



# A multi-product MPS optimization under risk

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## Abstract

In this work, an MPS optimization model is developed to maximize the expected profit using GA under demand uncertainty. The model is built for @RiskOptimizer in MS Excel. The customer demands have been assumed to follow the normal distribution of a standard deviation related to their mean values with a ratio called demand variability. The effects of demand variability on the profit mean, profit variation and the processing time have been studied.

**Keywords:** MPS; Genetic Algorithm; @ RiskOptimizer.

## 1. Introduction

Genetic algorithm (GA) is used in optimization problems for the generation of high-quality solutions. GA approaches are developed to find the optimal or near-optimal solution. Holland [1], Michalewicz [2], Gen and Chneg [3], Davis [4], and Goldberg [5] gave a detailed discussion on GA in their books.

MPS problems have been solved using Differential Evolution technique [6]. Al-Ashhab, M. S. et. Al. [7] developed a multi-objective and multi-product MPS optimization model using the lexicographic procedure to maximize the total profit for a single product chain [8].

In this paper, a robust optimization MPS model is developed to maximize the total expected profit. The developed model has been solved using @RiskOptimizer and formulated in MS Excel. This work is an extension of the work done by M. Al-Ashhab and H. Fadag [9]

## 2. Model formulation

### Sets:

S, C, and P: sets of suppliers, customers, and products.

T: number of planning periods.

### Parameters:

$F_i$ : fixed cost,

$DEMAND_{cpt}$ : demand of customer c from product p in period t,

$IIf_p$ : the initial inventory of product p,

$FIIf_p$ : the final stock of product p,

$P_{pct}$ : the unit price of product p at customer c in period t,

$W_p$ : the weight of product p,

$MH_p$ : processing hours for product p,

$D_{ij}$ : distance facilities i and j,

$CAP_{st}$ : supplier capacity in period t,

$CAPM_{fi}$ : raw material store,

$CAPH_{fi}$ : manufacturing capacity of the factory in hours,

$CAPFS_{fi}$ : final product storing capacity,

MatCost: material cost,

$MC_{fi}$ : manufacturing cost,

$MH_p$ : Required processing hours for product p,

NUCCf: non-utilized capacity cost per hour,

SCPU<sub>p</sub>: shortage cost per unit per period,

HC: holding cost per unit weight per period at the factory store,

$B_s$ : batch size from supplier s,

$Bf_p$ : batch size from the factory for product p,

TC: transportation cost per unit per kilometre,

**Decision Variables:**

- $Q_{ijpt}$ : number of batches transported from facility  $i$  to  $j$  for product  $p$  in period  $t$ ,  
 $Iff_{pt}$ : number of batches transported to the factory store for product  $p$  in period  $t$ ,  
 $Ifc_{cpt}$ : number of batches transported from the store to customer  $c$  for product  $p$  in period  $t$ ,  
 $Rf_{pt}$ : the remaining inventory of the period  $t$  at the store of the factory for product  $p$ .  
 $CSL_c$ : Customer Service Level of customer  $c$ .

The profit is calculated by subtracting the total cost from the total revenue given in Equation 1.

$$\text{Total Revenue} = \sum_{c \in C} \sum_{p \in P} \sum_{t \in T} (Q_{fc_{cpt}} + Ifc_{cpt}) Bf_p P_{pst} \quad (1)$$

The cost elements are equated in Equation (2-8)

$$\text{Fixed costs} = Ff \quad (2)$$

$$\text{Material cost} = \sum_{s \in S} \sum_{t \in T} Q_{st} B_s \text{MatCost}_{st} + \sum_{p \in P} Iff_p W_p \text{MatCost}_{st} - \sum_{p \in P} Ff_p W_p \text{MatCost}_{st} \quad (3)$$

$$\begin{aligned} \text{Manufacturing costs} = & \sum_{c \in C} \sum_{p \in P} \sum_{t \in T} Q_{cpt} Bf_p MH_p Mc_c + \sum_{p \in P} \sum_{t \in T} Iff_{pt} Bf_p MH_p Mc_c + \\ & \sum_{p \in P} Iff_p MH_p Mc_c - \sum_{p \in P} Ff_p MH_p Mc_c \end{aligned} \quad (4)$$

$$\text{Non - Utilized capacity cost} = \sum_{t \in T} ((CAPH_{ft}) L_f - \sum_{p \in P} \sum_{c \in C} (Q_{fc_{cpt}} Bf_p MH_p) - \sum_{p \in P} (Iff_{pt} Bf_p MH_p)) NUCCf \quad (5)$$

$$\text{Shortage cost} = \sum_{p \in P} \sum_{c \in C} (\sum_{t \in T} \sum_{i \in T} \text{DEMAND}_{cpt} - \sum_{i \in T} (Q_{icpt} + Ifc_{cpt}) Bf_p) SCPU_p \quad (6)$$

$$\text{Transportation costs} = \sum_{t \in T} \sum_{s \in S} Q_{st} B_s Tc D_{st} + \sum_{p \in P} \sum_{t \in T} \sum_{c \in C} (Q_{cpt} + Ifc_{cpt}) Bf_p W_p Tc D_{c} \quad (7)$$

$$\text{Holding cost} = \sum_{p \in P} Iff_{pt} Bf_p W_p HC + \sum_{p \in P} \sum_{t=1}^{T-1} Rf_{pt} Bf_p W_p HC \quad (8)$$

Constraints (9-13) ensure balancing of the factory and its store

$$\sum_{s \in S} Q_{sf_t} B_s = \sum_{c \in C} \sum_{p \in P} Q_{fc_{cpt}} Bf_p W_p + Iff_{pt} Bf_p W_p, \forall t \in T \quad (9)$$

$$Iff_{pt} Bf_p + Iff_p = Rf_{pt} Bf_p + \sum_{c \in C} Ifc_{cpt} Bf_p, \forall p \in P \quad (10)$$

$$Iff_{pt} Bf_p + Rf_{p(t-1)} Bf_p = Rf_{pt} Bf_p + \sum_{c \in C} Ifc_{cpt} Bf_p, \forall t \in 2 \rightarrow T, \forall p \in P \quad (11)$$

$$Rf_{pt} Bf_p = Ff_p, \forall p \in P \quad (12)$$

$$\begin{aligned} (Q_{fc_{cpt}} + Ifc_{cpt}) Bf_p & \leq \text{DEMAND}_{cpt} + \\ \sum_{i \in T} \text{DEMAND}_{cp(i-1)} - \sum_{d \in T} (Q_{fc_{cp(i-1)}} + Ifc_{cp(i-1)}) Bf_p, & \forall t \in T, \forall c \in C, \forall p \in P \end{aligned} \quad (13)$$

Constraints (14-17) ensure that the capacities of all facilities will not be exceeded.

$$Q_{sf_t} B_s \leq CAP_{st} L_s, \forall t \in T, \forall s \in S \quad (14)$$

$$\sum_{s \in S} Q_{sf_t} B_s \leq CAPM_{ft} L_t, \forall t \in T \quad (15)$$

$$(\sum_{c \in C} \sum_{p \in P} Q_{fc_{cpt}} Bf_p + \sum_{p \in P} Iff_{pt} Bf_p) MH_p \leq CAPH_{ft} L_t, \forall t \in T \quad (16)$$

$$\sum_{p \in P} Rf_{pt} Bf_p W_p \leq CAPFS_t L_t, \forall t \in T \quad (17)$$

Fig. 1 presents the model definition in @ RiskOptimizer in which, Cell CL24 contains the output. Cells J2:CL2 include variables. Cells CN32:CN91 and CP32:CP91 include both capacity and balancing constraints.

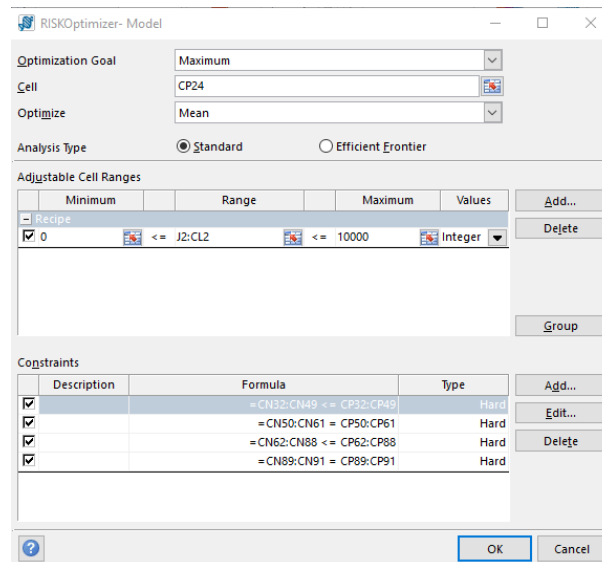


Fig. 1: Model Definition in @RiskOptimizer.

### 3. Computational results

In these experiments, the demands are assumed to follow the normal distribution that has two parameters; the mean and the standard deviation, the standard deviation is given as a ratio to the mean; this ratio of the standard deviation to the mean is referred to as the variability. For simplicity, the variability is assumed to be the same for all demands in the range of 0 up to 0.5 with a step of 0.05. Also, the level of confidence is assumed to be 0.95. The needs of each customer for all products in the three periods are 770, 590, and 300 items/period

Table 1: Model Parameters

Parameter	Value	Parameter	Value
Number of Periods	3	Supplier batch size	10
Number of products	3	Factory Batch size	1
Fixed costs (\$)	50,000	Holding cost per period (\$/kg)	5
Factory capacity in hours (hrs.)	12,000	The capacity of each supplier (Kg)	12,000
Weight of products 1, 2, 3 (Kg)	1,2,3	Transportation cost per Km per Kg (\$)	0.001
Price of Products	100,150, 200	Machining time of products 1, 2, 3 (hrs.)	1,2,3
Material Cost (\$/kg)	10	Capacity of Raw Material Store (Kg)	10,000
Manufacturing Cost (\$/hr.)	10	Capacity of Factory Store (Kg)	2,000
Initial Inventory of Products	50,100,150	Final Inventory of Products	100,150,200

#### 2.1. Analysis of results

Initially, the problem has been solved for zero variability (deterministic demand), and the solver has achieved the optimal solution of 1,482,026 \$ in 99 trial as shown in Fig. 2.

RISKOptimizer stops simulating scenarios in three manners; Trials, Time and or progress (see Fig. 3). Assigning number of trials stops the optimization process when this number of trials have been executed. And assigning a specific time stops the optimization after the given time has elapsed. While assigning the progress terminates optimization when the improvement in the target in a designated number of trials is less than the specified amount.

Solving the problem of 5% variability gave uncertain total profit that has been summarized in Table 2.

The output mean of the total profit has been affected by the demands of customers. The demand (Inputs) are ranked according to their effects on the overall profit and presented in Fig. 4 and tabulated in Table 3.

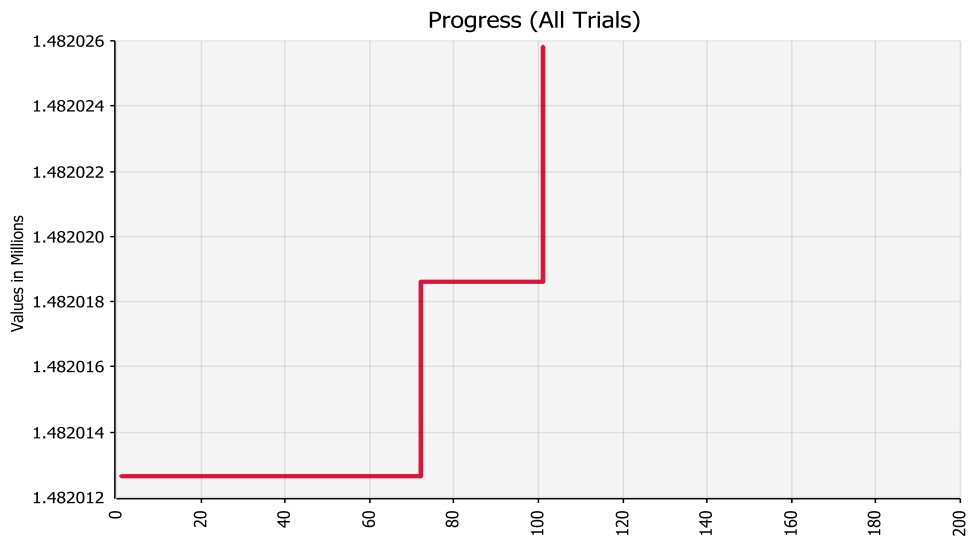


Fig. 2: Zero Variability Solution.

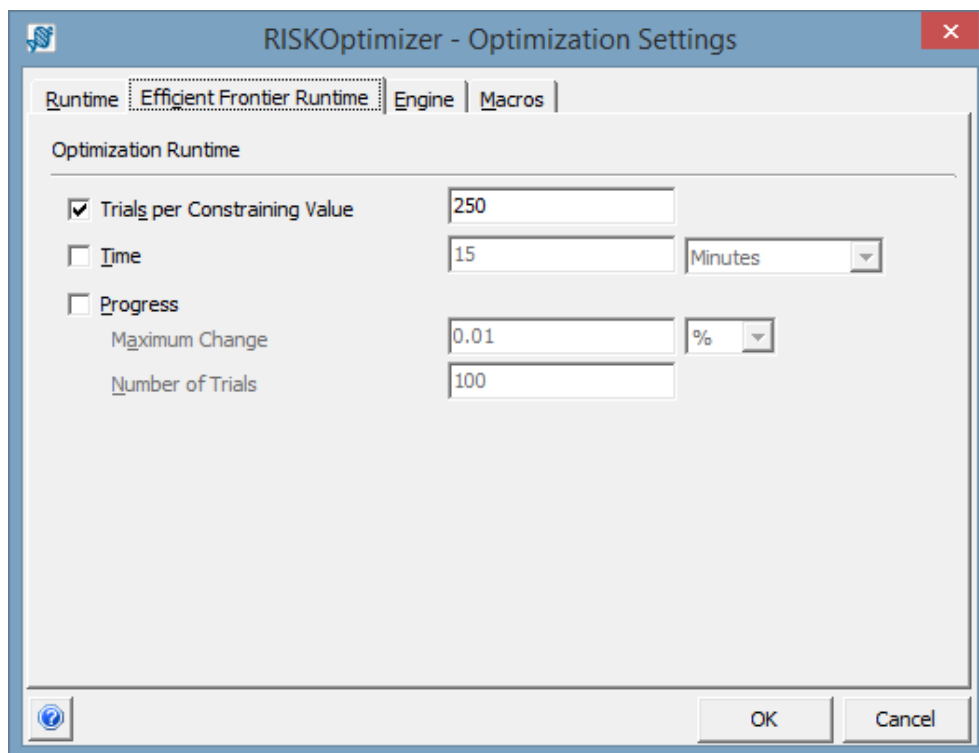


Fig. 3: Optimization Settings Dialog.

Table 2: Summary Statistics for the Resulted Profit

Statistics		Percentile	
Minimum	1,313,182	5%	1,319,619
Maximum	1,334,498	10%	1,320,128
Mean	1,326,105	15%	1,320,914
Std Dev	4,313	20%	1,322,077
Variance	18598304	25%	1,322,928
Skewness	-0.213426052	30%	1,323,847
Kurtosis	2.714774277	35%	1,324,338
Median	1,325,996	40%	1,325,011
Mode	1,325,305	45%	1,325,318
Left X	1,319,619	50%	1,325,996
Left P	5%	55%	1,326,943
Right X	1,332,776	60%	1,327,408
Right P	95%	65%	1,328,233
Diff X	13,157	70%	1,328,571
Diff P	90%	75%	1,329,064
#Errors	0	80%	1,329,536
Filter Min	Off	85%	1,330,775
Filter Max	Off	90%	1,332,007
#Filtered	0	95%	1,332,776

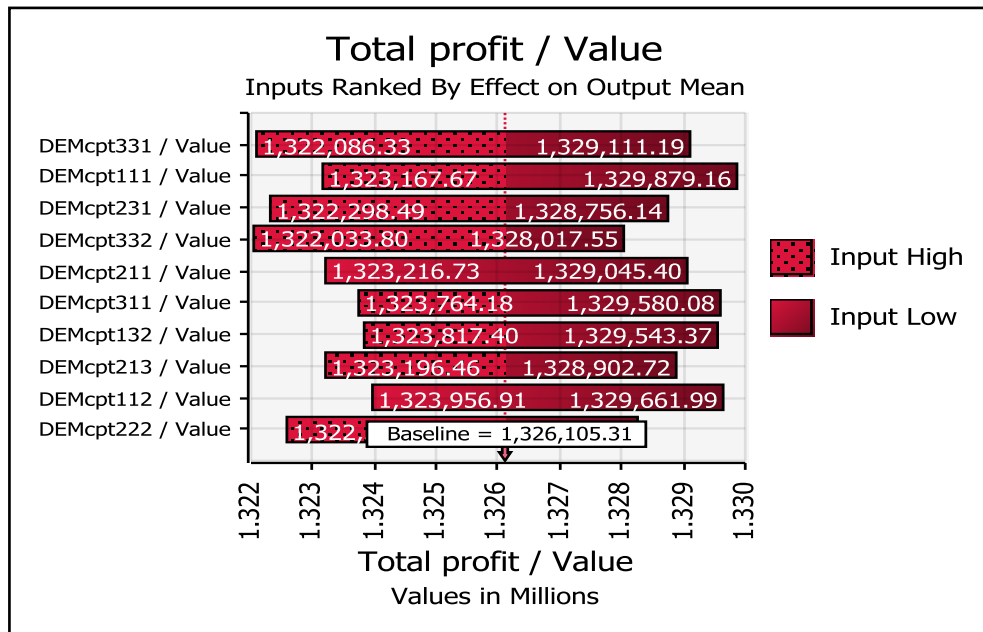


Fig. 4: Effect of Customer Demands on the Total Profit.

Table 3: Effect of Customer Demands on the Total Profit

Rank	Name	Lower	Upper
1	DEMcpt331	1,322,086	1,329,111
2	DEMcpt111	1,323,168	1,329,879
3	DEMcpt231	1,322,298	1,328,756
4	DEMcpt332	1,322,034	1,328,018
5	DEMcpt211	1,323,217	1,329,045
6	DEMcpt311	1,323,764	1,329,580
7	DEMcpt132	1,323,817	1,329,543
8	DEMcpt213	1,323,196	1,328,903
9	DEMcpt112	1,323,957	1,329,662
10	DEMcpt222	1,322,587	1,328,275

### 2.2. Effect of demand variability

The effect of demand variability has been studied by changing the demand variability from zero to 50% with a step of 5%. And the statistics or results are presented in

Table 4.

Increasing the demand variability decreases the expected profit as shown in Fig. 5 which declare the terrible effect of demand variation on the stability of the organization. Also, the increase of demand variability increases the standard deviation of the profit as shown in Fig. 6 which means that the difference between the minimum and maximum values of the profit increases as depicted in Fig. 7.

Table 4: Results Statistics

Exp. No.	Variability %	No. of Trials	Processing Time	Expected Profit	Goal Cell Statistics		
					Std. Dev.	Min.	Max.
1	0	99	0:01:19	1,482,026	-	1,482,026	1,482,026
2	5	467	0:02:36	1,326,105	4,313	1,313,182	1,334,498
3	10	165	0:01:24	1,025,981	8,034	1,005,362	1,056,936
4	15	492	0:03:03	999,088	12,407	973,370	1,032,920
5	20	492	0:02:15	889,361	14,450	849,281	921,705
6	25	120	0:01:18	283,505	18,215	235,612	325,911
7	30	495	0:02:17	760,305	22,212	695,792	814,226
8	35	485	0:02:23	292,535	27,766	208,979	376,057
9	40	468	0:02:17	294,095	32,123	207,724	370,564
10	45	498	0:02:07	36,892	42,829	(58,943)	154,338
11	50	488	0:02:10	147,051	40,343	51,151	241,366

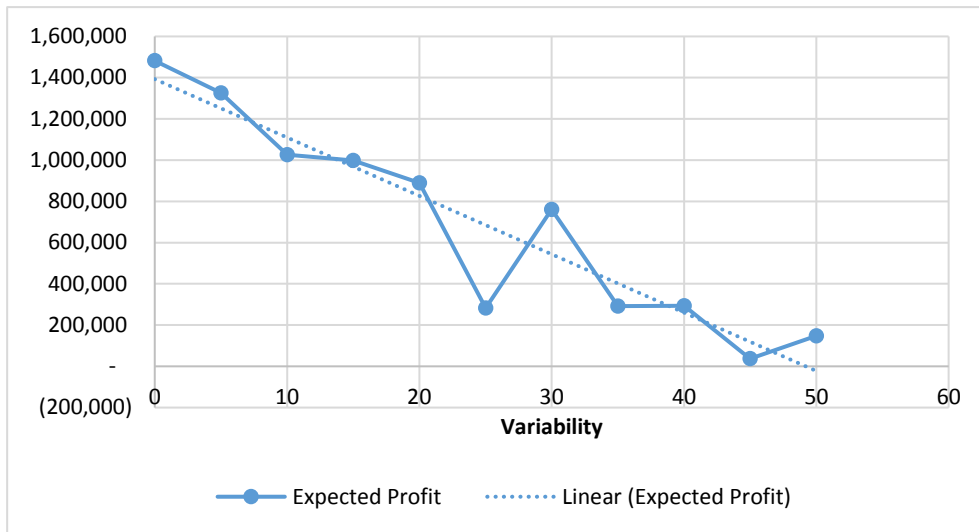


Fig. 5: The Relationship between Demand Variability and the Optimal Expected Profit.

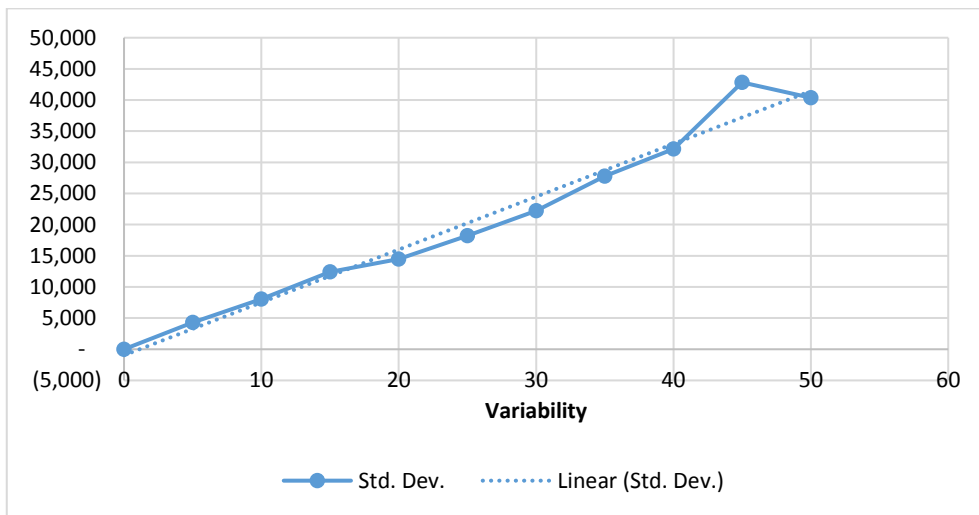


Fig. 6: The Relationship between Demand Variability and Standard Deviation the Total Profit.

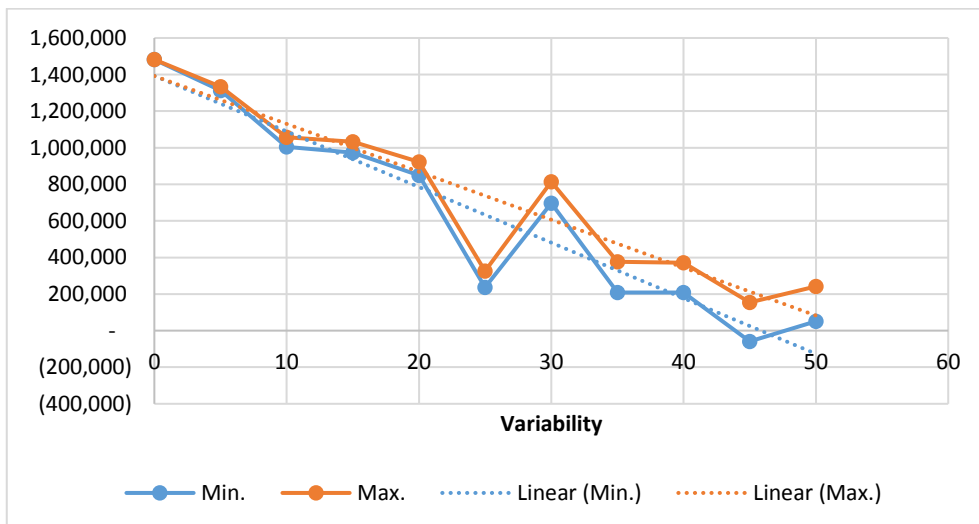


Fig. 7: The Relationship between Demand Variability and Max. and Min. Values of the Resulted Profit.

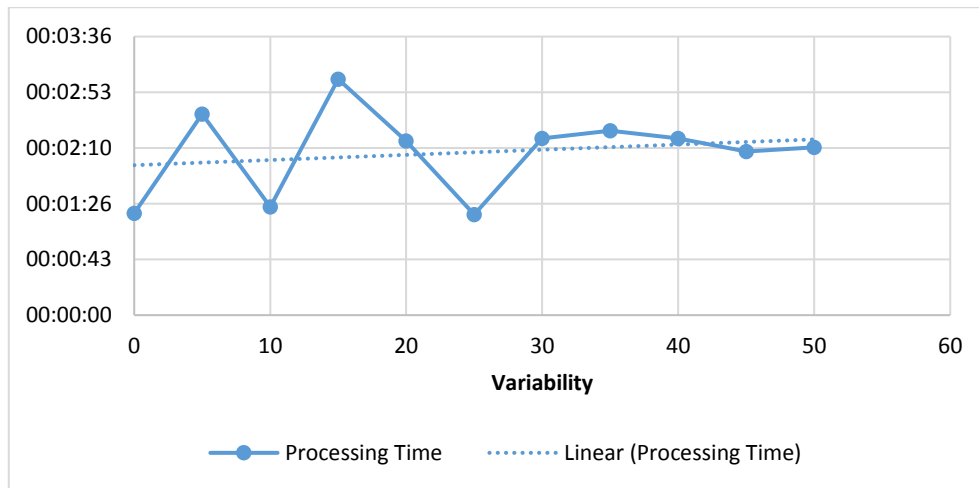


Fig. 8: The Relationship between Demand Variability and the Processing Time to Achieve the Optimal Solution.

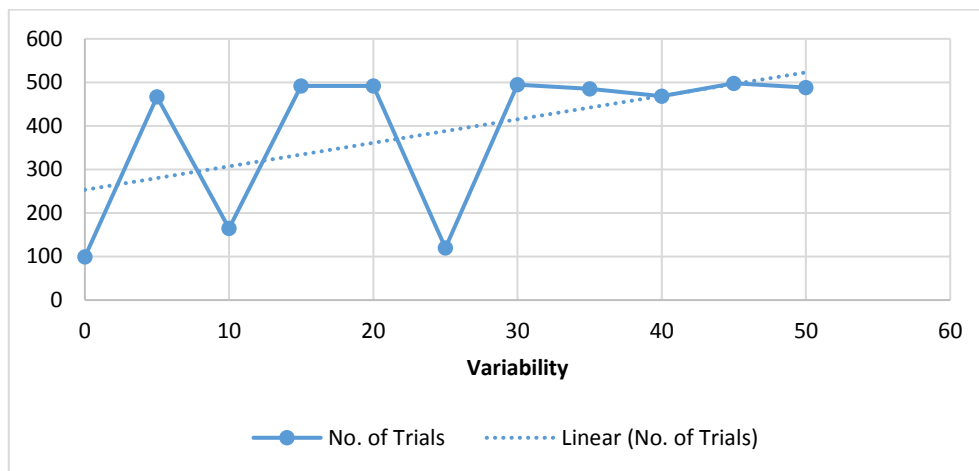


Fig. 9: The Relationship between Demand Variability and the No. of Trials to Achieve the Optimal Solution.

Fig. 8 shows that the increase of demand variability increases slightly the processing time to get the optimal solution this may be due to the increase of the number of trials required to get this solution as presented in Fig. 9.

### 2.3. Effect of stopping conditions

The problem has been solved two times at 0.01 and 0.005 progress in studying the effect of the stopping conditions. The results of these two experiments are tabulated in Table 5. The maximum progress change of all trials of the two solutions is shown in Fig. 10 and Fig. 12. While Fig. 11 and Fig. 13 represent the distribution of the outputs of them. Referring to Fig. 10, it is clear to notice that stopping the optimization at 0.01 % maximum progress change does not assure getting a suitable solution as setting the maximum progress change at 0.005% where the graph flattens out for a while before this condition is met and @RISKOptimizer stopped as shown in Fig. 12.

Table 5: Results of 0.01 and 0.005 Progress and 5% Variability

Progress	No. of Trials	Processing Time	Expected Profit	Goal Cell Statistics			
				Mean	Std. Dev.	Min.	Max.
0.01	467	0:02:36	1,326,105	1,326,105	4,313	1,313,182	1,334,498
0.005	20225	2:17:06	1,445,935	1,445,935	4,069	1,438,462	1,456,283

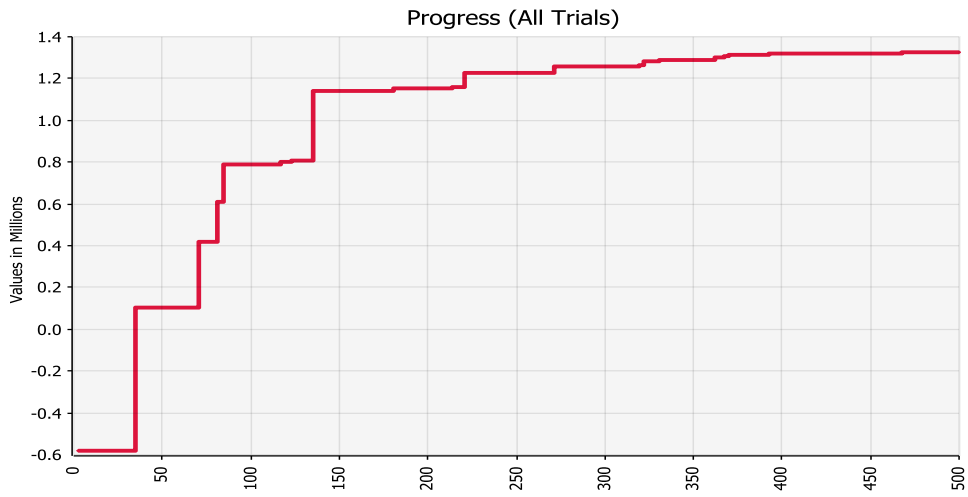


Fig. 10: The Progress for 0.05 Variability and 0.01 Progress.

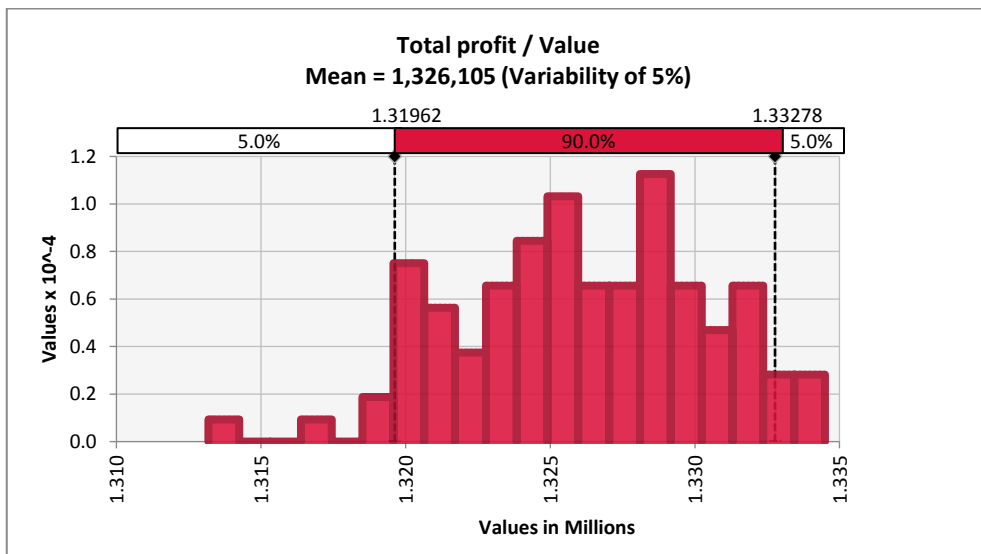


Fig. 11: The Progress for 0.05 Variability and 0.01% Max Progress.

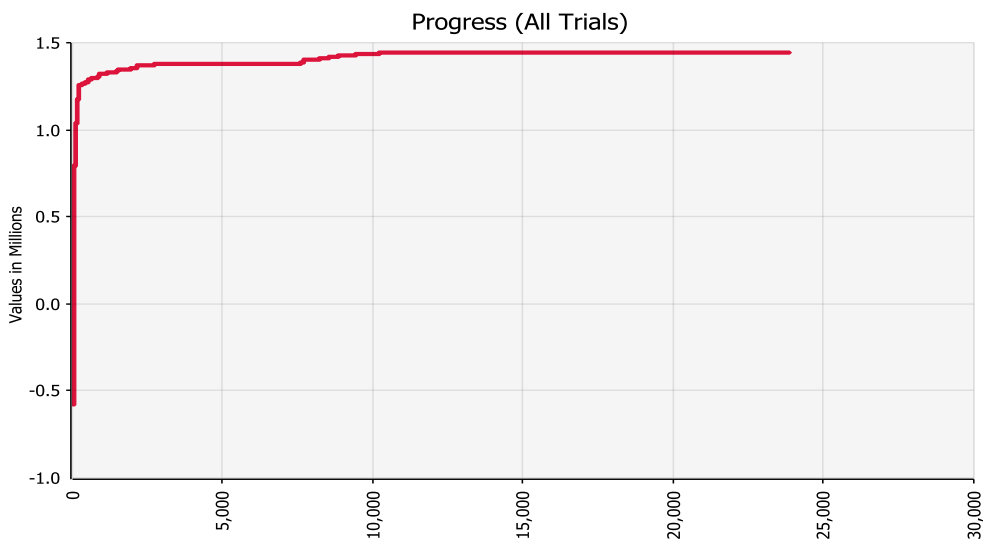


Fig. 12: The Progress for 0.05 Variability and 0.005% Progress.



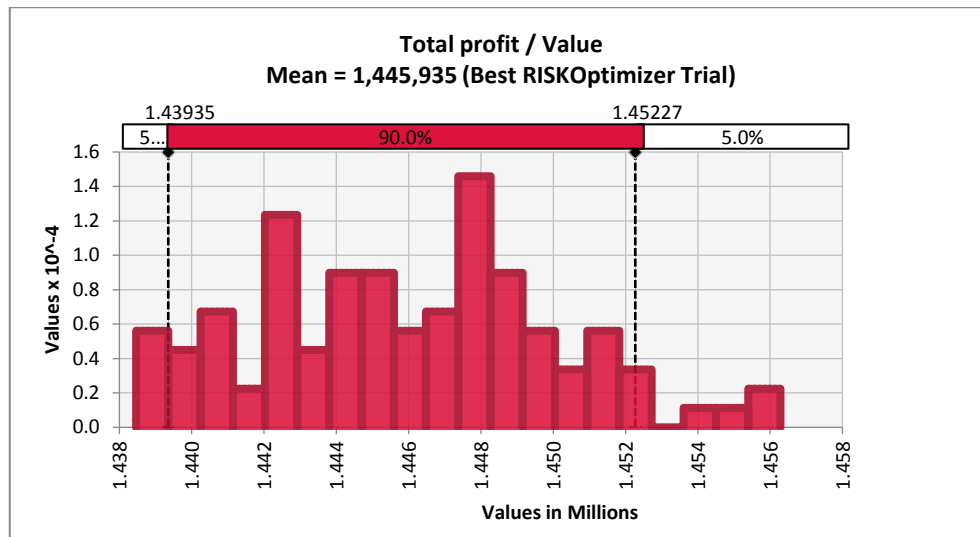


Fig. 13: The Total Profit Variation for 0.05 Variability and 0.005% Progress.

## 4. Conclusion

In this work, an MPS optimization model is developed to maximize the total expected profit using GA under demand uncertainty. The model is built for @ RiskOptimizer in MS Excel. The customer demands have been assumed to follow the normal distribution of a standard deviation related to their mean values with a ratio called demand variability. The effects of demand variability on the profit mean, profit variation and the processing time have been studied.

It may be concluded that:

- 1) Increasing the demand variability has a bad effect on the organization profit, so it is recommended to set some policies to flatten the customer demand.
- 2) Setting the stopping options of the solver has a great effect on the stability and accuracy of the resulted values.

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