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Research paper

A New Method for the Order Reduction of Multivariable Systems Using Bilinear Transformation and Time Moments Matching Technique

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Abstract

This paper proposes a new order reduction procedure for high order continuous-time MIMO systems. The denominator of the low order model is obtained using a Bilinear transformation whereas the Moment matching method is used to obtain the numerator. The reduced order system obtained by this method gives better approximation than some of existing methods.

Keywords: *Control Systems, Order Reduction, Multi Variable Systems*

1. Introduction:

Several order reduction methods for high order systems reduction are available in literature, Many familiar and important high order MIMO systems reduction techniques are proposed in literature [1-20]. In this paper an attempt is successfully made to suggest a procedure for the order reduction of high order MIMO systems. The procedure is computationally very simple, straight forward and based on application of Bilinear transformation and Time moments matching technique.

2. Proposed Method:

Consider the stable, n^{th} order, linear, continuous-time MIMO system given by

$$G_n(s)=\frac{\begin{bmatrix} N_{n1}(s) \\ N_{n2}(s) \end{bmatrix}}{D_n(s)}$$

Where the common denominator $D_n(s)$ is in the form of

$$D_n(s) = a_n s^n + a_{n-1}s^{n-1} + ... + a_1 s + a_0 \text{ and let}$$

$$N_1(s) = b_{1n-1}s^{n-1} + b_{1n-2}s^{n-2} .... + b_{10}$$

$$N_2(s) = b_{2n-1}s^{n-1} + b_{2n-2}s^{n-2} .... + b_{20}$$

$$G_1(s)=\frac{N_1(S)}{D_n(s)}=\frac{b_{1n-1}s^{n-1}+b_{1n-2}s^{n-2}........+b_{10}}{a_n s^n+a_{n-1}s^{n-1}+........+a_1 s+a_0}$$

$$G_2(s)=\frac{N_2(S)}{D_n(s)}=\frac{b_{2n-1}s^{n-1}+b_{2n-2}s^{n-2}........+b_{20}}{a_n s^n+a_{n-1}s^{n-1}+........+a_1 s+a_0}$$

In general, consider

$$G(s) = \frac{b_{n-1}s^{n-1}+\cdots......b_1 s^1+b_0}{a_n s^n+\cdots............a_1 1+a_0}$$

Then the k^{th} order reduced model is proposed as

$$R_k(s) = \frac{d_{k-1}s^{k-1}+\cdots..........d_1 s+d_0}{s^k+e_{k-1}s^{k-1}+\cdots.........e_1 s+e_0}$$

For $R_K(s)$ to be stable, Bilinear transformation is applied on G(s) and hence the poles of $R_k(s)$ will be in the unit circle.
Applying transformation to G(s) with

$$s = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$$

where T is the sampling time interval = 2.

$$H(z) = \frac{\beta_n z^n+\cdots............\beta_1 z+\beta_0}{\alpha_n z^n+\cdots............\alpha_1+\alpha_0}$$

The Markov parameters m_i*(i=0,1,……)* are obtained as

$$m_i = \frac{1}{\alpha_n}\left\{\beta_{n-i} - \sum_{j=0}^{i-1} m_j \alpha_{n+j-i}\right\};$$

$\beta_j = 0$; for $j < 0$ where $H(z) = \frac{m_i}{z^i}$;

Step3:

Let

$$M=\begin{bmatrix} m_k & m_{k-1}\dots\dots & m_2 & m_1 \\ m_{k+1} & m_k\dots\dots\dots & m_3 & m_2 \\ m_{k+2} & m_{k+1}\dots\dots & m_4 & m_3 \\ \dots\dots & \dots\dots & \dots\dots\dots\dots & \\ \dots\dots\dots & \dots\dots & \dots\dots\dots\dots & \end{bmatrix}$$

$$m=\begin{bmatrix} -m_{k+1} \\ -m_{k+2} \\ -m_{k+3} \\ \vdots \end{bmatrix}$$

For a stable system, $m_i \to 0$ as $i\to\infty$, so that M and m can be truncated at a suitable point ($i > 2k$). The least-squares solution is given by

$\delta = [M^T M]^{-1} M^T m$

which in turn specifies the pole locations in the z-domain of the reduced-order model.

$\delta = [\delta_{k-1} \quad \delta_{k-2}\dots\dots\delta_2 \quad \delta_1]^T$

is the vector of the k^{th} order reduced denominator.

The reduced order denominator is defined as

$D_k(z) = z^k + \sum_{i=0}^{k-1} \delta_i z^i$

Retaining the Markov parameters of $H(z)$ in $H_k(z)$, $N_k(z)$ is obtained

$H_k(z) = N_k(z)/D_k(z)$;

Applying the inverse bilinear transformation,

$z = \frac{1+\frac{1}{2}sT}{1-\frac{1}{2}sT}$

$$G_k(s)=\frac{\begin{bmatrix} N_{k1}(s) \\ N_{k2}(s) \end{bmatrix}}{D_k(s)}$$

EXAMPLE 1:

Consider the original system defined by

$$G_n(s)=\frac{\begin{bmatrix} N_{n1}(s) \\ N_{n2}(s) \end{bmatrix}}{D_n(s)}=\frac{\begin{bmatrix} s^3+10s^2+40s+24 \\ s^3+7s^2+24s+24 \end{bmatrix}}{s^4+10s^3+35s^2+50s+24}$$

Where

$$G_1(s)=\frac{N_1(S)}{D_n(s)}=\frac{b_{1n-1}s^{n-1}+b_{1n-2}s^{n-2}\dots\dots\dots+b_{10}}{a_n s^n+a_{n-1}s^{n-1}+\dots\dots\dots+a_1 s+a_0}$$

(I[st] Output)

$$=\frac{s^3+10s^2+40s+24}{s^4+10s^3+35s^2+50s+24}$$

$$=\frac{(s+2.7306-2.853i)(s+2.7306+2.853i)(s+1.5388)}{(s+4)(s+3)(s+2)(s+1)}$$

Similarly,

$$G_2(s)=\frac{N_2(S)}{D_n(s)}=\frac{b_{2n-1}s^{n-1}+b_{2n-2}s^{n-2}\dots\dots\dots+b_{20}}{a_n s^n+a_{n-1}s^{n-1}+\dots\dots\dots+a_1 s+a_0}$$

(2[nd] Output)

$$=\frac{s^3+7s^2+24s+24}{s^4+10s^3+35s^2+50s+24}$$

$$=\frac{(s+4.639-3.433i)(s+4.639+3.433i)(s+0.7204)}{(s+4)(s+3)(s+2)(s+1)}$$

It is proposed to obtain a Second order reduced model for the original MIMO system G(s) using the proposed method.

3. Application of Proposed reduction procedure:

For I[st] Output:

Using bilinear transformation on $G_1(s)$ with

$s = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$

where T is the sampling time interval = 2.

$H_1(z) = \frac{56z^4+142z^3+130z^2+50z+8}{120z^4+172z^3+80z^2+12z}$

For k=2, the Markov parameters m_i for (i>2k) of H(z) are calculated using the long division method, where k is the order of reduced transfer function.

They are obtained as:

$m_0 = 0.467$; m_1=0.51444; m_2=0.03489;

m_3=-0.0229; m_4=0.01815; m_5= -0.01423

Then the least-squares solution of the linear set is obtained as

$\delta = \begin{bmatrix} -0.066105 \\ -0.114033 \end{bmatrix}$

Then, the reduced order denominator is

$D_k(z) = z^k + \sum_{i=0}^{k-1} \delta_i z^i$.

For k= 2 , $D_2(z)= z^2 - 0.066105z - 0.114033$;

For $z = \frac{1+s}{1-s}$;

$D_2(s) = s^2 + 2.34024s + 0.861145$

The reduced order numerator is obtained by matching the original system time moments to the reduced system time moments given by

$T_1 = 1$; T_2=1.0833; and

b_0=0.861145 ; b_1=1.4073;

Using the proposed method, the second order system (I[st] Output) is

$$R_{21}(s)=\frac{1.4073s+0.861145}{s^2+2.3402s+0.861145}$$

$$=\frac{(s+0.615)}{(s+1.8827)(s+0.4573)}$$

Similarly,

For II[nd] Output :

Using the proposed method, the second order system (II[nd] Output) is

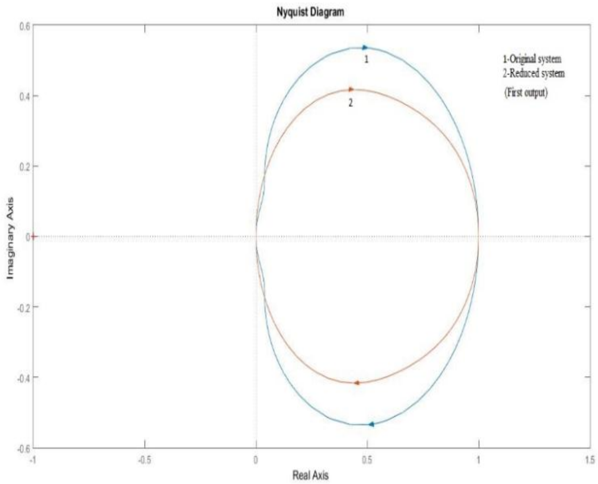
$$R_{22}(s)=\frac{2.136s+0.861145}{s^2+2.3402s+0.861145}$$

$$=\frac{(s+0.40)}{(s+1.8827)(s+0.4573)}$$

the second order system is obtained as

$$R_2(s)=\frac{\begin{bmatrix} 1.4073s+0.861145 \\ 2.136s+0.861145 \end{bmatrix}}{s^2+2.3402s+0.861145}$$

Fig.1 and Fig.2 depict the frequency responses and step responses of original MIMO system G(s) and its MIMO Model $R_2(s)$ obtained by the proposed reduction method.



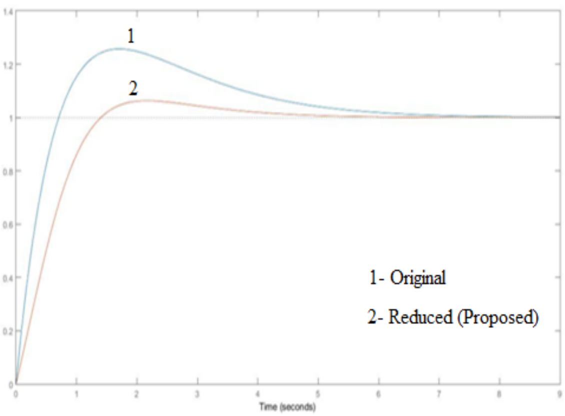
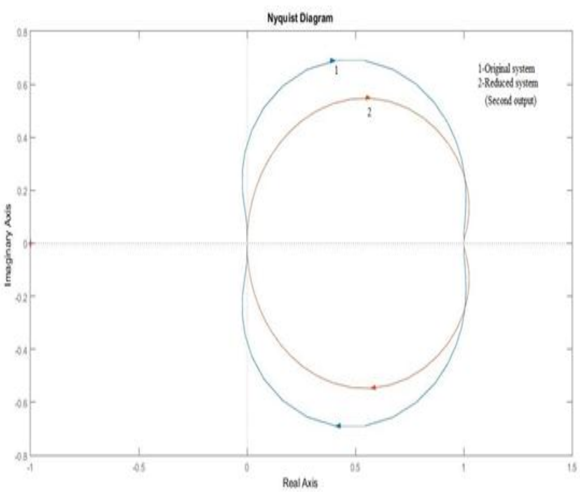


Fig.1 (Ist Output)



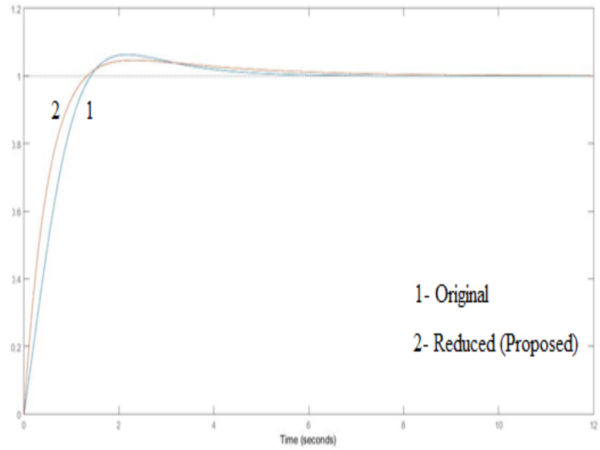


Fig.2 (IInd Output)

3. Comparison with other methods :

EXAMPLE 2:
Consider the original system defined by

$$G_n(s)=\frac{\begin{bmatrix} N_{n1}(s) \\ N_{n2}(s) \end{bmatrix}}{D_n(s)}=\frac{\begin{bmatrix} 58s^3+984s^2+2700s+4400 \\ 36s^3+924s^2+2660s+7520 \end{bmatrix}}{2s^4+36s^3+204s^2+360s+240}$$

It is proposed to obtain a Second order reduced model for the original MIMO system G(s) using the proposed method and compare it with the models obtained by two other existing methods [3 , 6].

4. Application of Proposed reduction procedure:

$$G_1(s)=\frac{58s^3+984s^2+2700s+4400}{2s^4+36s^3+204s^2+360s+240}$$

$$G_2(s)=\frac{36s^3+924s^2+2660s+7520}{2s^4+36s^3+204s^2+360s+240}$$

Using bilinear transformation on $G_1(s)$ and $G_2(s)$with

$s = \frac{2}{T}\left(\frac{Z-1}{Z+1}\right)$

where T is the sampling time interval = 2.

$H_1(z) = \frac{8142z^4+22884z^3+24432z^2+12316z+2626}{842z^4+1600z^3+1044z^2+304z+50}$

and

$H_2(z) = \frac{16460z^4+40668z^3+43292z^2+24832z+5748}{842z^4+1600z^3+1044z^2+304z+50}$

Then the second order MIMO system is obtained as

$$R_2(s)=\frac{\begin{bmatrix} -6.4+40.82 \\ -29.1s+71.08 \end{bmatrix}}{s^2+1.62s+2.269}$$ (proposed method)

$$R^{|}_2(s) = \frac{\begin{bmatrix} 17.4s + 4.2 \\ 28s + 7.3 \end{bmatrix}}{s^2 + 1.156s + 0.241}$$ **(CFE Method)[3]**

$$R^{||}_2(s) = \frac{\begin{bmatrix} 4.824s + 21.82 \\ 4.529s + 37.30 \end{bmatrix}}{s^2 + 1.785s + 1.1903}$$ (**R Prasad Method**) **[6]**

Fig.3 and Fig.4 depict the comparison of frequency responses and step responses of original MIMO system G(s) and its MIMO Model $R_2(s)$ obtained by the proposed reduction method and CFE method [3] and R.Prasad method [6].

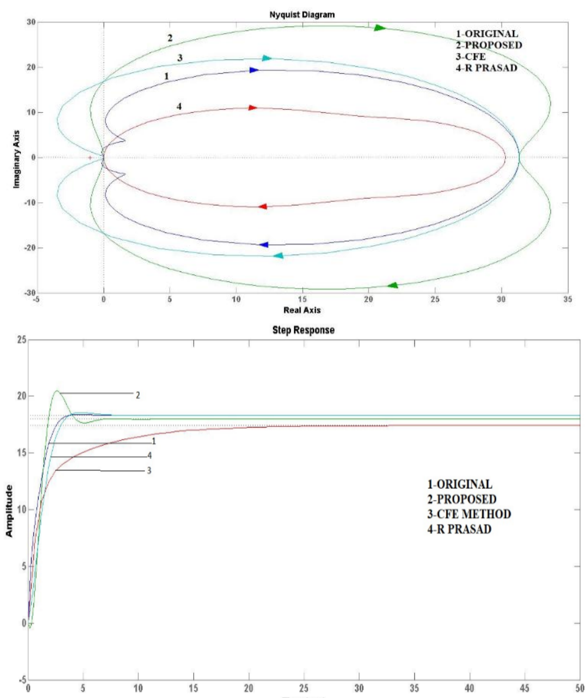
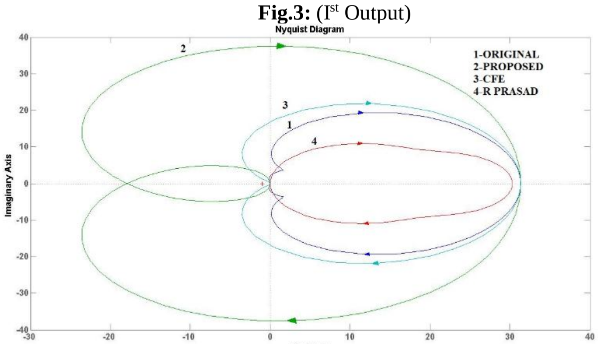


Fig.3: (I[st] Output)



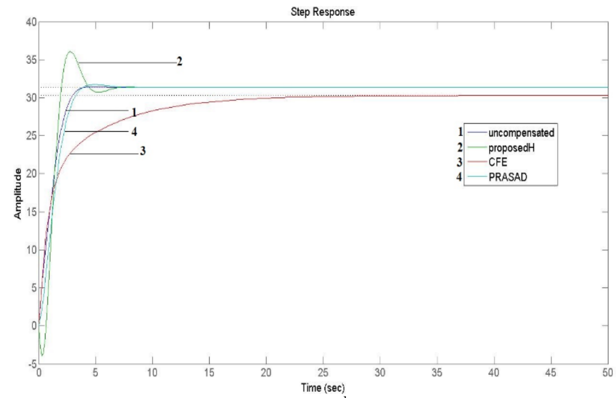


Fig.4: (II[nd] Output)

5. Conclusion:

A new stability-preserving method for MIMO systems reduction is presented. The method is based on application of Bilinear transformation and Moment matching technique. The original system characteristics are more accurately approximated in the low order models than in the case of many existing methods. The proposed procedure is explained through typical numerical examples. The proposed method is also compared with two other existing methods.

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