

# Performance enhancement of MIMO detectors using wavelet de-noising filters

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## Abstract

Multiple-Input Multiple-Output (MIMO) technology has attracted great attention in many wireless communication systems. It provides significant enhancement in the spectral efficiency, throughput, and link reliability. There are numerous MIMO signal detection techniques that have been studied in the previous decades such as Maximum Likelihood (ML), Zero Forcing (ZF), Minimum Mean Square Error (MMSE) detectors, etc. It is well known that the additive and multiplicative noise in the information signal can significantly degrade the performance of MIMO detectors. During the last few years, the noise problem has been the focus of much research, and its solution could lead to profound improvements in symbol error rate performance of the MIMO detectors. In this paper, ML, ZF, and MMSE based wavelet de-noising detectors are proposed. In these techniques, the noise contaminated signals from each receiving antenna element are de-noised individually in parallel to boost the SNR of each branch. The de-noised signals are applied directly to the desired signal detector. The simulation results revealed that the proposed detectors constructed on de-noising basis achieve better symbol error rate (SER) performance than that of systems currently in use.

**Keywords:** Multiple Input- Multiple Output; Maximum Likelihood; Zero Forcing; Minimum Mean Square Error Detector; Wavelet De-noising.

## 1. Introduction

MIMO detection is one of the most studied themes in wireless communication systems. MIMO detection has been a vital area of research because of its significance in applications such as: wireless communications, smart antennas, and radar systems. Great efforts are exerted in the field of performance enhancement of MIMO detectors in terms of symbol error rate (SER) and complexity. Mixing different types of MIMO detectors in one technique is one of the most followed trends nowadays. In [1], a low complexity MIMO detection technique based on a hybrid combination between ZF, ML, and SIC is introduced. This technique has provided much better performance than the individual ZF and SIC. Linear detectors, for the most part taking into account the zero-forcing or minimum mean squared error criteria, are low in complexity yet poor in symbol error rate performance. Ordered successive interference cancellation detectors, which extricate the transmitted symbols one-by one according to the post-detection signal-to-noise ratio (SNR) and perform successive interference elimination, can accomplish better error-rate performance with increased complexity. Nevertheless, there is still a big performance crevice between these suboptimal detectors and the Maximum Likelihood one [2].

It is well known that the additive and multiplicative noise in the information signal can significantly degrade the performance of MIMO detectors. During the last few years, the noise problem has been the focus of much research, and its solution could lead to profound improvements in error rate performance of the MIMO detectors. The challenging goal of the signal de-noising process is to recover the desired signal from its noisy version.

In this paper, ML, ZF, and MMSE based wavelet de-noising detectors are proposed. In these techniques, the noise contaminated

signals from each receiving antenna element are de-noised individually in parallel to boost the SNR of each branch. The de-noised signals are applied directly to the desired signal detector.

## 2. MIMO signal model

Multiple-Input Multiple-Output wireless systems utilize numerous transmit and receive antennas to increase the transmission data rate through spatial multiplexing or to enhance the system reliability as far as bit error rate (BER) performance utilizing space-time codes (STCs) for diversity maximization. MIMO systems abuse multipath propagation to accomplish these advantages, without the cost of extra bandwidth. Consider a  $N_R \times N_T$  MIMO system; the received signal  $y$  can be expressed as;

$$y = Hx + n \quad (1)$$

Which can be represented in matrix form as takes after;

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N_R} \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1N_T} \\ h_{21} & h_{22} & & h_{2N_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_R1} & h_{N_R2} & & h_{N_RN_T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{N_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N_R} \end{bmatrix} \quad (2)$$

Where  $N_T$  represents the number of transmitting antennas,  $N_R$  represents the number of receiving antennas where  $N_R > N_T$ .  $y$  is the  $N_R \times 1$  received signal vector which can be expressed as  $y = [y_1 y_2 \dots \dots \dots y_{N_R}]^T$ .  $H$  denotes the  $N_R \times N_T$  channel matrix whose elements  $h(i, j)$  represent the flat fading complex channel response from the  $j^{\text{th}}$  transmitting antenna to the  $i^{\text{th}}$  receiving antenna.  $x$  Denotes the spatially multiplexed data symbols emitted

from the transmitting antennas such  $x = [x_1 x_2 \dots x_{N_T}]^T$ . And  $n$  is an  $N_R \times 1$  complex additive white Gaussian noise vector with zero mean and variance  $\sigma_n$ .

### 3. MIMO detectors

In this section, the data detection problem for MIMO system and several detection algorithms are presented.

#### 3.1. Maximum likelihood detector

ML detection ascertains the Euclidean separation distance between the received signal vector and the result of all conceivable transmitted signal vectors with the given channel  $H$ , and finds the one with the minimum distance [3]. Let  $C$  and  $N_T$  denote a set of signal constellation symbol points and a number of transmit antennas, respectively. At that point, ML detection determines the estimate of the transmitted signal vector  $x$  as

$$\hat{x} = \arg \min_{x \in X^N} \|y - Hx\|^2 \quad (3)$$

Where  $\|y - Hx\|^2$  compares to the ML metric. The ML technique accomplishes the optimal performance as the most extreme a posteriori (MAP) detection when all the transmitted vectors are similarly likely [4]. However, its complexity increments exponentially as modulation order and/or the number of transmit antennas increase [4]. The required number of ML metric calculation is  $|C|^{N_T}$ , that is, the complexity of metric calculation exponentially increases with the number of antennas.

#### 3.2. Zero forcing detector

Zero-Forcing detector is one of the most commonly used linear detectors as it has low complexity which gives the estimate of  $x$  as [3]:

$$\hat{x}_{zf} = W_{zf} y = x + (H^H H)^{-1} H^H n = x + \hat{z}_{zf} \quad (4)$$

The detector thus forces the interference to zero. The matrix  $W_{zf}$  nullifying the interference is

$$W_{zf} = (H^H H)^{-1} H^H \quad (5)$$

Thus the processed noise is  $\hat{n}_{zf} = W_{zf} n = (H^H H)^{-1} H^H n$ . Here  $H(\cdot)$  denotes the Hermitian transpose operation. ZF detection algorithm is a linear detection algorithm since it carries on a linear filter separating different data streams to perform decoding autonomously on every stream, therefore eliminating the multi-stream interference [3]. The drawback of ZF detection is retarded BER performance due to noise enhancement [3]. The additive white Gaussian noise (AWGN)  $n$  loses its whiteness property. It is enhanced and correlated across the data streams. As the SNR increases, ZF solution  $\hat{x}$  turns out to will probably match with the ML solution vector.

#### 3.3. Minimum mean square error detector

MMSE detector estimates the transmitted vector  $x$  by applying the linear transformation to the received vector  $y$ . It finds out the estimate  $\hat{x}_{MMSE}$  of the transmitted symbol vector  $x$  as [3]:

$$\begin{aligned} \hat{x}_{MMSE} &= W_{MMSE} y \\ &= (H^H H + \sigma_n^2 I)^{-1} H^H y \\ &= \hat{x} + (H^H H + \sigma_n^2 I)^{-1} H^H n \\ &= \hat{x} + \hat{n}_{MMSE} \end{aligned} \quad (6)$$

The MMSE weighting matrix  $W_{MMSE}$  is used to maximize the post-detection signal-to-interference plus noise ratio (SINR) [3]. Furthermore, MMSE receiver requires the statistical information of noise  $\sigma_n^2$ . MMSE detectors balance the noise enhancement and multi-stream interference by minimizing the total error. Its BER performance is superior to ZF detection due to mitigating the noise enhancement.

### 4. Wavelet de-noising of MIMO signal

Removing noise from the signal is the key idea that can be achieved via Wavelet de-noising. Wavelet is a wave-like variation with the ability of representing a signal in the time-frequency plane. This variation or oscillation has amplitude that varies starting from a zero level, and then it increases or decreases incrementally, and finally back gradually to zero. Wavelet possess specific properties which fit them to digital signal processing. Moreover, they are considered as a mathematical tool for analyzing time-variant signals or transient phenomena. Wavelet based de-noising filters may be implemented via various methods. Discrete Wavelet Transform (DWT) is one common method. The wavelet theory is based on representing a general function using an infinite series expansion in terms of a basic mother wavelet function  $\psi$  [5]. The technique which is examined in this paper is Wavelet de-noising by Thresholding.

Let  $y_i$  be the received signal at each antenna element at the receiving end that is contaminated by additive white Gaussian noise  $n_i$  with variance  $\sigma_n^2$  as expressed in Eq.2. Hence  $y_i$  can be represented as:

$$y_i = \sum_{j=1}^{N_T} h_{ij} x_j + \sigma_n n_i, \quad (i = 1, 2, \dots, N_R) \quad (7)$$

The main target is to find a function  $f$  from the noisy signal  $y_i$  that satisfies:

$$\hat{f}_i = \min_{\hat{f}_i} \|\hat{f}_i - f_i\|_2 \quad (8)$$

Where  $f_i = \sum_{j=1}^{N_T} h_{ij} x_j$  which in practice is unknown function.  $\hat{f}_i$  is the estimation of  $y_i$  such that  $\hat{f}_i = \hat{f}_i(y_i)$ . If  $C_{km}$  are the wavelet coefficients of  $y_i$ , where  $k$  denotes the decomposition level and  $m$  is the index of the coefficient in this level. Then the transformation of Eq. (7) in the wavelet domain can be expressed as in [6, 7]:

$$C_{km} = w_{km} + \sigma_n \tilde{u}_{km} \quad (9)$$

Where  $C_{km} = W y_i^T$  are considered the wavelet coefficients of  $y_i$ ,  $w_{km} = W f_i^T$  which represent the uncontaminated wavelet coefficients of the function  $f_i$ ,  $\tilde{u}_{km} = W n_i^T$ ,  $W$  denotes a  $K \times K$  Discrete Wavelet Transform (DWT) matrix. The wavelet coefficients can be divided to approximation and detail coefficients. Some of these coefficients belonged to a distorted version of the matrix. So, to recover the function ( $\hat{f}_i$ ) from the noisy signal  $y_i$ , firstly obtaining its clear coefficients. These desired coefficients can be obtained by deleting the coefficients that have small magnitude as they represent pure noise. This process is called Wavelet Thresholding.

Wavelet thresholding method is applied just to the detail coefficients  $d_{km}$  of  $C_{km}$ , and it isn't necessary to be applied on the approximation coefficients  $c_{km}$ , since the  $c_{km}$  represent 'low-frequency' terms that usually include important information of the data. Also, the approximation coefficients aren't sensitive to noise. The thresholding concept can be described as; the process of zeroing all the coefficients whose magnitude values are less than a certain threshold  $\lambda$ , and keeping or modifying the other coefficients [6]. The wavelet thresholding operation can be represented as a diagonal filtering operation in the wavelet domain.

Next, the thresholded wavelet coefficients values will be obtained in two methods. The first one is the Hard Thresholding, and its equation is as described as in [6]:

$$h(d_{km}) = \begin{cases} 0, & \text{if } |d_{km}| \leq \lambda \\ d_{km}, & \text{if } |d_{km}| > \lambda \end{cases} \quad (10)$$

While, the other one is the Soft Thresholding, and its equation is as described:

$$h(d_{km}) = \begin{cases} 0, & \text{if } |d_{km}| \leq \lambda \\ d_{km} - \lambda, & \text{if } d_{km} > \lambda \\ d_{km} + \lambda, & \text{if } d_{km} < -\lambda \end{cases} \quad (11)$$

Hard thresholding nulls out all the coefficients values smaller than  $\lambda$ . If the magnitude of a coefficient is only somewhat less than  $\lambda$ , then this value is set to zero, while a coefficient whose magnitude is only slightly greater than  $\lambda$  is kept unchanged. So, hard thresholding creates discontinuities and it is not suitable for removing the noise. Soft thresholding or the kill follows the same manner of hard threshold, but, subtracts  $\lambda$  from the values larger than  $\lambda$ . Unlike hard thresholding, soft thresholding causes continuities in the resulting signal [6].

According to Donoho, David L's method [6, 8, and 9], the threshold estimates  $\lambda$  for denoising the signal is given by:

$$\lambda = \sigma_n \sqrt{2 \log(K)} \quad (12)$$

This threshold rule called universal threshold (VisuShrink), where  $K$  is the number of samples. Also, the computed thresholds require knowledge of the noise variance  $\sigma_n$  which can be calculated as shown below [6] [10]:

$$\sigma_n = \frac{\text{median}(\{|d_{k-1,m}|\}; m=0,1,\dots,2^{K-1}-1)}{0.6745} \quad (13)$$

Where the factor in the denominator is the scale factor which depends on the distribution of  $d_{km}$ , and is equal to 0.6745 for normally distributed data. Finally, estimate the desired signal using inverse discrete wavelet transform (IDWT).

### 5. Proposed MIMO detection techniques

MIMO technology has pulled in extraordinary consideration in numerous wireless communication systems. It provides significant enhancement in the spectral efficiency, throughput, and link reliability. There are numerous MIMO signal detection techniques that have been studied in the previous decades such as Maximum Likelihood (ML), Zero Forcing (ZF), Minimum Mean Square Error (MMSE) detector, etc. It is well known that the additive and multiplicative noise in the information signal can significantly degrades the performance of MIMO detectors. During the last few years the noise problem has been the focus of much research, and its solution could lead to profound improvements in error rate performance of the MIMO detectors. In this paper, ML, ZF, and MMSE based wavelet de-noising detectors are proposed. In these techniques, the noise contaminated signals from each receiving antenna element are de-noised individually in parallel to boost the SNR of each branch. In this work, the Wavelet de-noising filter is utilized to remove the noise from the received signals. The de-noised signals are applied directly to the desired signal detector as shown in figure 1.

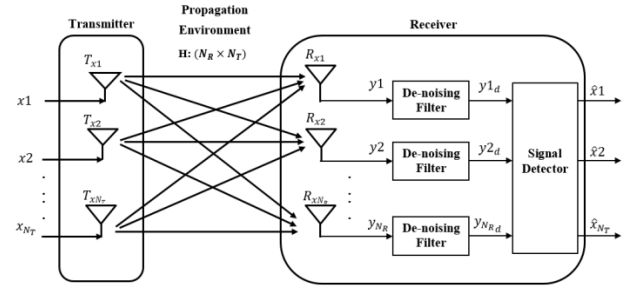


Fig. 1: The Block Diagram of the Proposed MIMO Detector.

### 6. Simulation results

In the previous sections, several MIMO detection algorithms are introduced. All of these algorithms are analyzed with realistic channel knowledge at both transmitting and receiving ends. Consider an  $N_T \times N_R$  MIMO channel model where  $N_T = 4$  and  $N_R = 12$  are the number of transmitting and receiving antennas respectively. A 4-QAM modulated signal is utilized. The symbol error rate (SER) performance of the proposed detectors are measured over SNR range from  $-12$  dB to  $0$  dB. SER performance comparison of the classical ML, ZF, and MMSE detectors and the proposed detectors are introduced. Figure (2), figure (3), and figure (4), show the SER performances of the proposed ML detector, ZF detector, and MMSE detector based on wavelet de-noising versus the classical ML detector, ZF detector, and MMSE detector for a  $4 \times 12$  MIMO system with 4-QAM modulation. It is clear from the comparisons that the proposed techniques provide better performance than the classical detectors.

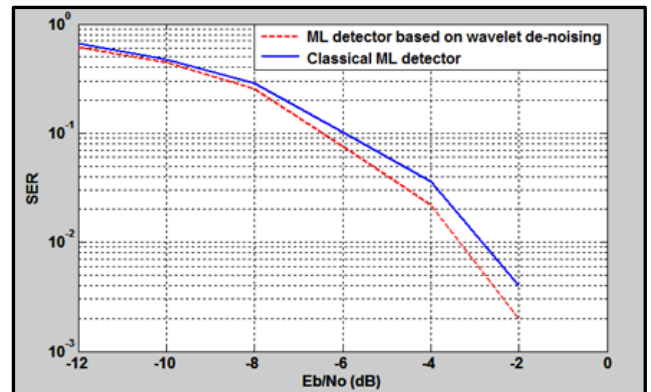


Fig. 2: Comparison between the Proposed ML Detector Based on Wavelet De-Noising and the Classical ML Detector.

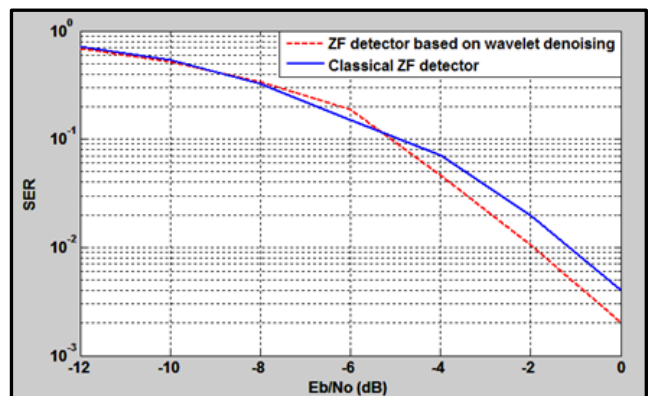


Fig. 3: Comparison between the Proposed ZF Detector Based on Wavelet De-Noising and the Classical ZF Detector.

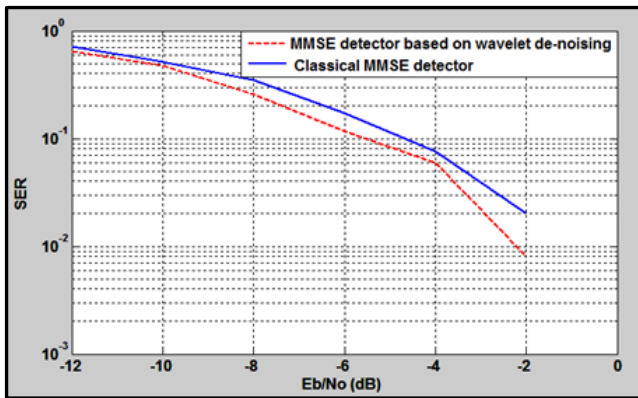


Fig. 4: Comparison between the Proposed MMSE Detector Based on Wavelet De-Noising and the Classical MMSE Detector.

## 7. Conclusion

During the last few years, the noise problem has been the focus of much research, and its solution could lead to profound improvements in error rate performance of the MIMO detectors. In this paper, ML, ZF, and MMSE based wavelet de-noising detectors are proposed. In these techniques, the noise contaminated signals from each receiving antenna element are de-noised individually in parallel to boost the SNR of each branch. The de-noised signals are applied directly to the desired signal detector. The proposed algorithms are analyzed with realistic channel knowledge at both transmitting and receiving ends. Consider an  $N_T \times N_R$  MIMO channel model where  $N_T = 4$  and  $N_R = 12$  are the number of transmitting and receiving antennas respectively. A 4-QAM modulated signal is utilized. The simulation results revealed that the proposed detectors outstand the classical detectors performance.

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