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Research paper

Magnetohydrodynamic slip flow and heat transfer over a nonlinear shrinking surface in a heat generating fluid

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Abstract

In this paper, the problem of steady slip magnetohydrodynamic (MHD) boundary layer flow and heat transfer over a nonlinear permeable shrinking surface in a heat generating fluid is studied. The transformed boundary layer equations are then solved numerically using the bvp4c function in MATLAB solver. Numerical results are obtained for various values of the magnetic parameter, the slip parameter and the suction parameter. The skin friction coefficients, the heat transfer coefficients, as well as the velocity and temperature profiles for various values of parameters are also obtained and discussed.

Keywords: *Heat Generation; Heat Transfer; Magnetohydrodynamic; Shrinking Surface; Slip Flow*.

1. Introduction

In the problems of shrinking and stretching surfaces, the main assumption of many studies is the "no slip condition" on the surface. However, in certain situations this assumption is no longer valid. The flow of a viscous fluid with slip past a stretching surface has been discussed by many authors in different aspects [1 – 6]. The slip magnetohydrodynamic (MHD) flow and heat transfer problem over a nonlinear permeable stretching surface with chemical reaction has been studied by Yazdi et al. [3]. Later, Reddy et al. [4] investigated a two-dimensional MHD boundary layer flow of a Maxwell nanofluid over an exponentially stretching surface in the presence of velocity slip and convective boundary conditions. Hayat et al. [5] solved numerically via Runge-Kutta-Fehlberg method the problem of heat transfers in MHD stagnation-point flow of Cross fluid towards a stretching surface. Recently, Sravanthi and Gorla [6] examined the effects of heat source/sink and chemical reaction on a steady MHD Maxwell nanofluid over a porous exponentially stretching sheet with suction and blowing using the homotopy analysis method (HAM).

Meanwhile, for the shrinking surface, several researchers have studied the slip problems, namely, Jain and Choudhary [7], Nandeppanavar [8] and Ahmad et al. [9]. Jain and Choudhary [7] studied the effect of magnetohydrodynamic on boundary layer flow over an exponentially shrinking permeable sheet with slip condition, placed at the bottom of fluid saturated porous medium, where the setup was subjected to suction to contain the vorticity in the boundary layer. Later, Nandeppanavar [8] considered the effect of second order slip of viscous flow and heat transfer over a shrinking sheet for both constant and prescribed surface temperatures in his research. Further, Ahmad et al. [9] investigated the heat transfer of MHD boundary layer flow over a shrinking sheet with the effect of thermal slip. They transformed the nonlinear partial differential equations into nonlinear ordinary differential equations using the Lie group analysis.

On the other hand, slip problems can also be found in many other aspects, for example, Muhammad and Makinde [10] discussed the thermodynamics irreversibility of an unsteady MHD mixed convection with slip and thermal radiation over a permeable surface. Further, Eegunjobi and Makinde [11] investigated the combined effects of magnetic field, buoyancy force, velocity slip, suction and injection, porous medium permeability, thermal radiation absorption, viscous and Joule heating on MHD mixed convection flow of Casson fluid in a vertical channel. Later, Shashikumar et al. [12] studied the Casson nanofluid flow between parallel plates with the effects of second order slip and nonlinear thermal radiation. The problem of an unsteady MHD mixed convection flow of nanofluid in a stagnation region of an impulsively rotating sphere has been studied by Ahmed and Rashed [13]. Ahmed and Rashed [13] used the Buongiorno's model and considered the effects of slip, thermal radiation and convective boundary conditions.

The aim of this present study is to solve the problem of slip MHD flow and heat transfer in a heat generating fluid over a nonlinear permeable shrinking surface. To the best of our knowledge, this specific problem has not been considered before.

2. Mathematical formulation

Consider a steady MHD flow and heat transfer towards a nonlinear permeable shrinking surface in a heat generating electrically conducting fluid. It is assumed that the velocity of the shrinking surface is $u_w(x)$, while the velocity of the slip flow is $u_s(x)$ and it is also assumed that the mass transfer velocity is $v_w(x)$. Further, it is assumed that the temperature of the surface (sheet) is $T_w(x)$, while the uniform temperature of the ambient fluid is T_∞. The fluid is bounded by the shrinking sheet at $y = 0$ and the flow occupies the space $y > 0$. A variable magnetic field of strength $B(x)$ is applied in the transverse direction to the flow. The schematic diagram of the problem is shown in Figure 1.

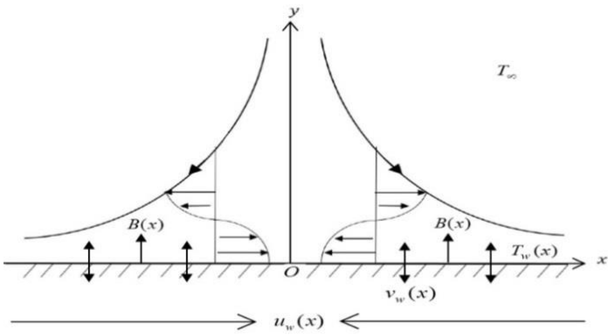

Fig. 1: Physical Model and Coordinate System.

The electric and induced magnetic fields are negligibly small. Following Yazdi et al. [3] and Merkin [14], the basic equations for the problem under consideration by applying the boundary layer approximations can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2(x)}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} - \frac{Q_0}{\rho c_p}(T - T_\infty)^p \tag{3}$$

subject to the boundary conditions

$$u = \pm[u_w(x) + u_s(x)], v = v_w(x), T = T_w(x) \text{ at } y = 0$$
$$u \to 0, T \to T_\infty \text{ as } y \to \infty \tag{4}$$

where ν is the kinematic viscosity, ρ is the fluid density, σ is the electrical conductivity, α is the thermal diffusivity of the fluid, c_p is the specific heat capacity, Q_0 is the volumetric ratio of generation of heat and p is the exponent. The last term of Eq. (3) refers to the local power-law temperature dependence (see Merkin [15]). The slip velocity $u_s(x)$ is assumed to be proportional to the local wall shear stress as follows (see Gal-el-Hak [16]):

$$u_s(x) = l(x)\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{5}$$

where $l(x)$ is slip as a proportional constant of the velocity slip.
In order that Eqs. (1)-(3) subject to the boundary conditions (4) admit a similarity solution, we assume that $u_w(x), T_w(x), B(x)$ and $l(x)$ are given as (see Yazdi et al. [3]),

$$u_w(x) = u_0 x^m, T_w(x) = T_\infty + T_0 x^n,$$
$$B(x) = B_0 x^{(m-1)/2}, l(x) = K x^{(1-m)/2} \tag{6}$$

where u_0 is a constant rate parameter of the stretching/shrinking surface velocity, m is the nonlinear shrinking parameter, n is a constant, T_0 is the characteristic temperature, B_0 is the constant applied magnetic field and $K = [2\nu/(m+1)u_0]^{1/2}$ is the constant slip parameter. By using a similarity transformation, the basic equations (1)-(3) are transformed into ordinary differential equations. Thus, the mathematical analysis of the problem can be simplified by introducing the following dimensionless variables (7):

$$u = u_w(x)f'(\eta), v = -\sqrt{\frac{(m+1)\nu u_w(x)}{2x}}\left[f(\eta) + \frac{m-1}{m+1}\eta f'(\eta)\right]$$
$$\theta(\eta) = \frac{T - T_\infty}{T_w(x) - T_\infty}, \eta = \sqrt{\frac{(m+1)u_w(x)}{2\nu x}}\,y \tag{7}$$

Substituting (7) into Eqs. (2) and (3), we obtain the following nonlinear similarity equations:

$$f''' + ff'' - \left(\frac{2m}{m+1}\right)f'^2 - \left(\frac{2M^2}{m+1}\right)f' = 0 \tag{8}$$

$$\frac{1}{Pr}\theta'' + f\theta' - \left(\frac{2n}{m+1}\right)f'\theta + Qx^{(p-1)n-m+1}\theta^p = 0. \quad (9)$$

The boundary conditions (4) now become

$$f(0) = s, f'(0) = \pm[1 + Kf''(0)], \theta(0) = 1$$
$$f'(\eta) \to 0, \theta(\eta) \to 0 \text{ as } \eta \to \infty \quad (10)$$

where s is the suction/injection parameter, M is the magnetic parameter, Q is the Eckert number and Pr is the Prandtl number. Note that s negative is for mass injection and s positive is for mass suction.
It is noticed that for the energy equation (9) to have a similarity solution, we have to take

$$n = \frac{m-1}{p-1}. \quad (11)$$

and it becomes

$$\frac{1}{Pr}\theta'' + f\theta' - \left(\frac{2n}{m+1}\right)f'\theta + Q\theta^p = 0. \quad (12)$$

3. Results and discussion

The nonlinear ordinary differential equations (8) and (12) subject to the boundary conditions (10) have been solved numerically by the bvp4c function in MATLAB software. The equations are solved simultaneously. The present numerical results are compared with the results by Javed et al. [17] and Yazdi et al. [3] to check on the accuracy of the results obtained. Javed et al. [17] solved their problem numerically using the finite-difference scheme, known as the Keller box method, while Yazdi et al. [3] used the Dormand-Prince pair and shooting method. The comparison results for the velocity gradients are shown in Table 1 and it is found that there is a very good agreement between the present results and the previously published ones.

Table 1: Comparison of the Velocity Gradient at the Wall $|f''(0)|$ for the Stretching Case

m	s	K	M	Javed et al. [17]	Yazdi et al. [3]	Present
0	0	0	0	0.6275	0.6275	0.6275
0.2	0	0	0	0.7668	0.7667	0.7667
0.5	0	0	0	0.8895	0.8896	0.8895
0.75	0	0	0	0.9539	0.9540	0.9538

Figures 2 and 3 show the variations of the skin friction coefficient $f''(0)$ with suction parameter, s, for different values of the magnetic parameter, M, and the velocity profiles for various M, respectively. It is found that the skin friction coefficient in Figure 2 increases with M due to the increase in Lorentz force, which opposes the flow. This leads to the increase in the skin friction coefficient and decelerates the velocity profiles as shown in Figure 3. However, in Figure 4, the opposite effect occurs for the local Nusselt number when the magnetic parameter is applied. It is observed that both the skin friction coefficient and also the local Nusselt number increase with the suction parameter, s. The changes in the local Nusselt number for various magnetic parameter M is not significant as suction parameter s increases. This is due to magnetic parameter M and suction parameter s affect the fluid flow directly, but not the temperature, as can be seen clearly in Eqs. (8) – (9) and boundary conditions (10).

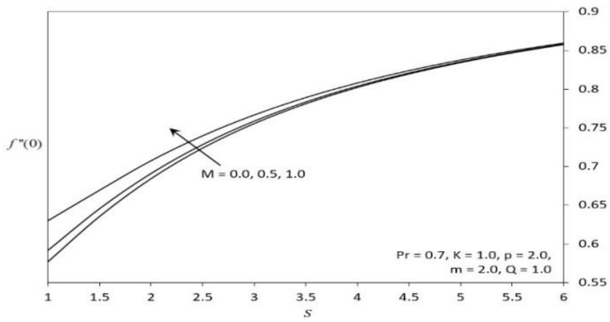

Fig. 2: Variation of the Skin Friction Coefficient with s for Various Values of M.

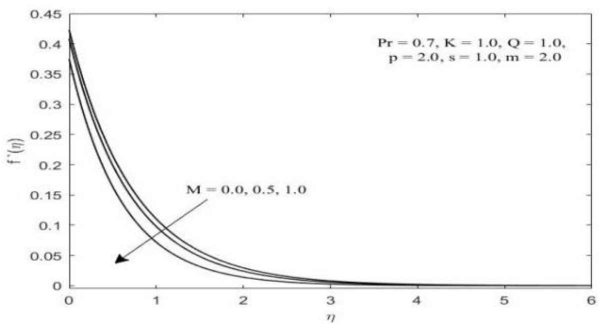

Fig. 3: Velocity Profiles for Various Values of M.

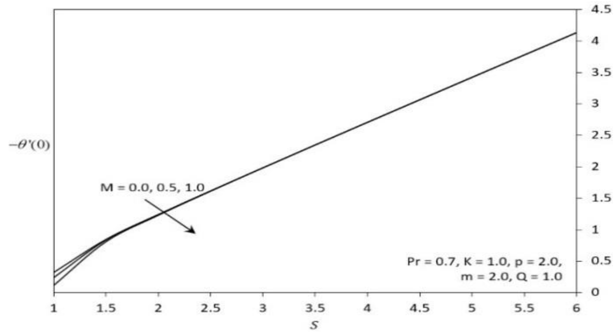

Fig. 4: Variation of the Local Nusselt Number with s for Various Values of M.

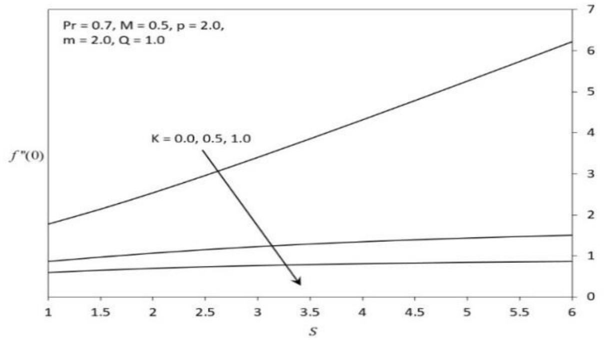

Fig. 5: Variation of the Skin Friction Coefficient with s for Various Values of K.

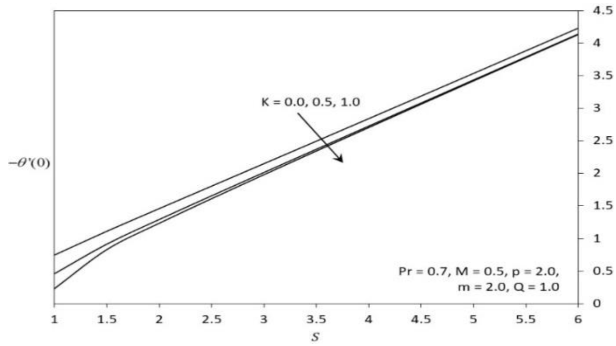

Fig. 6: Variation of the Local Nusselt Number with s for Various Values of K.

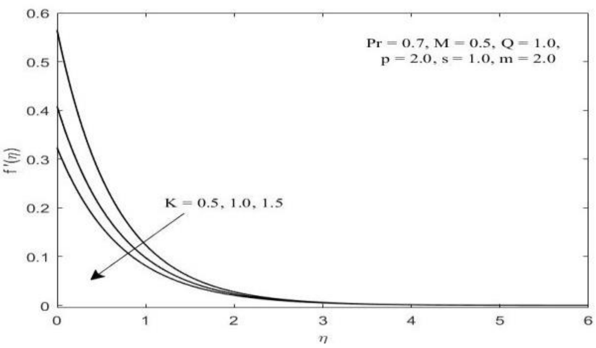

Fig. 7: Velocity Profiles for Various Values of K.

The effects of the slip parameter, K and the suction parameter, s on the skin friction coefficient and the local Nusselt number are displayed in Figures 5 and 6, respectively. Both the skin friction coefficient and the local Nusselt number are found to decrease as K increases. Physically, the increase in slip parameter has reduced the velocity gradient as displayed in Figure 7. Due to this phenomenon, this has decreased the fluid activity and the heat transfer rate on the surface. The changes in the heat transfer rate on the surface in Figure 6 becomes less significant when the value of s becomes larger.

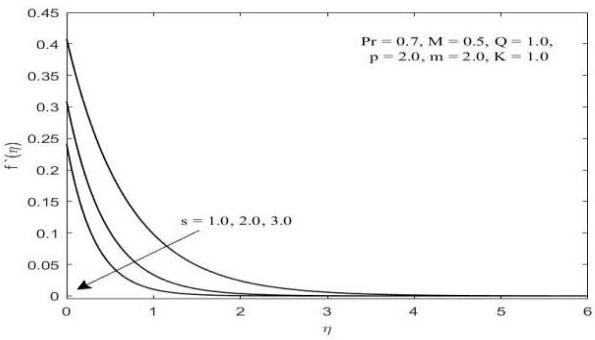

Fig. 8: Velocity Profile for Various Values of s.

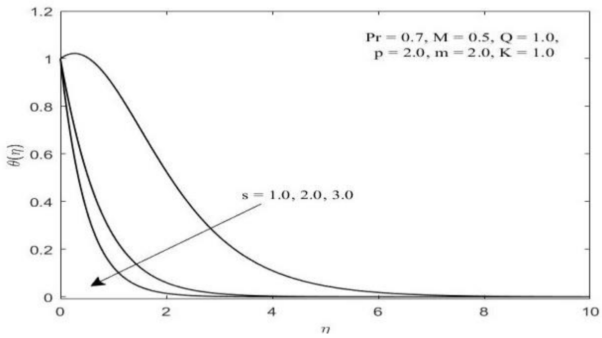

Fig. 9: Temperature Profiles for Various Values of s.

The effects of suction parameter s on the velocity profiles and temperature profiles are shown in Figures 8 and 9, respectively. Suction parameter has the effect of thinning the velocity and temperature profiles. This is due to suction reduces the fluid flow and increases resistance to the transport phenomena, hence thinning the profiles. Therefore, this increases the surface heat transfer, as displayed in Figure 10. Figure 10 also shows the effect of the Eckert number, Q on the heat transfer rate at the surface with other parameters been fixed. An increment in Q reduces heat in the fluid flow. Consequently, the rate of heat transfer at the surface decreases with Q.

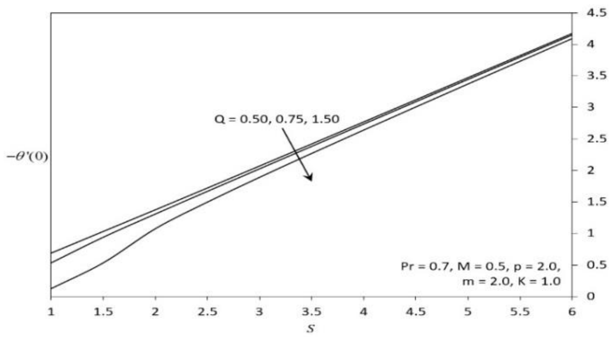

Fig. 10: Variation of the Local Nusselt Number with s for Various Values of Q.

4. Conclusion

A study is performed for the problem of steady slip magnetohydrodynamic (MHD) boundary layer flow and heat transfer over a nonlinear shrinking surface. The effect of all related parameters are studied numerically via bvp4c function built in MATLAB solver. The increase of magnetic parameter will lead to an increase of the skin friction coefficient, however, the magnetic parameter decreases the surface heat transfer. In this present study, the skin friction coefficient and the surface heat transfer increase with the suction parameter. When the value of suction parameter becomes larger, the changes in the skin friction coefficient and the surface heat transfer are not that significant as magnetic parameter, slip parameter and the Eckert number are applied. It is also observed that higher magnetic parameter, slip parameter and suction parameter has the effect of thinning the boundary layer.

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Nomenclature

Roman letters

B_0 = constant applied magnetic field

$B(x)$ = variable magnetic field

c_p = specific heat capacity
$f(\eta)$ = dimensionless stream function
$f''(0)$ = skin friction coefficient
K = constant slip parameter
l(x) = slip as a proportional constant of the velocity slip
m = nonlinear shrinking parameter
M = magnetic parameter
n = constant
p = exponent
Pr = Prandtl number
Q = Eckert number
Q_0 = volumetric ratio of generation of heat
s = suction/injection parameter
T = fluid temperature
T_0 = characteristic temperature
$T_w(x)$ = temperature of the surface
T_∞ = uniform temperature of the ambient fluid
u_0 = constant rate parameter of the shrinking surface
$u_s(x)$ = velocity of the slip flow
$u_w(x)$ = velocity of the shrinking surface
u, v = velocity components along the x and y directions, respectively
$v_w(x)$ = mass transfer velocity
x, y = Cartesian coordinates along the surface and normal to it, respectively

Greek symbols
α = thermal diffusivity of the fluid
η = similarity variable
$\theta(\eta)$ = dimensionless temperature
$-\theta'(0)$ = local Nusselt number
ν = kinematic viscosity
ρ = density
σ = electrical conductivity

Subscripts
w = condition at the surface
∞ = condition outside of boundary layer

Superscript
$'$ = differentiation with respect to η

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