x,t)\frac{\partial w_{m-1-j}(x,t)}{\partial x}, \quad (31)$$

$$R_{4,m}(u_{m-1},v_{m-1}) = \frac{\partial p_{m-1}(x,t)}{\partial t} - \frac{\partial^3 p_{m-1}(x,t)}{\partial x^3} - 3\sum_{j=0}^{m-1} u_j(x,t)\frac{\partial p_{m-1-j}(x,t)}{\partial x}. \quad (32)$$

For simplicity, we choose $H_i = 1$and$\hbar_i = h, \quad i = 1,2,3,4$.
Obviously, the solution of the m-th order deformation equations for$m \geq 1$becomes

$$u_m(x,t) = \chi_m u_{m-1}(x,t) + hL^{-1}[R_{1,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1})], \quad (33)$$

$$v_m(x,t) = \chi_m v_{m-1}(x,t) + hL^{-1}[R_{2,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1})], \quad (34)$$

$$w_m(x,t) = \chi_m w_{m-1}(x,t) + hL^{-1}[R_{3,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1})], \quad (35)$$
$$p_m(x,t) = \chi_m p_{m-1}(x,t) + hL^{-1}[R_{4,m}(u_{m-1},v_{m-1},w_{m-1},p_{m-1})]. \quad (36)$$

We choose the initial guesses of the solutionsas

$$u_0(x,t) = u(x,0),\ v_0(x,t) = v(x,0),\ w_0(x,t) = w(x,0),\ p_0(x,t) = p(x,0). \quad (37)$$

From now onwards, throughout we will use $\sinh[mx+\gamma t]^n$ for $(\sinh(mx+\gamma t))^n$ and $\cosh[mx+\gamma t]^n$ for $(\cosh(mx+\gamma t))^n$.
We, therefore, successively obtain the various approximations as follows:

$$u_0(x,t) + u_1(x,t) = \left\{-\frac{c}{3} + \frac{2d^2}{3} - 4cd^3 ht\,\mathrm{sech}[dx]^2\tanh[dx] - 2d^2\tanh[dx]^2\right\},$$

$$v_0(x,t)+v_1(x,t)=\Big\{-\frac{1}{2}cd^2\text{sech}[dx]^2+\frac{3}{4}e\text{sech}[dx]^2+\frac{1}{2}cd^2\cosh[3dx]\text{sech}[dx]^3+\frac{1}{4}e\cosh[3dx]\text{sech}[dx]^3+4c^2d^3ht\text{sech}[dx]^2\tanh[dx]\Big\},$$

$$w_0(x,t)+w_1(x,t)=\Big\{\frac{1}{2}a\text{sech}[dx]^2+bcdht\text{sech}[dx]^2+\frac{1}{2}a\cosh[2dx]\text{sech}[dx]^2+\frac{1}{2}b\text{sech}[dx]^2\sinh[2dx]\Big\},$$

$$p_0(x,t)+p_1(x,t)=\Big\{\frac{ac^2d^2\text{sech}[dx]^2}{2b^2}+\frac{2acd^4\text{sech}[dx]^2}{b^2}+\frac{ad^2e\text{sech}[dx]^2}{b^2}-\frac{c^3d^3ht\text{sech}[dx]^2}{b}-\frac{4c^2d^5ht\text{sech}[dx]^2}{b}-\frac{2cd^3eht\text{sech}[dx]^2}{b}+\frac{ac^2d^2\cosh[2dx]\text{sech}[dx]^2}{2b^2}+\frac{2acd^4\cosh[2dx]\text{sech}[dx]^2}{b^2}+\frac{ad^2e\cosh[2dx]\text{sech}[dx]^2}{b^2}-\frac{c^2d^2\text{sech}[dx]^2\sinh[2dx]}{2b}-\frac{2cd^4\text{sech}[dx]^2\sinh[2dx]}{b}-\frac{d^2e\text{sech}[dx]^2\sinh[2dx]}{b}\Big\},$$

$$u_0(x,t)+u_1(x,t)+u_2(x,t)=\Big\{-\frac{c}{3}+\frac{2d^2}{3}-4c^2d^4h^2t^2\text{sech}[dx]^4+2c^2d^4h^2t^2\cosh[2dx]\text{sech}[dx]^4-8cd^3ht\text{sech}[dx]^2\tanh[dx]-4cd^3h^2t\text{sech}[dx]^2\tanh[dx]-2d^2\tanh[dx]^2\Big\},$$

$$v_0(x,t)+v_1(x,t)+v_2(x,t)=\Big\{-\frac{1}{4}cd^2\text{sech}[dx]^4+\frac{3}{8}e\text{sech}[dx]^4+4c^3d^4h^2t^2\text{sech}[dx]^4+\frac{1}{2}e\cosh[2dx]\text{sech}[dx]^4-2c^3d^4h^2t^2\cosh[2dx]\text{sech}[dx]^4+\frac{1}{4}cd^2\cosh[4dx]\text{sech}[dx]^4+\frac{1}{8}e\cosh[4dx]\text{sech}[dx]^4+4c^2d^3ht\text{sech}[dx]^4\sinh[2dx]+2c^2d^3h^2t\text{sech}[dx]^4\sinh[2dx]\Big\},$$

$$w_0(x,t)+w_1(x,t)+w_2(x,t)=\{\frac{3}{4}a\text{sech}[dx]^2+2bcdht\text{sech}[dx]^2+bcdh^2t\text{sech}[dx]^2+\frac{1}{4}a\cosh[3dx]\text{sech}[dx]^3+\frac{1}{2}b\text{sech}[dx]^2\tanh[dx]-bc^2d^2h^2t^2\text{sech}[dx]^2\tanh[dx]+\frac{1}{2}b\cosh[2dx]\text{sech}[dx]^2\tanh[dx]\},$$

$$p_0(x,t)+p_1(x,t)+p_2(x,t)=\{\frac{3ac^2d^2\text{sech}[dx]^2}{4b^2}+\frac{3acd^4\text{sech}[dx]^2}{b^2}+\frac{3ad^2e\text{sech}[dx]^2}{2b^2}-\frac{2c^3d^3ht\text{sech}[dx]^2}{b}-\frac{8c^2d^5ht\text{sech}[dx]^2}{b}-\frac{4cd^3eht\text{sech}[dx]^2}{b}-\frac{c^3d^3h^2t\text{sech}[dx]^2}{b}-\frac{4c^2d^5h^2t\text{sech}[dx]^2}{b}-\frac{2cd^3eh^2t\text{sech}[dx]^2}{b}+\frac{2cd^4\text{sech}[dx]^2\tanh[dx]}{b}\frac{ac^2d^2\cosh[3dx]\text{sech}[dx]^3}{4b^2}+\frac{acd^4\cosh[3dx]\text{sech}[dx]^3}{b^2}+\frac{ad^2e\cosh[3dx]\text{sech}[dx]^3}{2b^2}-\frac{c^2d^2\text{sech}[dx]^2\tanh[dx]}{2b}-\frac{d^2e\text{sech}[dx]^2\tanh[dx]}{b}+\frac{c^4d^4h^2t^2\text{sech}[dx]^2\tanh[dx]}{b}+\frac{4c^3d^6h^2t^2\text{sech}[dx]^2\tanh[dx]}{b}+\frac{2c^2d^4eh^2t^2\text{sech}[dx]^2\tanh[dx]}{b}-\frac{c^2d^2\cosh[2dx]\text{sech}[dx]^2\tanh[dx]}{2b}-\frac{2cd^4\cosh[2dx]\text{sech}[dx]^2\tanh[dx]}{b}-\frac{d^2e\cosh[2dx]\text{sech}[dx]^2\tanh[dx]}{b}\},$$

and so on. Solving the above set of equations we obtain $u_i(x,t)$, $v_i(x,t)$, $w_i(x,t)$, $p_i(x,t) (i=1,2)$. Since the other computed terms$u_i(x,t), v_i(x,t), w_i(x,t)$, $p_i(x,t)$ $i=3,4$ ...involve very large expressions, we have not written these terms here. However, we have obtained five term approximate solutions in this paper, and they are given by

$$u(x,t)=\sum_{i=0}^{4}u_i(x,t), \tag{38}$$

$$v(x,t)=\sum_{i=0}^{4}v_i(x,t), \tag{39}$$

$$w(x,t)=\sum_{i=0}^{4}w_i(x,t), \tag{40}$$

$$p(x,t)=\sum_{i=0}^{4}p_i(x,t). \tag{41}$$

4. Convergence Analysis and Numerical Solutions

The series solutions of the functions$u(x,t)$, $v(x,t)$, $w(x,t)$and$p(x,t)$ are given in (38)-(41). The convergence of these series and rate of the approximation for the homotopy analysis method strongly depends upon the value of the auxiliary parameter h, also known as convergence control parameter. In general, by means of the so called h-curve, it is straight forward to choose a proper value of h to control the convergence of the approximation series. To find the range of admissible values of h, h-curves of$u_{tt}(1,0)$, $v_{tt}(1,0)$, $w_{tt}(1,0)$and$p_{tt}(1,0)$obtained by the four term approximation of the HAM are plotted in Figure 1. From this figure, the valid regions of h correspond to the line segment nearly parallel to the horizontal axis. We should note that by choosing a good enough initial guess and good enough auxiliary linear operator, one can get accurate approximations by only a few terms. However, even if the initial guess and auxiliary linear operator are not good enough but reasonable, one can still get convergent results by properly choosing the auxiliary parameter h.

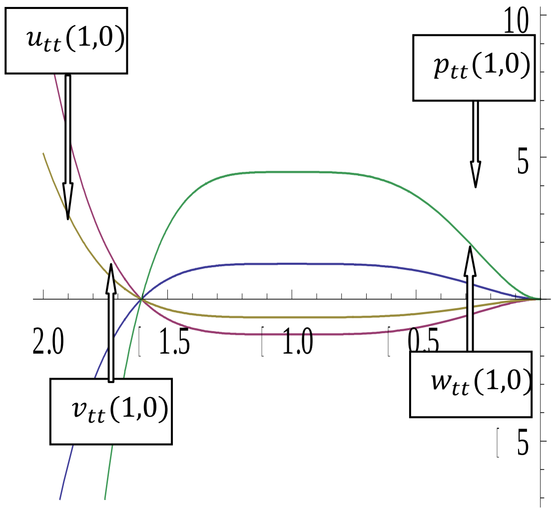
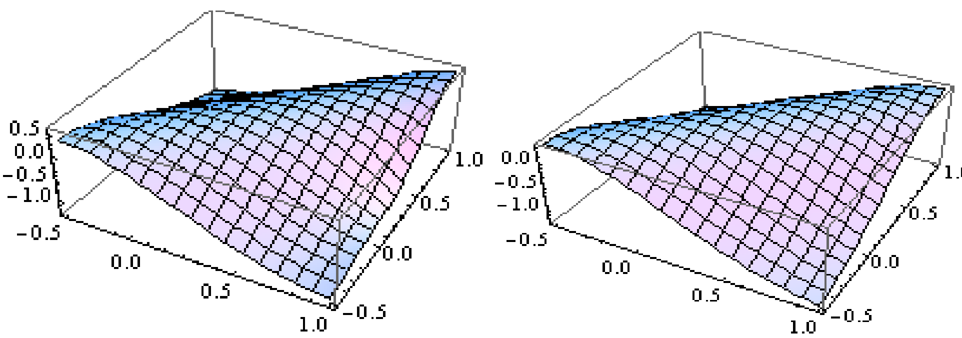


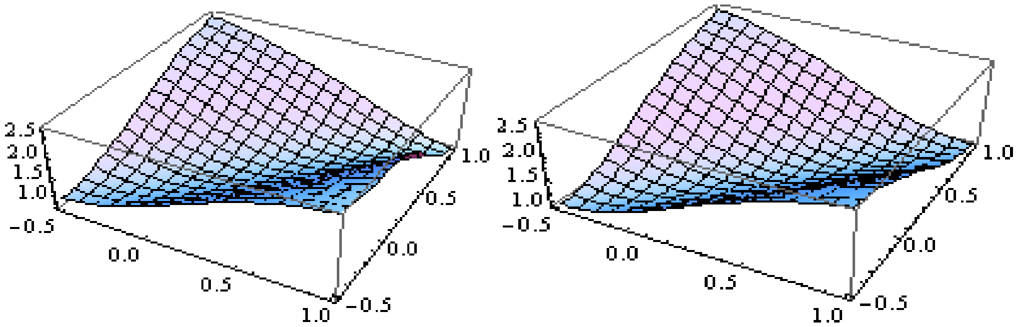
Figure 1: The h curves of $u_{tt}(1,0)$, $v_{tt}(1,0)$, $w_{tt}(1,0)$ and $p_{tt}(1,0)$obtained by the five term approximation of the HAM corresponding to $a = b = c = d = e = 1$.

To demonstrate the efficiency of the HAM for this problem, we compare the approximate solutions u(x, t), v(x, t), w(x, t) and p(x, t), with the exact solutions (6). Here all the analysis has been made for $a = b = c = d = e = 1$.

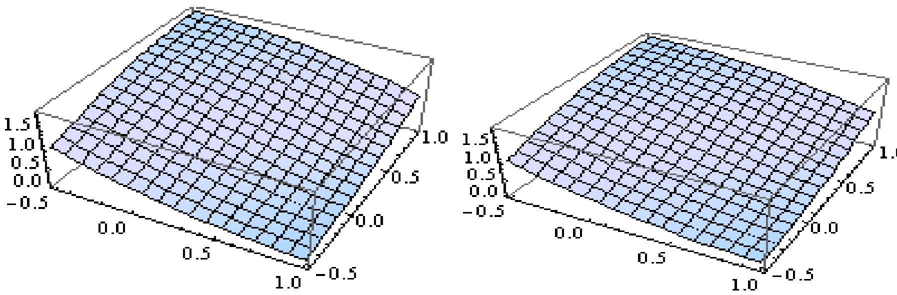
In Figure 1 the convergence region may be taken where all the h-curves are parallel or nearly parallel to x-axis. We have chosen$h = -0.7$ as this is the most appropriate value of h for this problem. The comparison of the computed value of the fifth order approximationby HAM with the exact solutions is presented inthe following figures and tables:



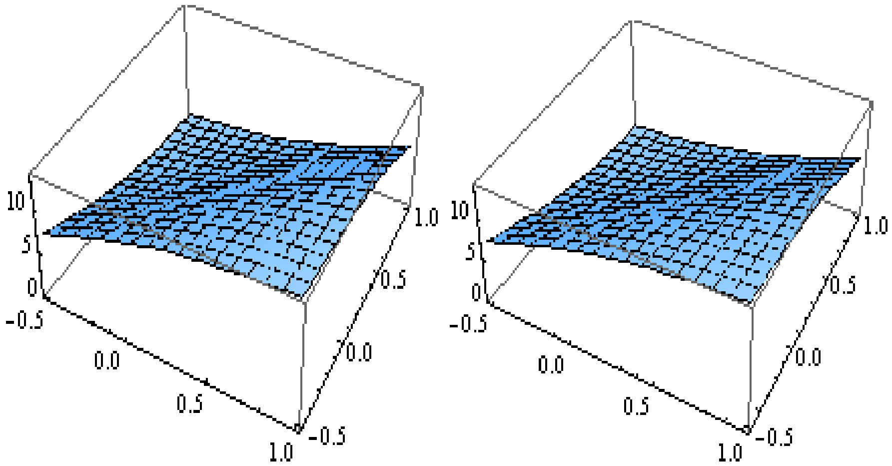
(a) (b)



(c) (d)



(e) (f)



(g) (h)

Figure 2. **(a)** u(x,t) computed by HAM, **(b)** u(x,t) exact, **(c)** v(x,t) computed by HAM, **(d)** v(x,t) exact, **(e)** w(x,t) computed by HAM, **(f)** w(x,t) exact, **(g)** p(x,t) computed by HAM, **(h)** p(x,t) exact.

Table 1: Absolute errors for u(x, t) obtained by the 5th-order approximate solution of the HAM for $h = -0.7$.

x	0.2	0.4	0.6	0.8	1.0
5	2.08871E-06	1.32235E-05	4.89777E-05	1.39951E-04	3.41506E-4
6	2.82817E-07	1.79110E-06	6.63638E-06	1.89712E-05	4.63175E-5
7	3.82777E-08	2.42426E-07	8.98283E-07	2.56804E-06	6.27022E-6
8	5.18037E-09	3.28093E-08	1.21572E-07	3.47557E-07	8.48616E-07
9	7.01088E-10	4.44026E-09	1.64530E-08	4.70370E-08	1.14848E-07
10	9.48823E-11	6.00925E-10	2.22668E-09	6.36577E-09	1.5543E-08

Table 2: Absolute errors for v(x, t) obtained by the 5th-order approximate solution of the HAM for $h = -0.7$.

x	0.2	0.4	0.6	0.8	1.0
5	2.08871E-06	1.32235E-05	4.89777E-05	1.39951E-04	3.41506E-4
6	2.82817E-07	1.79110E-06	6.63638E-06	1.89712E-05	4.63175E-5
7	3.82777E-08	2.42426E-07	8.98283E-07	2.56804E-06	6.27022E-6
8	5.18037E-09	3.28093E-08	1.21572E-07	3.47557E-07	8.48616E-07
9	7.01088E-10	4.44026E-09	1.64530E-08	4.70370E-08	1.14848E-07
10	9.48823E-11	6.00925E-10	2.22668E-09	6.36577E-09	1.5543E-08

Table 3: Absolute errors for w(x, t) obtained by the 5th-order approximate solution of the HAM for $h = -0.7$.

x	0.2	0.4	0.6	0.8	1.0
5	5.22328E-07	3.30747E-06	1.22529E-05	3.50207E-05	8.54830E-05
6	7.07070E-08	4.47804E-07	1.65925E-06	4.74341E-06	1.15813E-05
7	9.56947E-09	6.06070E-08	2.24574E-07	6.42022E-07	1.56759E-06
8	1.29509E-09	8.20233E-09	3.03931E-08	8.68896E-08	2.12155E-07
9	1.75273E-10	1.11007E-09	4.11326E-09	1.17592E-08	2.87121E-08
10	2.37212E-11	1.50232E-10	5.56670E-10	1.59144E-09	3.88576E-09

Table 4: Absolute errors for p(x, t) obtained by the 5th-order approximate solution of the HAM for $h = -0.7$.

x	0.2	0.4	0.6	0.8	1.0
5	3.65630E-06	2.31523E-05	8.57705E-05	2.45145E-04	5.98381E-04
6	4.94949E-07	3.13462E-06	1.16148E-05	3.32039E-05	8.10693E-05
7	6.69863E-08	4.24249E-07	1.57201E-06	4.49416E-06	1.09731E-05
8	9.06565E-09	5.74163E-08	2.12751E-07	6.08227E-07	1.48508E-06
9	1.22690E-10	7.77046E-09	2.87928E-08	8.23147E-08	2.00985E-07
10	1.66043E-10	1.05162E-09	3.89668E-09	1.11401E-08	2.72003E-08

A very good agreement between the results of the HAM and exact solutions is observed, which confirms the validity of the HAM.

5. Conclusion

In this research work, we have successfully applied the homotopy analysis method(HAM) to obtain an approximate analytic solution of the Ito system of partial differential equations arising in mathematical Physics. The solution is found in the form of a convergent series with easily computed terms. The results obtained are compared with the exact solutionsshowing a very good agreement even using only few terms of the recursive relations. Different from all previous analytic methods, one can ensure the convergence of series solution of strongly nonlinear problems by means of choosing a proper value of the convergence-control parameter h. This is an obvious advantage of the HAM.In general, this method provides highly accurate numerical solutions and can be applied to wide class of nonlinear problems. Homotopy analysis method does not require small or large parameters which are needed by the perturbation methods. Also the method avoids linearization and physically unrealistic assumptions. The results demonstrate reliability and efficiency of the homotopy analysis method.

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