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Research paper

Modeling of a Bernoulli-type data quality control process using hidden Markov chains

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Abstract

This article seeks to employ hidden Markov chains in quality control area in general and on binary-data process in particular. A hidden Markov model (HMM) has been applied on a Bernoulli-type data process to monitor its stability over time. In quality control respect, binary variables are widely used when an inspected item is classified into either conforming or nonconforming as per some specific specifications. In this article, we present a new scheme to monitor a quality process yielding binary outcomes or variables such that a new variable is proposed to regulate/evaluate process stability as time passes by. This variable determines the process probability of being statistically in control at each point in time and can be calculated using the developed hidden Markov model. As a result, it was found that it is straightforward to obtain inferences about process stability whether or not it is statistically in-control which, in turn, helps making decisions associated with actions needed when the process goes in an out-of-control way. Furthermore, unlike control charts, the judgment on the process state depends on the entire observation sequence not the current sample only.

Keywords: *Markov Chains; Hidden Markov Models; Control Charts; Attribute Data.*

1. Introduction

Researchers always searching variant methodologies to improve productivity of operations and production systems quality and quantity wise. Evaluation and analysis of productivity of a Just-In-Time (JIT)–Kanban production system is investigated by AL-Tahat M. D. et al. [1], where the processing do not always meet the requirement of the quality. AL-Tahat M. D. [2] applied another methodology for Kanban based production systems, where a queuing network model is formulated. Al-Momani K. R. et al. [3] investigated the impact of maintenance management on productivity for in King Abdullah University Hospital (KAUH).; Usefulness of the mean time between failures (MTBF), the mean time to repair (MTTR) and availability (A) of the equipment were considered. Some factors that affect productivity have been modeled by Al-Refaie A. et al. [4] using structural equation mathematics; several factors were considered including; knowledge management, organization learning, customer relationship management, employee performance, innovation and business performance. The Activity-Based Cost Estimation methodology is applied productivity cost by AL-Tahat M. D. et al. [5] for better estimation of productivity of a foundry system in terms of production cost of the produced steel castings. Aljanaideh O. et al. [6] investigated productivity of magnetostrictive actuator in mechatronic systems, where hysteresis model that can describe rate and bias effects of the harmonic magnetic fields on hysteresis nonlinearities is proposed. Dalalah D. et al. [7] used fuzzy theory to solve a multiple objectives decision-making problem, the weights of mutually dependent criteria, of a group of professionals is estimated based on cause- effect. AL-Tahat M. D. et al. [8] presented a statistical structural equation model that analyses the impact of Concurrent Engineering (CE) in the productivity of some Jordanian industrial sectors, results are validated by a system dynamics model, then the true CE trade-offs are investigated. AL-Tahat M. D. et al. [9] predicted the relationships between types of failures encountered in tablet production of medicine, Ordinal Logistic Regression Modeling (OLRM) is followed and the impact on productivity of medicine is analyzed. Mohammad D. Al-Tahat et al. [10] presented a continuous-time Markov chain of a multi-stage multi-product serial production line; based on CONstant work-in-process (CONWIP) control mechanism of production the steady state behavior of the system is analyzed.

Unlike the aforementioned methodologies, authors in this paper presented another methodology for productivity improvements; a hidden Markov model (HMM) is applied on a Bernoulli-type data process to monitor the stability of productivity in terms of quality over time.

Quality control charting is becoming a necessity to ensure high quality of products and services. Control chart is the typical tool amongst the collection of the statistical process control (SPC) tools to monitor quality characteristics over time. In control charting theory, one of the fundamental assumptions of quality controlling is the independence, which assumes that samples or items, drawn from an online process. However, many researchers have showed that that assumption is no longer appropriate for all production processes. For example, Bhat and La [11] provide new attribute control limits fitting a Markovian process and Lai et al. [12] show the effect of serial correlation on high-quality processes using Markov models.

The idea behind this article is that it assumes a dependency relation between possible states of a quality process. In other words, it employs the Markovian property, which assumes that the current state of a system depends only on the previous one, between the system states not the samples themselves as a system state itself produces samples. Namely, the system states are statistically in-control and out-of- control from the quality perspective.

Usage of Markov chains and even hidden Markov models are not contemporary in quality field. Demiralp and Moghimihadji [13] study the application of hidden Markov Model HMM on variable quality characteristics, which eventually results in dropping or ignoring normality assumption when monitoring such quality characteristics. In the same fashion several authors attempt to tackle the problem of autocorrelation emerging among samples under inspection using such models, see for example, Gadre et al. [14], Shepherd et al. [15], Mousavi, and Reynolds [16].
In addition, the literature has showed low efforts regarding control chart development in the direction of binary data in general. Therefore, it is expected through the article that the developed scheme will largely contribute to monitor binary variables and provide some distinct analysis and information about process under inspection.

2. Hidden markov models

HMM can be considered as an extension of Markov chains, which have been highlighted as the most discussed stochastic processes over the recent years. As HMM theory relates to Markov chains, it is plausible to discuss Markov chains first. Informally, a Markov chain is a stochastic process that has the Markovian property which states that the current state of a system is only dependent of the previous one regardless all other past states. Formally, we say,

$$\Pr(Z_t = i | Z_0, \ldots, z_{t-1}) = \Pr(Z_t = i | z_{t-1}) \forall t, \forall z \quad (1)$$

Normally, a Markov chain is parameterized in the form of a stochastic matrix called transition probability matrix and represented graphically in the form of the so-called state diagram. Figure 1 shows an example of three-state Markov chain with transition parameters.

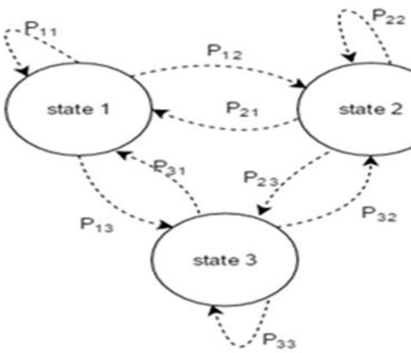

Fig. 1: Three-State Markov Chain.

From Figure 1, it is easy to gather transition probabilities in a matrix form T as follows

$$T = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{13} & P_{32} & P_{33} \end{bmatrix} \quad (2)$$

Broadly speaking, Markov chains are very strong statistical tools and have many real life applications such as weather forecasting, Rank Page and web search and information theory. However, someone can argue how we can reason about any Markovian process's states if we don't actually observe the states themselves. In fact, Markov chains fail to deal with such above-mentioned scenario; the most fitting solution that handles this scenario is an HMM. Rabiner [17] defines an HMM as "An HMM is a doubly stochastic process with an underlying stochastic process that is not observable, but can only be observed through another set of stochastic processes that produce the sequence of observed symbols". An HMM allows us to talk about both observed events and hidden events that we think of as causal factors in our probabilistic model as well. In Figure 2, the structure of an HMM and its assumptions are shown.

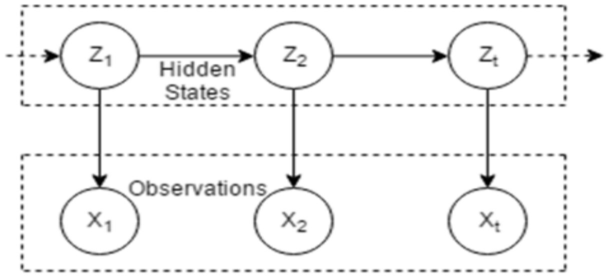

Fig. 2: Trellis Diagram of HMM.

From Figure 2, it is straightforward to extract the assumptions of an HMM. It can be seen that the observations are independent and this assumption is usually called as the independence output assumption. Besides, the hidden states are connected to each other in such a way each state depends only on the earlier state. Finally, given a latent state at a specific point in time, the produced observation at that time is conditional on that state. Now, after detailing the assumptions, a question that comes to mind is how to parameterize or characterize an HMM? Generally, there are five elements that define an HMM regularly, Degirmenci [18]:

1) Number of states in the model, M: which is the number of all possible discrete states that a model or a process can have and can switch from one to another over the passage of time
2) Number of distinct observations, X: at each step in time, a hidden state will generate an observation or a random variable whose values are finite and discrete
3) Transition probability matrix, T: which contains all probabilities quantifying the chance of moving from each state to others
4) Emission probability matrix, E: which compiles the all conditional probabilities yielding the observations given all possible hidden states
5) Initial state probabilities, Π : these probabilities describe the chance of beginning at each possible hidden state or at time 0

Using these parameters the mathematical formulation of an HMM can be found in Rabiner's [19] article. The formula is usually represented in the triplet $\lambda = (T, E, \Pi)$ as follows

$$\Pr(X | \pi, T, E) = \sum_{K=1}^{M} \pi_{K_0} \prod_{t=1}^{T} P_{K_{t-1}K_t} . e_{K_t}(X_t) \quad (3)$$

$e_{k_t}(X_t) = \Pr(X_t \mid K_t = i)$Where, X is the observation sequence, K represents the possible state space which belongs to 1, 2, 3, 4, M, and $e_{k_t}(X_t)$ is the probability of producing X_t given the state i , mathematically we say and finally $P_{k_{t-1}k_t}$ is the transition probability from one state to another.
Now, this model remains relatively ridiculous unless the three fundamental problems are solved. The first problem is called the evaluation problem, by convention, which concerns with evaluating the above-mentioned discrete joint distribution in an efficient way, the second is the estimation problem, which focuses on updating, or adjusting the parameter model and decoding is the third one that deals with discovering the hidden state path.
Several algorithms can tackle these problems but the most popular ones are forward algorithm, Baum-Welch algorithm and Viterbi algorithm respectively.
The next section employs an HMM in terms of quality aspect and shows a quality model to monitor processes of binary variables. The quality model is completely derived from the HMM theory and its mathematical background.

3. The proposed process model

We assume that we have an online process that produces products or items that can be classified into either conforming or nonconforming in accordance to some specifications. In addition, it is equally important to mention that the produced units are independent on each other and the process states, by assumption, are dependent and form a Markov chain. In addition to that, the process states belong to a two-state space, which are specifically statistically in-control and statistically out-of-control. We similarly assume a homogenous (stationary) discrete-time discrete-space Markov chain. The above-mentioned considerations are summarized in Figure 3.

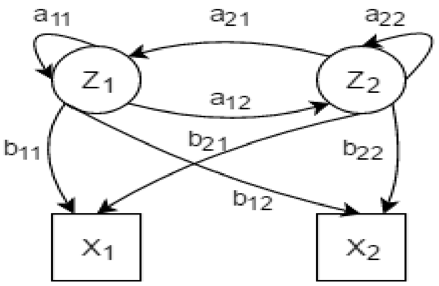

Fig. 3: An HMM for Quality Control Process.

Where Z_1 and $Z_2 \in \{in, out\}$ which are the hidden variables and X_1 and $X_2 \in \{C, NC\}$ which represent observed outputs. The abbreviations in, out, C, NC are in control, out of control, conforming and nonconforming respectively. Furthermore, the parameters in Figure 3 can be matrix formed as next,

$$A = \begin{matrix} & in & out \\ in & a_{11} & a_{12} \\ out & a_{21} & a_{22} \end{matrix} \tag{4}$$

$$B = \begin{matrix} & C & NC \\ in & b_{11} & b_{12} \\ out & b_{21} & b_{22} \end{matrix} \tag{5}$$

Where, A and B are the transition probability and emission probability matrices respectively. For explanation, a_{12} is the probability of moving from an in-control to out of control state and b_{12} is the probability of producing unit given that the process is in control. To finalize the model parameters it is quietly considerable to determine the initial probabilities of each hidden state that is:

$$\Pr(Z_1 = in) = \pi_1 \quad , \quad \Pr(Z_1 = out) = \pi_2 \tag{6}$$

It is also important to note that the summation of all initial probabilities is unity, that is $\pi_1 + \pi_2 = 1$, these probabilities must be estimated arbitrarily at time zero or based on operational experience. In control charting theory, either way variables or attributes, an under-control quality characteristic is mainly plotted as a point of the sample mean on the control chart and if a point exceeds the control limits or if there is a nonrandom pattern , then that would be a signal of an out-of- control condition. Here we propose a new variable to monitor quality, which calculates the process probability of being statistically in control.

Let us denote this variable as $\gamma_t(Z = z)$ which represent the posterior probability of state z at time t. In addition, we are interested in measuring this variable when the process is expected to be statistically in control that is $Z = in$ at each time t. Then, the posterior probability can easily be computed using the following equation:

$$\gamma_t(Z = in) = P_\lambda(Z_t = in|X) = \frac{\alpha_t(Z = in)\beta_t(Z = in)}{P_\lambda(\boldsymbol{X})} \tag{7}$$

Where

$$\alpha_t(Z = in) = \sum_{r \in Z} \alpha_{t-1}(r) a_{r1} b_1(x_t) \qquad \forall\, t = 2, \dots, T \tag{8}$$

$$\beta_t(Z = in) = \sum_{r \in Z} a_{1r} b_r(x_{t+1}) \beta_{t+1}(r) \;\forall\, t = T-1, \cdots, 1 \tag{9}$$

The variable $r \in \{1 = in,\ 2 = out\}$, the quantity $b_r(x_t)$ is the conditional probability of producing x given a state z at time t and $P_\lambda(\boldsymbol{X})$ is the probability of a particular sequence X. Given that $\alpha_1(Z = z) = \pi_z b_z(x_1)$ and $\beta_T(Z = z) = 1 \quad \forall z \in \{in = 1, out = 2\}$.

As working on monitoring the posterior probability $\gamma_t(Z = in)$, we can get a scheme that shows probabilistically the process status over time. Such a scheme can be described as the cardiograph of a process or machine and will be shown in a numerical analysis in section 5 of this paper.

4. Hidden-variable chain analysis

One key advantage of HMMs is to predict the status of a system at the next step of time t+1 as long as HMM parameters are known or estimated. This prediction is purely dependent on the current observation sequence and that hidden state at time t. As we are interested in finding the probability of being in the stable (i.e., statistically in-control) state in the next point of time, we mathematically derive the conditional distribution as:

$$\Pr(Z_{t+1} = in \mid \boldsymbol{X}_t) = \Pr(Z_{t+1} = in \mid Z_t, \boldsymbol{X}_t) \tag{10}$$

$$\Pr(Z_{t+1} = in \mid Z_t, \boldsymbol{X}_t) = \sum_{Z_t} \Pr(Z_{t+1} \mid Z_t) \Pr(Z_t \mid \boldsymbol{X}_t) \tag{11}$$

$$= \Pr(Z_{t+1} = in \mid Z_t = out) * \Pr(Z_t = in \mid \boldsymbol{X}_t) + \Pr(Z_{t+1} = in \mid Z_t = out) * \Pr(Z_t = out \mid \boldsymbol{X}_t) \tag{12}$$

Using the fact $\gamma_t(Z = in) + \gamma_t(Z = out) = 1$, we obtain

$$\Pr(Z_{t+1} = in \mid Z_t, \boldsymbol{X}_t) = a_{11}\gamma_t(Z = in) + a_{21}[1 - \gamma_t(Z = in)] \tag{13}$$

We previously mentioned that the hidden-variable stochastic process constitutes a Markov chain. In view of Markov chains and to get the whole perception, one can analyze this chain in terms of long run probabilities, mean first return time, and mean first passage time.

Recalling that we have a transition probability matrix A, one can calculate the long run probabilities as follows:

$$\lim_{n \to \infty} A^n = \begin{bmatrix} \frac{a_{21}}{a_{12} + a_{21}} & \frac{a_{12}}{a_{12} + a_{21}} \\ \frac{a_{21}}{a_{12} + a_{21}} & \frac{a_{12}}{a_{12} + a_{21}} \end{bmatrix}, \text{ Where } 0 < a_{12}, a_{21} < 1 \tag{14}$$

That means the limiting probabilities of being in an in-control state L_{in} and out-of-control state L_{out} are:

$$[L_{in} \quad L_{out}] = \left[\frac{a_{21}}{a_{12} + a_{21}} \quad \frac{a_{12}}{a_{12} + a_{21}}\right] \tag{15}$$

As long as the Markov chain is ergodic, one can easily calculate the so-called mean first return time, which is the average number of transitions needed by a system to return to state i for the first time. Once the steady state probabilities are evaluated, it is simple to express the expected first return time (μ_{ii}) of state i as follows:

$$\mu_{ii} = \frac{1}{L_i}$$

In the similar fashion, the mean first-passage time (μ_{ii}), which is considered as a key quantity that must be studied in the theory of stochastic processes, can also be determined. This variable concerns with projecting the number of transitions required to reach state j from state i for the first time. Using the formula below and assuming an m-by-m matrix,

$$[\mu_{ij}] = (I - N_j)^{-1}\mathbf{1} \qquad for\ i \neq j \tag{16}$$

Where

I = $(m-1)$ identity matrix

N_j = transition matrix A less its row j^{th} and j^{th} column of target state j

$\mathbf{1}$ = $(m-1)$ column vector with all elements are unity

We get the following formulas

$$\mu_{21} = \frac{1}{1-a_{22}} = \frac{1}{a_{21}} \text{ and } \mu_{12} = \frac{1}{1-a_{11}} = \frac{1}{a_{12}} \tag{17}$$

Where (μ_{21}) represents the average number of time units needed to reach an in-control state from an out-of-control state for the first time. While the average number of time units or moves needed to embrace an out-of-control state from an in-control state for the first time is (μ_{12}) Indeed, (μ_{12}) is the same as what is referred to as the Average Run Length (ARL) in terms of quality control respect.
In case of absorbing state occurs, which can be defined as a state that cannot be left as soon as the system enters that state, we study the next. Here in our model it does make sense to assume that the system will be absorbed by the out-of-control state not the other one. With this assumption, the transition matrix A will have the below form and the state diagram is depicted in Figure 4:

$$A = \begin{matrix} & in & out \\ in & a_{11} & a_{12} \\ out & 0 & 1 \end{matrix} \tag{18}$$

It is obvious that the probability of transitioning from the out state to the in state is zero that is; once the out state is entered, it cannot be left.

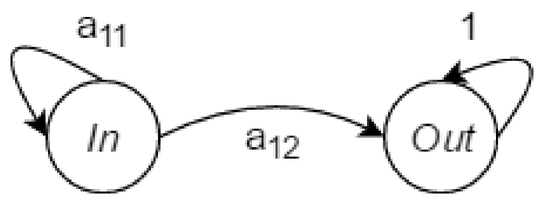

Fig. 4: State Diagram with Absorbing State.

In absorbing-state analysis, practitioners are usually interested in determining two unknowns: the probability of absorption and the expected time to absorption. To obtain these variables it is conventionally agreed to put the transition matrix in the following canonical form:

$$A = \begin{matrix} & TR. & ABS. \\ TR. & \mathbf{Q} & \mathbf{R} \\ ABS. & \mathbf{0} & \mathbf{I} \end{matrix} \tag{19}$$

Where I is an (r x r) identity matrix, 0 is an (r x t) zero matrix, R is a nonzero (t x r) matrix and Q is an (t x t) matrix. The first t states are transient and the last r are absorbing.
Kemeny et al. [20] provide a complete derivation regarding the probability of absorption. They prove that $Q^t = 0$ as $t \to \infty$, i.e., the probability that the process will be absorbed is 1. Now, turning to the second variable representing the expected number of times to absorption, they also gave a proof to this and found that $E = Nc$ where E is the column vector whose i[th] entry is E_i, N is called the fundamental matrix of A and $N = (I - Q)^{-1}$ and c is column vector whose all elements are 1.
Applying these calculations in our proposed model and partitioning the A matrix we get,

$$A = \begin{matrix} & TR. & ABS. \\ TR. & a_{11} & a_{12} \\ ABS. & 0 & 1 \end{matrix} \tag{20}$$

First, we calculate $N = (1 - a_{11})^{-1} = \frac{1}{1-a_{11}}$ and c is 1. So we get

$$Expectd\ time\ to\ absorption = \frac{1}{1-a_{11}} = \frac{1}{a_{12}} \tag{21}$$

It clear that the expected time to absorption equalizes the average number of transitions of directing from an in-control state to an out-of-control state for the first time (μ_{12}).

5. Numerical analysis

Let us assume a process that produces units, which can be classified into two categories; conforming, and nonconforming products. For the purpose of illustration, we suppose that we have the following 100-point sequence listed in Table 1. This sequence was generated by MATLAB under the following parameters:

$$A = \begin{matrix} & in & out \\ in & 0.9751 & 0.0249 \\ out & 0.1276 & 0.8734 \end{matrix}$$

$$B = \begin{matrix} & C & NC \\ in & 0.8631 & 0.1369 \\ out & 0.4725 & 0.5275 \end{matrix}$$

$$[\pi_{1_{in}} \quad \pi_{1_{out}}] = [0.9 \quad 0.1]$$

Table 1: 100-Point Set Generated by MATLAB

item	Attr.	item	Attr.	item	Attr.	item	Attr.	item	Attr.	item	Attr.	item	Attr.
1	C	16	C	31	NC	46	C	61	C	76	C	91	C
2	C	17	NC	32	NC	47	C	62	C	77	C	92	C
3	C	18	C	33	C	48	C	63	C	78	C	93	C
4	C	19	C	34	NC	49	C	64	C	79	C	94	C
5	C	20	C	35	NC	50	C	65	C	80	C	95	C
6	NC	21	C	36	NC	51	C	66	C	81	C	96	C
7	C	22	C	37	C	52	C	67	NC	82	C	97	C
8	C	23	C	38	C	53	C	68	C	83	C	98	C
9	NC	24	C	39	NC	54	C	69	C	84	C	99	C
10	C	25	NC	40	C	55	NC	70	C	85	NC	100	C
11	C	26	NC	41	C	56	C	71	C	86	NC		
12	C	27	C	42	C	57	NC	72	C	87	C		
13	C	28	C	43	C	58	C	73	C	88	C		

14	C	29	NC	44	C	59	C	74	C	89	NC
15	NC	30	C	45	C	60	C	75	NC	90	C

As we interested in detecting statistically out-of-control signals, we plot the posterior probabilities versus time/items. Figure 5 shows the probability of being in statistically in control.

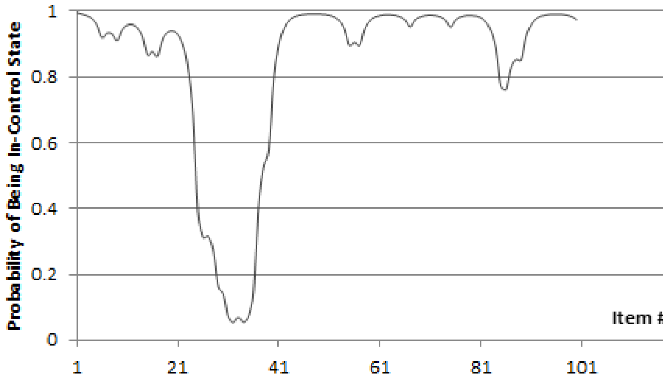

Fig. 5: Posterior Probabilities over Time.

It seems form Figure 5 that the process goes out of statistical stability from item 25 to 36. Assuming the process has experienced an out-of-control condition at that period, we can remove those items from the observation sequence to fulfill some stability in the process. This action is required to help us monitor future observations in analog to control charting theory. Figure 6 illustrates the process in much more stable way.

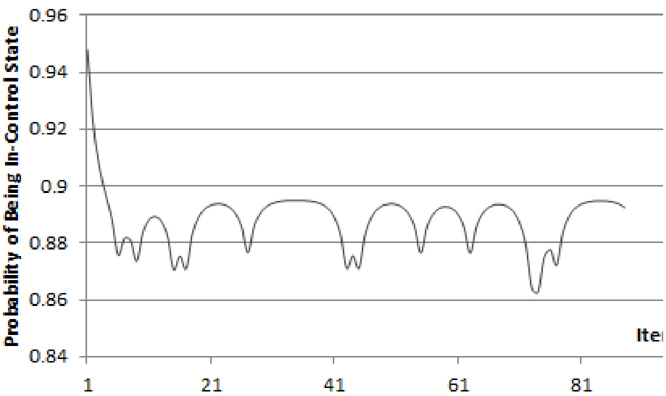

Fig. 6: Removing Seem-Like Out of Control Items.

Now, we generate another data set with different parameters to validate the ability to detect shifts. Table 2 lists 50-point data set generated under the following parameters:

$$A = \begin{matrix} & in & out \\ in & 0.7 & 0.3 \\ out & 0.4 & 0.6 \end{matrix}$$

$$B = \begin{matrix} & C & NC \\ in & 0.75 & 0.25 \\ out & 0.15 & 0.85 \end{matrix}$$

$$[\pi_{1_{in}} \quad \pi_{1_{out}}] = [0.7 \quad 0.3]$$

Table 1: 50-Point Data Set Generated by MATLAB with Different Parameters

item	Attr.	item	Attr.	item	Attr.	item	Attr.	item	Attr.	item	Attr.	item	Attr.
1	NC	9	NC	17	C	25	NC	33	C	41	C	49	C
2	NC	10	NC	18	C	26	NC	34	C	42	C	50	NC
3	C	11	NC	19	C	27	NC	35	NC	43	NC		
4	NC	12	NC	20	NC	28	C	36	NC	44	NC		
5	C	13	NC	21	NC	29	C	37	NC	45	NC		
6	C	14	NC	22	C	30	C	38	NC	46	C		
7	NC	15	NC	23	NC	31	NC	39	NC	47	C		
8	C	16	NC	24	NC	32	NC	40	C	48	C		

When we added the new generated data set to the stabilized/refined one, we get Figure 7.

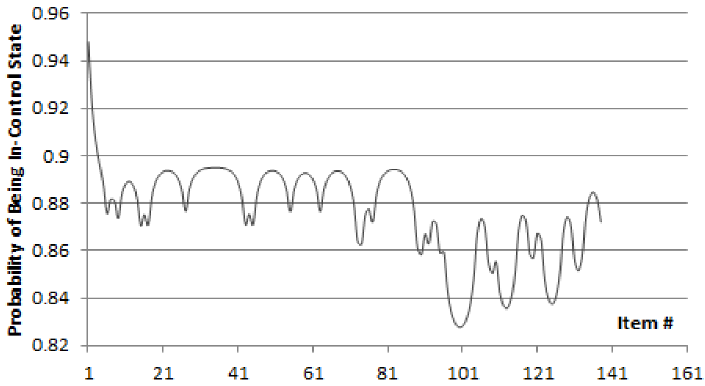

Fig. 7: Shift Detection in Process.

It can be seen that the new generated sequence does not statistically belong to the old one that is the process exhibited to some out-of-control conditions.

6. Conclusions and future work

The proposed model employs Markov chains theory as a base for this article. An HMM, a by-product of Markov chains was employed and applied in quality control field as discussed earlier. As a result, it was found that the suggested model is superior to detect the out-of-control signals. It has the capability of monitoring process stability over time and detecting changes in process parameters. It judges the state of a process numerically rather than the purely subjective human judgment emerging in control charting theory. Further, the proposed model takes into account all the past available observation sequence to make a decision regarding the current process's state and distinctively offers a computing tool to predict the next state of a production process stochastically. Somehow, it is expected that the suggested model will be with a great benefit in healthcare sectors as they involve many attribute data.

The model discussed in this article is dramatically built on the assumption of that observed variables are independent and the system states form a Markov chain. One can generalize the proposed model to take into consideration the autocorrelation phenomenon among the observed variables as time passes by. So this idea is highly recommended for future work. Figure 8 shows the Bayesian network of the generalized idea based on observations dependency.

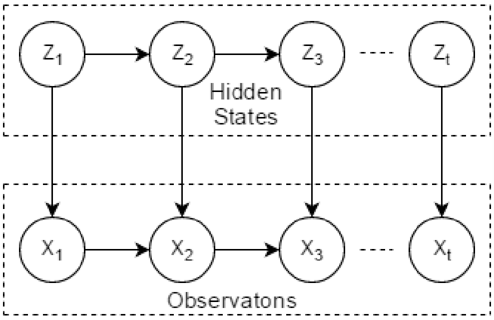

Fig. 8: A Proposed Model with Observation Dependency Relation.

In figure 8, it is illustrated that there is a conditional or dependence relation between observed items/samples. This conditional property generalizes the model to take into account the effect of autocorrelation among items/samples.

In addition to that, it is highly recommended applying HMMs in the case of continuous variables. No doubt, this requires much more complicated equations and derivations, but this application will be dynamic in quality control field. That means usage of probability density functions instead of probability mass functions to describe the observed samples. HMMs will be usefully applicable in quality control to both variable and attribute data throughout this perspective. Figure 9 depicts how to model variable data with an HMM.

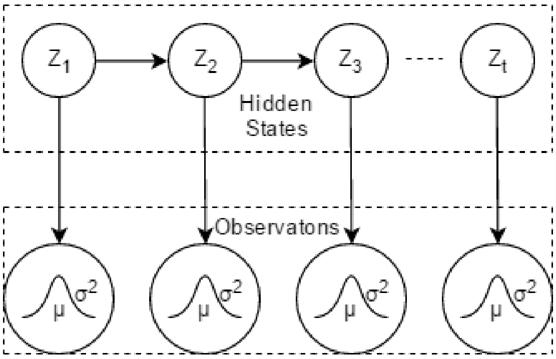

Fig. 9: An HMM with Normally Distributed and Independent Variables.

It is essential to mention that the two graphs (Figure 8 and 9) could be combined together to simulate the most likely case emerging in production situations. This combination will assume a correlation or dependency between continuous observed variables under control. Mathematically, the combination is exactly described by the so-called continuous joint distributions.

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