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Research paper

Lattice ordered interval-valued hesitant fuzzy soft sets in decision making problem

Ar. Pandipriya[1], J. Vimala[2*] and S. Sabeena Begam[3]

[1,2,3] *Department of Mathematics, Alagappa University, Karaikudi, Tamilnadu India*
[*] *Corresponding author E-mail: vimaljey@alagappauniversity.ac.in*

Abstract

In 2015, Haidong Zhang enhanced the idea of Hesitant Fuzzy Sets and Soft Sets into Interval-Valued Hesitant Fuzzy Soft Sets along with its properties. In this present work, we establish the Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Sets and examined its vital properties with examples. Eventually the application of Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Set to decision making problem is established by means of elegant algorithm.

Keywords: *Soft sets; Fuzzy sets; Hesitant Fuzzy Soft Sets; Interval-Valued Hesitant Fuzzy Soft Sets; Lattice Ordered Interval-Valued Hesitant Fuzyy Soft sets*

1. Introduction

L.A.Zadeh [30] proffered the most suitable mathematical kit to deal with lack of certainty as Fuzzy Sets. The conception of Soft Set theroy has several applications in various domains. The result of Soft Set was instigated by Molodtsov [15, 16] for dealing with intricated problems of various uncertainties and P.K.Maji [13] analysed the notion of decision making concepts in Soft Sets. The Lattice Ordered Soft Group is introduced by L.Vijayalashmi and J.Vimala in 2017 [22]. In 2010, Pinaki [17] prolonged the generalization of Fuzzy Soft Sets. In 2016, J.Vimala, J.Arockia Reeta [21] initiated the Lattice Ordered Fuzzy Soft Group. Also Distributive and Modular Lattice Ordered Fuzzy Soft Group and its duality [2] is established in 2017 and multiset theory is applied to the concept of lattice in [1]. In 2009, Torra and Narukawa [19, 20] offered Hesitant Fuzzy Set. In 2013, Hesitant Fuzzy Soft Set was analysed by Babitha and John [3] and some of its basic properties were studied. Also, Zheng [31], Wang [24] and Jiang-qiang [11] explored the idea of decision making over Hesitant Fuzzy Soft Sets. In 2009, Xibei [25] studied the Interval-Valued Fuzzy Set by combined with Soft Set and Feng Feng [8] propounded some more operations and applications of Interval-Valued Fuzzy Soft Sets in 2010. In 2010, Yuncheng [29] explored Interval-Valued Intuitionistic Fuzzy Soft Sets along with its axioms. In 2010, Bivas Dinda [5] redefined operations of Interval-Valued Intuitionistic Fuzzy Soft Sets. In 2011, Shawkat [18] commenced Interval-Valued Fuzzy Soft Sets with fuzzy parameters and aggregation operators technique. In 2015 [9], Haidong Zhang offered the notion of Interval-Valued Hesitant Fuzzy Soft Sets and technique of decision making problems are conferred by Xindong peng [26]. Also, Manash Jyoti [14] commenced some more operators on it.

In this present work, discussion on Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Sets has been carried out along with its pertinent operations such as union and intersection.

This work has been compiled as follows. In section 2, some preliminary definitions of Interval-Valued Hesitant Fuzzy Soft sets are given. In section 3, Lattice Ordered Interval-Valued Hesitant Fuzzy Soft sets has been established and some of its operations are studied. In section 4, An efficient algorithm is constructed to solve decision making problems through Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Sets.

Throughout this paper, we use $\mathscr{L}-\mathscr{IVHFSS}$ in place of Lattice Ordered Interval-Valued Hesitant Fuzzy Soft sets.

2. Preliminaries

In this segment, we analyse vital definitions of Soft Sets, Hesitant Fuzzy Soft Sets, Interval-Valued Hesitant Fuzzy Sets and Interval-Valued Hesitant Fuzzy Soft Sets. Let U and E represents universal set and set of parameters respectively in this part.

Definition 2.1. *[15] A Soft Set on* U *is defined by a pair* (F,E)*, where* F *is mapping as* $\mathrm{F}\colon \mathrm{E}\to \mathrm{P}(\mathrm{U})$

Example 2.2. *Let* $\mathrm{U}=\{h_1,h_2,h_3\}$ *be a set of houses and* $\mathrm{E}=\{e_1,e_2,e_3\}$ *be a set of parameters represents beautiful, cheap and large in size respectively. Then a soft set* $(\mathrm{F},\mathrm{E})=\{\mathrm{F}(e_1)=\{h_1,h_2\},\mathrm{F}(e_2)=\{h_2,h_3\},\mathrm{F}(e_3)=\{h_1,h_3\}\}$ *represents the category of houses satisfies the conditions of parameters.*

Definition 2.3. *[13] Let* $F(\mathrm{U})$ *represents the set of all fuzzy sets on* U*. Then a Fuzzy Soft Set over* U *is a pair* (F,E)*, where* $\mathrm{F}\colon \mathrm{E}\to F(\mathrm{U})$ *is a mapping.*

Example 2.4. *Let* $\mathrm{U}=\{h_1,h_2,h_3\}$ *be a set of houses and* $\mathrm{E}=\{e_1,e_2,e_3\}$ *be a set of parameters represents beautiful, cheap and large in size respectively. Then a fuzzy soft set* $(\mathrm{F},\mathrm{E})=\{\mathrm{F}(e_1)=\{<h_1,0.8>,$

$< h_2, 0.7 >, < h_3, 0.2 >\}$,
$\mathrm{F}(e_2) = \{< h_1, 0.3 >, < h_2, 0.7 >$,
$< h_3, 0.9 >\}, \mathrm{F}(e_3) = \{< h_1, 0.6 >$,
$< h_2, 0.2 >, < h_3, 0.7 >\}\}$ *represents how much the houses satisfies the given conditions of parameters.*

Definition 2.5. *[19, 20] A Hesitant Fuzzy Set* F *over* U *is declared interms of a function with its range is a subset of* $[0,1]$ *whenever applied to* U. *i.e.,* $\mathrm{F} = \{< x, h_{\mathrm{F}}(x) > | x \in \mathrm{U}\}$, *where* $h_{\mathrm{F}}(x) \subseteq [0,1]$ *which gives membership degrees of the element* $x \in \mathrm{U}$ *to* F *and is called as hesitant fuzzy element. Also* H *denotes the set of all hesitant fuzzy elements.*

Example 2.6. *Let* $\mathrm{U} = \{x_1, x_2, x_3\}$ *be a universal set represents three participants in a dance competition and* $h_{\mathrm{F}}(x_1) = \{0.2, 0.4, 0.5\}$,
$h_{\mathrm{F}}(x_2) = \{0.3, 0.4, 0.5\}, h_{\mathrm{F}}(x_3) = \{0.3, 0.2, 0.5\}$ *be the hesitant fuzzy elements of* $x_i\,(i = 1,2,3)$ *to* F *represents marks of three judges of each participant respectively. Then a Hesitant Fuzzy Set*
$\mathrm{F} = \{< x_1, \{0.2, 0.4, 0.5\} >$,
$< x_2, \{0.3, 0.4, 0.5\} >$,
$< x_3, \{0.3, 0.2, 0.5\} >\}$

Definition 2.7. *K.V.B Let* $\mathrm{H}(\mathrm{U})$ *denotes the collection of all Hesitant Fuzzy Sets on* U. *A pair* (F, E) *is called a Hesitant Fuzzy Soft Set over* U, *where* $\mathrm{F} \colon \mathrm{E} \to \mathrm{H}(\mathrm{U})$ *is a mapping.*

Example 2.8. *Let* $\mathrm{U} = \{x_1, x_2, x_3\}$ *be a universal set represents three participants in a dance competition,* $\mathrm{E} = \{e_1, e_2\}$ *be a set of parameters represents choreography and perfection in dance respectively. Then a Hesitant Fuzzy Soft Set represents marks of each participant based on the parameters given by the two judges is as follows:*
$(\mathrm{F}, \mathrm{E}) = \{\mathrm{F}(e_1) = \{< x_1, \{0.8, 0.7\} >$,
$< x_2, \{0.7, 0.9\} >, < x_3, \{0.4, 0.2\} >\}, \mathrm{F}(e_2) = \{< x_1, \{0.3, 0.1\} >$,
$< x_2, \{0.7, 0.8\} >, < x_3, \{0.9, 0.6\} >\}\}$

Definition 2.9. *Chen Let* $D([0,1])$ *denotes the set of all closed intervals in* $[0,1]$. *An Interval-Valued Hesitant Fuzzy Set* F *over* U *is defined as* $\mathrm{F} = \{< x, h_{\mathrm{F}}(x) > | x \in \mathrm{U}\}$, *where* $h_{\mathrm{F}} \colon \mathrm{U} \to D([0,1])$ *denotes all possible interval-valued membership degrees of the element* $x \in \mathrm{U}$ *to the set* F. *Also*
$h_{\mathrm{F}}(x) = \{\nu | \nu = [\nu^L, \nu^U]$ *is an interval*$\}$ *is called as Interval-Valued Hesitant Fuzzy Element (IVHFE) and is denoted as* h. *The set of all Interval-Valued Hesitant Fuuzy Sets on* U *is denoted by* $IVHF(\mathrm{U})$.

Example 2.10. *Let* $\mathrm{U} = \{x_1, x_2\}$ *be a universal set represents two cities in India and* $h_{\mathrm{F}}(x_1) = \{[0.2, 0.3], [0.4, 0.5]\}, h_{\mathrm{F}}(x_2) = \{[0.5, 0.7], [0.4, 0.5]\}$ *be the IVHFEs of* $x_i, \{i = 1,2\}$ *to the set* F *represents the amount of rain fall in two consecutive days respectively. Then an Interval-Valued Hesitant Fuzzy Set represents the amount of rain fall in the given cities for two consecutive days is as follows:*
$\mathrm{F} = \{< x_1, \{[0.2, 0.3], [0.4, 0.5]\} >$,
$< x_2, \{[0.5, 0.7], [0.4, 0.5]\} >\}$

Definition 2.11. *The score function of an IVHFE* h *is defined as* $s(h) = \frac{\sum_{\gamma \in h} \gamma}{n(h)}$, *where* $n(h)$ *denotes the number of intervals in h.*

Definition 2.12. *HaZhg Let* $IVHF(\mathrm{U})$ *denotes the collection of all Interval-Valued Hesitant Fuzzy Sets on* U. *An Interval-Valued Hesitant Fuzzy Soft Set is defined as a pair* (F, E), *where* F *is such that* $\mathrm{F} \colon \mathrm{E} \to IVHF(\mathrm{U})$ *a mapping.*

Example 2.13. *Let* $\mathrm{U} = \{x_1, x_2\}$ *represents the participants of the dance competition and* $\mathrm{E} = \{e_1, e_2, e_3\}$ *represents perfection, choreography and creativity in their dance respectively. Then an Interval-Valued Hesitant Fuzzy Soft Set represents the performance level of each participant based on the given parameters by two judges is as follows:*
$(\mathrm{F}, \mathrm{P}) = \{\mathrm{F}(e_1) =$
$\{< x_1, \{[0.3, 0.5], [0.4, 0.5]\} >$,
$< x_2, \{[0.5, 0.7], [0.3, 0.4]\} >\}$,
$\mathrm{F}(e_2) = \{< x_1, \{[0.5, 0.6], [0.4, 0.5]\} >$,
$< x_2, \{[0.4, 0.6], [0.5, 0.6]\} >\}$,
$\mathrm{F}(e_3) = \{< x_1, \{[0.4, 0.5], [0.4, 0.5]\} >, \{x_2, \{[0.6, 0.7], [0.5, 0.7]\} >\}\}$

Definition 2.14. *HaZhg Let* A *and* B *be subsets of* E. *Then* (F, A) *is said to be an Interval-Valued Hesitant Fuzzy Soft subset of* (G, B) *if*

(i) $\mathrm{A} \subseteq \mathrm{B}$
(ii) $\gamma_{1_{\sigma(i)}} \leq \gamma_{2_{\sigma(i)}}$, *where* $\gamma_{1_{\sigma(i)}}, \gamma_{2_{\sigma(i)}}$ *stands for the largest interval in the* $h_{\mathrm{F}(e_i)}(x)$ *and* $h_{\mathrm{G}(e_i)}(x)$ *respectively.*

Definition 2.15. *HaZhg Let* (F, E) *be an Interval-Valued Hesitant Fuzzy Soft Set. Then its complement is defined by* $(\mathrm{F}^c, \mathrm{E})$, *where* $\mathrm{F}^c \colon \mathrm{E} \to IVHF(\mathrm{U})$ *such that* $\mathrm{F}^c(e)$ *is the complement of Interval-Valued Hesitant Fuzzy Set* $\mathrm{F}(e)$ *on* U.

Definition 2.16. *HaZhg Let* A *and* B *be subsets of* E. *The union of two Interval-Valued Hesitant Fuzzy Soft Sets* (F, A) *and* (G, B) *is defined as* $(\mathrm{F}, \mathrm{A}) \cup (\mathrm{G}, \mathrm{B}) = (\mathrm{H}, \mathrm{C})$, *where* $\mathrm{C} = \mathrm{A} \cup \mathrm{B}$ *and*

$$h(\mathrm{H}(e)) = \begin{cases} h(F(e)) & \text{if } e \in A - B \\ h(G(e)) & \text{if } e \in B - A \\ h(F(e)) \cup h(G(e)) & \text{if } e \in A \cap B \end{cases}$$

Definition 2.17. *HaZhg Let* A *and* B *be subsets of* E. *The intersection of two Interval-Valued Hesitant Fuzzy Soft Sets* (F, A) *and* (G, B) *is defined as* $(\mathrm{F}, \mathrm{A}) \cap (\mathrm{G}, \mathrm{B}) = (\mathrm{H}, \mathrm{C})$, *where* $\mathrm{C} = \mathrm{A} \cap \mathrm{B}$ *and* $h(\mathrm{H}(e)) = h(\mathrm{F}(e)) \cap h(\mathrm{G}(e)), \forall e \in \mathrm{C}$.

3. Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Sets

In this segment, the conception of Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Sets has been established. Also some of its basic properties and theorems are discussed. We use U and $\mathscr{P}$ to denote universal set and parameter set respectively in this segment.

Definition 3.1. *Let* $(\mathscr{F}, \mathscr{P})$ *be an interval-valued hesitant fuzzy soft set. Then* $\mathscr{F}(e_i) \subseteq \mathscr{F}(e_j)$ *if* $s(h_{\mathscr{F}}(e_i)) \leq s(h_{\mathscr{F}}(e_j)), \forall e_i, e_j \in \mathscr{P}$, *where h is IVHFE and s is a score function.*

Definition 3.2. *Let* $(\mathscr{F}, \mathscr{P})$ *be an Interval-Valued Hesitant Fuzzy Soft Set. We say that* $(\mathscr{F}, \mathscr{P})$ *a Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Set* $(\mathscr{L} - \mathscr{IVHFSS})$ *if* $\mathscr{F}(e_i) \subseteq \mathscr{F}(e_j)$ *whenever* $e_i \leq e_j$,
$\forall e_i, e_j \in \mathscr{P}$.

Example 3.3. *Let* $U = \{x_1, x_2\}$ *represents bike, car respectively and* $\mathscr{P} = \{e_1, e_2, e_3\}$ *represents the years 2015, 2016 and 2017 respectively. Then* $(\mathscr{F}, \mathscr{P})$ *is a* $\mathscr{L} - \mathscr{IVHFSS}$ *which represents the sales of bike and car of two particular showrooms in the relevant years as follows,*
$(\mathscr{F}, \mathscr{P}) = \{\mathscr{F}(e_1) =$
$\{< x_1, \{[0.1, 0.3], [0.2, 0.4]\} >$,
$< x_2, \{[0.2, 0.4], [0.2, 0.3]\} >\}$,
$\mathscr{F}(e_2) = \{< x_1, \{[0.3, 0.4], [0.3, 0.5]\} >$,
$< x_2, \{[0.1, 0.4], [0.4, 0.7]\} >\}$,
$\mathscr{F}(e_3) = \{< x_1, \{[0.4, 0.5], [0.45, 0.6]\} >$,
$< x_2, \{[0.4, 0.6], [0.4, 0.6]\} >\}\}$.

Theorem 3.4. *If* $(\mathscr{F}, \mathscr{P})$ *is* $\mathscr{L} - \mathscr{IVHFSS}$, *then*

(i) $\mathscr{F}(e) \vee \mathscr{F}(e) = \mathscr{F}(e), \mathscr{F}(e) \wedge \mathscr{F}(e) = \mathscr{F}(e)$ *(Idempotency)*
(ii) $\mathscr{F}(e_i) \vee \mathscr{F}(e_j) = \mathscr{F}(e_j) \vee \mathscr{F}(e_i), \mathscr{F}(e_i) \wedge \mathscr{F}(e_j) = \mathscr{F}(e_j) \wedge \mathscr{F}(e_i)$
(Commutativity)

(iii) $\mathscr{F}(e_i) \vee (\mathscr{F}(e_j) \vee \mathscr{F}(e_k)) = (\mathscr{F}(e_i) \vee \mathscr{F}(e_j)) \vee \mathscr{F}(e_k)$, $\mathscr{F}(e_i) \wedge (\mathscr{F}(e_j) \wedge \mathscr{F}(e_k)) = (\mathscr{F}(e_i) \wedge \mathscr{F}(e_j)) \vee \mathscr{F}(e_k)$ *(Associativity)*

(iv) $\mathscr{F}(e_i) \vee (\mathscr{F}(e_i) \wedge \mathscr{F}(e_j)) = \mathscr{F}(e_i), \mathscr{F}(e_i) \wedge (\mathscr{F}(e_i) \vee \mathscr{F}(e_j)) = \mathscr{F}(e_i)$ *(Absorption)*

Proof

(i) Idempotency:

Clearly $\mathscr{F}(e) \vee \mathscr{F}(e) = \mathscr{F}(e)$ *and* $\mathscr{F}(e) \wedge \mathscr{F}(e) = \mathscr{F}(e)$.

(ii)Commutativity:

$\mathscr{F}(e_i) \vee \mathscr{F}(e_j) = \max\{\mathscr{F}(e_i), \mathscr{F}(e_j)\} = \max\{\mathscr{F}(e_j), \mathscr{F}(e_i)\} = \mathscr{F}(e_j) \vee \mathscr{F}(e_i)$ *and*

$\mathscr{F}(e_i) \wedge \mathscr{F}(e_j) = \min\{\mathscr{F}(e_i), \mathscr{F}(e_j)\} = \min\{\mathscr{F}(e_j), \mathscr{F}(e_i)\} = \mathscr{F}(e_j) \wedge \mathscr{F}(e_i)$.

(iii)Associativity:

Since $\mathscr{F}(e_i) \subseteq \mathscr{F}(e_j) \subseteq \mathscr{F}(e_k)$.

$\Rightarrow \mathscr{F}(e_j) \vee \mathscr{F}(e_k) = \mathscr{F}(e_k)$ *and* $\mathscr{F}(e_i) \vee \mathscr{F}(e_k) = \mathscr{F}(e_k)$

$\Rightarrow \mathscr{F}(e_i) \vee (\mathscr{F}(e_j) \vee \mathscr{F}(e_k)) = \mathscr{F}(e_k)$

Now, $\mathscr{F}(e_i) \vee \mathscr{F}(e_j) = \mathscr{F}(e_j)$

$\Rightarrow (\mathscr{F}(e_i) \vee \mathscr{F}(e_j)) \vee \mathscr{F}(e_k) = \mathscr{F}(e_j) \vee \mathscr{F}(e_k) = \mathscr{F}(e_k)$.

Hence we have $\mathscr{F}(e_i) \vee (\mathscr{F}(e_j) \vee \mathscr{F}(e_k)) = (\mathscr{F}(e_i) \vee \mathscr{F}(e_j)) \vee \mathscr{F}(e_k)$ *and*

similarly $\mathscr{F}(e_i) \wedge (\mathscr{F}(e_j) \wedge \mathscr{F}(e_k)) = (\mathscr{F}(e_i) \wedge \mathscr{F}(e_j)) \vee \mathscr{F}(e_k)$

(iv)Absorption:

Suppose $\mathscr{F}(e_i) \wedge \mathscr{F}(e_j) = \mathscr{F}(e_i)$

Then $\mathscr{F}(e_i) \vee (\mathscr{F}(e_i) \wedge \mathscr{F}(e_j)) = \mathscr{F}(e_i) \vee \mathscr{F}(e_i) = \mathscr{F}(e_i)$.

Also $\mathscr{F}(e_i) \vee \mathscr{F}(e_j) = \mathscr{F}(e_j)$

$\Rightarrow \mathscr{F}(e_i) \wedge (\mathscr{F}(e_i) \vee \mathscr{F}(e_j)) = \mathscr{F}(e_i) \wedge \mathscr{F}(e_j) = \mathscr{F}(e_i)$.

4. Decision making problems through $\mathscr{L}-\mathscr{IVHFSS}$

Three players are going to play a series of games each of which has 2 levels in it. Now we construct a $\mathscr{L}-\mathscr{IVHFSS}$ for this problem as follows: The alternatives x_1, x_2 and x_3 represents the three players and the parameters e_1, e_2 and e_3 denotes the three games repectively as e_1 is the easiest among all, e_2 is little difficult and e_3 is more difficult to play. Thus we compare e_1, e_2, e_3 by their difficulties as $e_1 < e_2 < e_3$. A membership function assigns a value $(interval)$ according to the time spent by the players and their performances separately for 2 levels. If time consumption is less and performance is good, then we assign a minimum interval value, i.e.,$[0.1, 0.2]$. Thus we can change the interval value depends on the time consuming and their performance in the respective time period. Thus we have $\mathscr{L}-\mathscr{IVHFSS}$ for this problem as

$(\mathscr{F}, \mathscr{P}) = \{\mathscr{F}(e_1)$

$= \{< x_1, [0.1, 0.2], [0.1, 0.3] >,$

$< x_2, [0.15, 0.2], [0.2, 0.4] >,$

$< x_3, [0.3, 0.5], [0.3, 0.5] >\},$

$\mathscr{F}(e_2) = \{< x_1, [0.1, 0.3], [0.15, 0.35] >,$

$< x_2, [0.23, 0.3], [0.3, 0.4] >,$

$< x_3, [0.4, 0.55], [0.4, 0.55] >\},$

$\mathscr{F}(e_3) = \{< x_1, [0.3, 0.5], [0.3, 0.6] >, < x_2, [0.31, 0.45], [0.4, 0.45] >,$

$< x_3, [0.45, 0.6], [0.45, 0.6] >\}\}$

Our aim is to find out which player played all the three games well in short period of time.We use the following algorithm to evaluate the optimal decision making by $\mathscr{L}-\mathscr{IVHFSS}$.

Algorithm

Step 1.

Write a $\mathscr{L}-\mathscr{IVHFSS}$ and find score value for each x_i with respect to each e_j, where $i = 1, 2...n$ and $j = 1, 2...m$

$(\mathscr{F}, \mathscr{P}) = \{\mathscr{F}(e_1) =$

$\{< x_1, [0.1, 0.2], [0.1, 0.3] >,$

$< x_2, [0.15, 0.2], [0.2, 0.4] >,$

$< x_3, [0.3, 0.5], [0.3, 0.5] >\},$

$\mathscr{F}(e_2) = \{< x_1, [0.1, 0.3], [0.15, 0.35] >,$

$< x_2, [0.23, 0.3], [0.3, 0.4] >,$

$< x_3, [0.4, 0.55], [0.4, 0.55] >\},$

$\mathscr{F}(e_3) = \{< x_1, [0.3, 0.5], [0.3, 0.6] >,$

$< x_2, [0.31, 0.45], [0.4, 0.45] >,$

$< x_3, [0.45, 0.6], [0.45, 0.6] >\}\}$

$s(h_1(x_1)) = \frac{1}{2}[0.2, 0.5] = [0.1, 0.25]$

$s(h_1(x_2)) = \frac{1}{2}[0.35, 0.6] = [0.175, 0.3]$

$s(h_1(x_3)) = \frac{1}{2}[0.6, 1] = [0.3, 0.5]$

$s(h_2(x_1)) = \frac{1}{2}[0.25, 0.65] = [0.125, 0.325]$

$s(h_2(x_2)) = \frac{1}{2}[0.53, 0.7] = [0.265, 0.35]$

$s(h_2(x_3)) = \frac{1}{2}[0.8, 1.1] = [0.4, 0.55]$

$s(h_3(x_1)) = \frac{1}{2}[0.6, 1.1] = [0.3, 0.55]$

$s(h_3(x_2)) = \frac{1}{2}[0.71, 0.9] = [0.355, 0.45]$

$s(h_3(x_3)) = \frac{1}{2}[0.9, 1.2] = [0.45, 0.6]$

Step 2.

Find the mid value for each score function value and write it as m_{ij}.

$$m_{ij} = \begin{bmatrix} 0.175 & 0.225 & 0.425 \\ 0.2375 & 0.3075 & 0.4025 \\ 0.4 & 0.475 & 0.525 \end{bmatrix}$$

Step 3.

Assign weight w_{ij} *for each* x_i *with respect to each* e_j *by* $+1$ *if corresponding midvalue* < 0.5 *and* 0 *if* ≥ 0.5.

$$w_{ij} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Step 4.

Compute $P_i = \sum_{j=1}^{n} (m_{ij}.w_{ij})$

$P_1 = (0.175)(1) + (0.225)(1) + (0.425)(1)$=0.825

$P_2 = (0.2375)(1) + (0.3075)(1) + (0.4025)(1)$=0.9475

$P_3 = (0.4)(1) + (0.475)(1) + (0.525)(0)$=0.875

Step 5.

Choose the maximum of P_i*s as the optimal value.*

$P_2 = 0.9475$ is the maximum value among all P_is.

Thus we have to select the optimal decision making as the second player x_2 who performed well in all three games in a short peroid of time.

5. Conclusion

In this manuscript, we proposed the conception of Lattice Ordered Interval-Valued Hesitant Fuzzy Soft sets. The basic operations such as union and intersection are discussed. Also an algorithm for decision making problems via Lattice Ordered Interval-Valued Hesitant Fuzzy Soft sets is executed with crisp example. We hope that our future work can be used to extend the algebraic properties of Lattice Ordered Interval-Valued Hesitant Fuzzy Soft Sets.

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