International Journal of Engineering & Technology, *5 (1) (2016) 25-28*

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v5i1.5493
Research paper

Identification of thermophysical characteristics of materials using heating probe

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Abstract

This paper investigates the numerical analysis to determine the thermo-physical characteristics of materials. This method is based on a heating probe kept at a constant temperature and maintained in contact with a cylindrical sample. The heat power dissipated in the sample is measured by the probe. The results address to identify simultaneously the thermal conductivity, the volumetric heat capacity and the heat transfer coefficient using the inverse problem.

Keywords: *Heating Probe; Inverse Problem; Power Measurements.*

1. Introduction

The design and the control of many systems using or producing energy require accurate knowledge of the thermal behavior of the materials and, consequently, their thermo-physical properties. The determination of these parameters by transient electro-thermal methods is the subject of many researches [1-6]. Some authors [2-4] proposed an estimation of effective thermal properties by using the hot wire method associated with a direct or an inverse method. Yi He [5] used the hot disk method for measuring the effective thermal conductivity. Gustafsson [6] used the hot strip method for measuring the effective thermal conductivity and diffusivity of media. Several experimental methods have also been presented. All these methods allow identifying the effective thermal conductivity of the whole medium. A measurement method of local thermal conductivity is the SThM (Scanning Thermal Microscopy). This method is presented using a calibration procedure [7], [8]. It consists of exciting the central point of a sample with a thin platinum wire maintained at a constant temperature with the help of a Wheatstone bridge. The power variation caused by the unbalanced bridge is then recorded. The calibration procedure consists of representing the power responses of different reference materials as function of their known thermal conductivity values. This function is supposed to be linear. The determination of the curve slope makes it possible to determine a local thermal conductivity.
In this numerical study, a 2D transient thermal model is associated with an inversion procedure in order to identify the local thermo-physical properties of materials using the hot point method. In this study, the heat transfer in the cylindrical sample is analyzed and the experiment is optimally designed.

2. Direct problem

2.1. System description

We consider a cylindrical sample of radius R and thickness H initially at the ambient temperature. The “hot point”, having a temperature T_p and a radius r_0, is maintained in contact with the front face (z=H) of the sample.

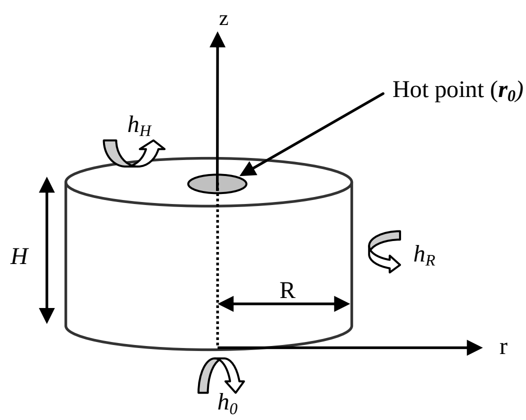

Fig.1: Experimental Setup.

The heat transfer is governed by the equation:

$$\rho Cp \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda \frac{\partial T}{\partial r}\right) + \frac{\partial}{\partial z}\left(\lambda \frac{\partial T}{\partial z}\right) \quad (1)$$

Initially, the sample is at the ambient temperature.

$$T(r,z,0) = T_{ex} \quad \text{at} \quad t = 0 \quad (2)$$

The sample is symmetry, we can write:

$$\frac{\partial T}{\partial r}(0,z) = 0 \quad \text{at} \quad r = 0 \quad (3)$$

$$-\lambda\frac{\partial T}{\partial r}=h(T-T_{ex}) \quad \text{at} \quad r=R \tag{4}$$

$$-\lambda\frac{\partial T}{\partial z}(r,0)=-h(T-T_{ex}) \quad \text{at} \quad z=0 \tag{5}$$

$$\begin{cases}-\lambda\frac{\partial T}{\partial z}=h\left(T-T_{ex}\right) \text{ for } & r\succ r_0 \text{ and } z=H\end{cases} \tag{6}$$

$$\begin{cases}-\lambda\frac{\partial T}{\partial z}=\frac{1}{R_C}\left(T-T_p\right) \text{ for } & r\le r_0 \text{ and } z=H\end{cases} \tag{7}$$

With:

R_C : Thermal contact resistance point/sample

r_0 : Heating probe radius

h: heat transfer coefficient

2.2. Numerical resolution

The system of equations (1-7) is solved numerically by the finite volume method. Uniform mesh (Δr and Δz are constant) is used. The resulting algebraic equations become:

$$a_{ij}T_{ij}=a_{i+1j}T_{i+1,j}+a_{i-1j}T_{i-1,j}+a_{ij+1}T_{ij+1}+a_{ij-1}T_{ij-1}+d_{ij} \tag{8}$$

The Gauss Seidel method is used to solve these algebraic equations.

To determine the optimal grid for which the model is invariant, we studied the influence of the mesh size and the time step on the variation of the calculated temperature (temperature measured in the center of the sample). In this analysis, we have found that from the mesh size 201x201 and the time step Δt=1s, the code becomes accurate.

3. Inverse problem

For this numerical simulation, the following properties of the sample are taken: R=30mm, H=10mm, $\lambda=386\frac{W}{mK}$, $\rho c_p=3429382\frac{J}{m^3K}$. A heating point is maintained at a constant temperature $T_p=330$ K and kept in contact in the center of the sample (r=0 and z=H). The following parameters are also used in the model: $T_{ex}=298.15K$, $h=12\frac{W}{m^2K}$, $R_C=10^{-4}\frac{m^2K}{W}$, the probe radius is r_0 = 0.2mm.The used mesh size is maxr=maxz=201 and the time step is Δt =1s. The parameters (ρc_p , λ , h) will be identified using the Levenberg- Marquardt algorithm by minimizing the least squares norm.

3.1. Heat power simulation

It is about simulating the measured power Φ by adding to the calculated power φ (from calculation of the temperature) an additive noise, having a normal distribution and constant standard deviation. In this case, the simulated signal is:

$$\Phi\left(t,z,r\right)=\phi\left(t,z,r\right)+\varepsilon \tag{9}$$

Φ is the required power to pass to the contact mode. It is written as:

$$\Phi=-\lambda S\frac{\partial T}{\partial z} \tag{10}$$

3.2. Least squares minimization

The problem consists of minimizing the ordinary least-squares norm, referring to the differences between the experimental and the calculated power [8], [9], [10].

$$J=\left[\Phi(q)-\varphi\right]^T\left[\Phi(q)-\varphi\right] \tag{11}$$

J is optimal if

$$\nabla J(q)=0 \tag{12}$$

This means that

$$2X^T\left[\Phi(q)-\varphi\right]=0 \tag{13}$$

Where $X=\left[\partial\Phi(q)/\partial q\right]$ is the sensitivity matrix [9].

To overcome these problems, a new diagonal matrix term $\mu^k\Omega^k$ is added to X^TX in order to damp oscillations and instabilities due to the ill-conditioned character of the problem. We obtain then the Levenberg–Marquardt method [10], [11]:

$$q^{k+1}=q^k+\left[\left(X^k\right)^T X^k+\mu^k\Omega^k\right]^{-1}\left(X^k\right)^T\left[\varphi-\Phi\left(q^k\right)\right] \tag{14}$$

This method has the advantage of varying smoothly between the steepest descent method, when the damping parameter μ^k is high, and the Gauss-Newton method by decreasing the value of μ^k .

3.3. Sensitivity analysis

In this part, we are going to study the relative variation of the heat flux to the thermo-physical parameters. The reduced sensitivity coefficients are calculated using the finite difference method:

$$\overline{X_{ij}}=\frac{\varphi_i\left(q_1,........,q_i+\delta q_1,.....q_p\right)-\varphi_i\left(q_1,........,q_i,........q_p\right)}{\partial q_i}q_i \;=X_{ij}q_i \tag{15}$$

The reduced sensitivity coefficients presented on figure 4 show that the thermal contact resistance and the volumetric heat capacity are correlated. It will be therefore impossible to identify them simultaneously. To solve this problem, it is necessary to fix one of the two parameters. We choose to fix the thermal contact resistance at a nominal value.

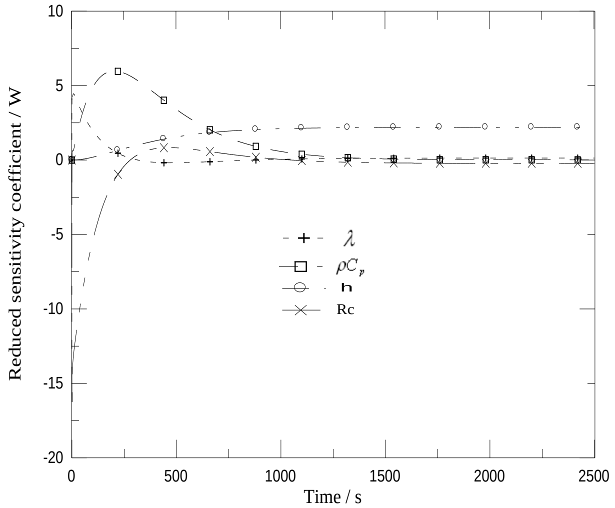

Fig. 4: Reduced Sensitivity Coefficients.

3.4. Optimal experiment design

The purpose of the optimal experiment design is to find the optimal conditions permitting to identify the unknown parameters with the lowest uncertainty.
Several optimality criteria exist, the mostly used one is the D criterion witch maximizes the determinant of the information matrix $X^T X$ [9].
In this work, another criterion is used to optimally design the experiment. It consists of analyzing the evolution of the relative confidence interval. This criterion gives more precise information about the identification uncertainty of each parameter.
This relative confidence interval is represented by the coefficient α defined by the following relation:

$$\alpha = \frac{2.576\sqrt{\text{variance (q)}}}{q} \quad (16)$$

This interval is given for 99% confidence by the relation:

$$-\alpha \le \frac{\hat{q}-q}{q} \le \alpha \quad (17)$$

With $\frac{\hat{q}-q}{q}$ is the relative difference between the exact and the estimated parameters.
The values of α are calculated when the algorithm has converged. This convergence is verified when the residual is comparable with the measurement noise ($J \approx N\sigma^2$).

3.5. Identification results

According to the experiment design, we choose the following parameters: r_0=0.2mm; R=20mm; H=20mm and $R_{C}=10^{-4}$m²K/W. The parameter identification is performed using these optimal parameters and the results are represented on table 1. It is shown that the residual is comparable with the measurement noise ($J \approx N\sigma^2$), the convergence is then verified. Furthermore, we can see that even when the initial values are very far from the exact values (20%), the parameters are estimated with a low uncertainty (lower than 0.07%).

Table.1: Parameter Identification Results with Power Measurement Noise $\sigma = 0.1$W

	Initial values	Estimated values	Exacted values	Variances	coef. α
ρcp	5000000	3428600	3429382	43555.21	0.019%
λ	100	386.01	386	$3.47\ 10^{-3}$	0.063%
h	3	11.99	12	$6.21\ 10^{-7}$	0.017%
quadratic criteria		221.05			
iterations Number		23			

4. Conclusion

This numerical study consists of identifying the thermo-physical properties of materials using power measurements from a transient electro-thermal method and a 2D thermal model associated with an inversion procedure. A sensitivity analysis has shown that the thermal contact resistance and the volumetric heat capacity are correlated. We have then chosen to fix the thermal contact resistance to a nominal value. We have also optimized the experiment by analyzing a coefficient representing the relative confidence interval of the different parameters. Optimal parameters have been then selected for the identification of the volumetric heat capacity, the thermal conductivity and the heat transfer coefficient. The identification is performed by minimizing the ordinary least-squares referring to the differences between the experimental and the calculated power, using the Levenberg-Marquardt method. The results were satisfactory, presenting a relative uncertainty lower than 0.07%.

Nomenclature

C_p : Sample heat capacity ($J.Kg^{-1}.K^{-1}$)
H : sample thickness (m)
h : heat transfer coefficient ($W.K^{-1}.m^{-2}$)
q : vector of parameters
r : radial coordinates (m)
R : sample radius (m)
R_C : thermal contact resistance point/sample (m^2.K/ W)
r_0 : Heating probe radius (m)
S : surface of the sample (m^2)
t : time (s)
t_{max} : Measurement time (s)
T_{ext} : Ambient temperature (K)
T : calculated temperature (K)
T_P : probe temperature (K)
J : Ordinary least-squares norm
Y_j : measured temperature (K)
X : sensitivity matrix
$\bar{X}$: Reduced sensitivity matrix
z : axial coordinates (m)

Greek letters

λ : Thermal conductivity of the sample ($W.m^{-1}.K^{-1}$)
ρ : Density of the sample ($Kg.m^{-3}$)
μ : Levenberg-Marquardt parameter
ε : Measurement noise (K)
Ω : Diagonal matrix
σ : Standard deviation of measurement noise (K)
Δt : Time step (s)
α : Confidence interval
Φ : Power transferred to the sample in contact mode (W)

Indices

I : Parameter indice
J : measurement number
K : iteration number

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